Polynomials:
Objective – Evaluate, add, subtract, multiply, and divide polynomials
Definition:
A Term is numbers or a product of numbers and/or variables. For example, 5x, 2y², -8, ab⁴c², etc. are all terms.
Polynomial expressions are often named based on the number of terms in the expression, which are separated by an operation sign. For example, a monomial has one term, such as 4y³. A binomial has two terms, such as a² – b². A trinomial has three terms, such as ax² + bx + c.
The term “Polynomial” means many terms. Monomials, binomials, and trinomials all fall under the category of a polynomial.

Let’s review negative variables and exponents.
-4² = -16 because the exponent is attached to the number 4
(-5)² = 25 because the term is surrounded by parenthesis, the term is subject to the exponent.
-3³ = -27
-(-2)³ = -(-8) = 8
Substitution method:
n² + 3n – 14: where n = -5
The entire term (-5) is being substituted for “n” so make sure you put parentheses around the term so you are simplifying it correctly.
(-5)² + 3(-5) – 14: Simplify
25 – 15 – 14 = -4

-5y⁴ – 10y³ – 7y² – y – 5: where y = -3
-5(-3)⁴ – 10(-3)³ – 7(-3)² –(-3) – 5: Simplify
-5(81) – 10(-27) – 7(9) –(-3) – 5: Evaluate
-405 + 270 – 63 + 3 – 5 = -200

-2x³ + 4x² – x + 11: where x = 2
-2(2)³ + 4(2)² – 2 + 11: Simplify
-2(8) + 4(4) – 2 + 11: Evaluate
-16 + 16 – 2 + 11 = 9
Adding polynomials:
What is a similar or like term?

**Similar terms** in algebra, sometimes referred to as like terms, are terms that contain the same base, variable, or variables raised to the same power.

For example:
2 + 3 are like terms and is equal to 5; x + x are like terms and is equal to 2x
3x – 6x are like terms and is equal to -3x; x^3 + x^3 = 2x^3

In algebraic expressions, terms cannot be combined unless they are similar terms. When adding polynomials, simply combine like terms in the expressions.

(4x^3 – 2x^2 + 8) + (3x^3 – 9x^2 – 11): Group like terms from largest exponent to smallest
4x^3 + 3x^3 – 2x^2 – 9x^2 + 8 – 11: Simplify
7x^3 - 11x^2 – 3

(6v + 8v^3) + (3 + 4v^3 – 3v): Group like terms from largest exponent to smallest
8v^3 + 4v^3 + 6v – 3v + 3: Simplify
12v^3 + 3v + 3

(7x^2 + 2x^4 + 7x^3) + (6x^3 – 8x^4 – 7x^2): Group like terms from largest exponent to smallest
2x^4 – 8x^4 + 7x^3 + 6x^3 + 7x^2 – 7x^2: Simplify
-6x^4 + 13x^3

Subtracting Polynomials:
When subtracting polynomials, there is one extra step you need to take. You need to distribute the negative sign to each term inside the second set of parenthesis.

(5x^2 – 2x + 7) – (3x^2 + 6x – 4): Distribute the negative sign to each term in the 2nd set of parenthesis and change the operation signs of each
term
5x^2 – 2x + 7 - 3x^2 - 6x + 4: Group like terms
5x^2 – 3x^2 – 2x – 6x + 7 + 4: Simplify
2x^2 – 8x + 11

(7y^2 + 5y^3) – (6y^3 – 5y^2): Distribute the negative sign to each term inside 2nd set of parenthesis and change operation signs of each term
7y^2 + 5y^3 – 6y^3 + 5y^2: Group like terms
5y^3 – 6y^3 + 7y^2 + 5y^2: Simplify
-y^3 + 12y^2
(4n^4 + 6) - (4n - 1 - n^4): Distribute the negative sign to each term inside 2nd set of parenthesis and change the operation signs of each term
4n^4 + 6 - 4n + 1 + n^4: Group like terms
4n^4 + n^4 - 4n + 6 + 1: Simplify
5n^4 - 4n + 7

(9x^3 - x^2 + 11) - (4x^2 - 2x^3 + 4): Distribute negative sign to each term inside 2nd set of parenthesis and change operation signs
9x^3 - x^2 + 11 - 4x^2 + 2x^3 - 4: Group like terms
9x^3 + 2x^3 - x^2 - 4x^2 + 11 - 4: Simplify
11x^3 - 5x^2 + 7

Multiplying Polynomials
When multiplying 2 monomials multiply any numbers and use the product rule for the variables.

(4x^3y^4z)(2x^2y^6z^3): Group like terms and use product rule
(4*2)(x^3*x^2)(y^4*y^6)(z^1*z^3): Evaluate
8x^5y^{10}z^4

(5a^2b^3c^8)(3a^4bc^5): Group like terms and use product rule
(5*3)(a^2*a^4)(b^3*b)(c^8c^5): Evaluate
15a^6b^4c^{13}

Multiplying monomial and a polynomial:
4x^3(5x^2 - 2x + 5): Distribute the 4x^3 and use product rule for like variables
(4*5)(x^{3+2}) + (4*-2)(x^{3+1}) + (4*5)(x^3): Simplify
20x^5 - 8x^4 + 20x^3

2a^3b(3ab^2 - 4a) Distribute 2a^3b and use the product rule for the variables
(2*3)(a^3 * a * b * b^2) + (2*-4)(a^3 * a * b): Simplify
6a^4b^3 - 8a^4b
Multiply binomials by distribution or FOIL

(x * 2x) + (x * -3) + (-4 * 2x) + (-4 * -3): Multiply terms inside parenthesis. Remember product rule.

(2x^2 – 3x – 8x + 12): Simplify and combine like terms
2x^2 – 11x + 12

(2y – 1)(6y + 2): Multiply binomials by distribution or FOIL
(2y * 6y) + (2y * 2) + (-1 * 6y) + (-1 * 2): Multiply terms inside parenthesis. Remember product rule.

(12y^2 + 4y – 6y – 2): Combine like terms
12y^2 – 2y – 2

Multiply binomials by distributing or using FOIL

(2x * x) + (2x * 5) + (-4 * x) + (-4 * 5): Multiply terms inside parenthesis. Remember product rule
2x^2 + 10x – 4x – 20: Simplify and combine like terms
2x^2 + 6x – 20

(9a – 4b)(a + 2b): Multiply binomials by distributing or using FOIL
(9a * a) + (9a * 2b) + (-4b * a) + (-4b * b): Multiply terms inside parenthesis. Remember product rule
9a^2 + 18ab – 4ab – 4b^2: Simplify and combine like terms
9a^2 + 14ab – 4b^2

Multiply binomials by distributing or using FOIL

2[(4a + 5)(-2a + 3)]: Multiply binomials by distributing or using FOIL
2(4a * -2a) + (4a + 3) + (5 * -2a) + (5 * 3): Multiply terms inside parenthesis. Remember product rule
2(-8a^2 + 12a – 10a + 15): Simplify and combine like terms
2(-8a^2 + 2a + 15): Distribute 2 to each term inside the parenthesis
-16a^2 + 4a + 30

It’s best if you distribute the 2 after you have simplified the product of 2 binomials. However, you can distribute the 2 first, but don’t distribute it to the terms in both set of the parenthesis. It is only distributed to the terms in the first set of parenthesis.
3(2x – y)(3x + 4y): Multiply binomials by distributing or using FOIL
3(2x * 3x) + (2x * 4y) + (-y * 3x) + (-y * 4y): Multiply terms inside parenthesis. Remember product rule.
3(6x^2 + 8xy – 3xy – 4y^2): Simplify and combine like terms
3(6x^2 + 5xy – 4y^2) Distribute 3 to each term inside parenthesis
18x^2 + 15xy – 12y^2

Polynomials: Multiplying Special Products
Objective: To recognize and use special product rules for a sum and difference of perfect squares to multiply polynomials.
Example:
(a + b)(a – b): Distribute (a + b)
a(a – b) + b(a – b): Distribute a and b
a^2 – ab + ab – b^2: Combine like terms
a^2 – ab + ab - b^2 = a^2 – b^2
When you combine the middle terms, –ab + ab, the result is zero. Instead of going through the distribution process, recognize when you have the sum and difference of the same binomial, square the first term, minus sign, and square the last term.
(x + 5)(x – 5): Recognize sum and difference of same binomial, so square both terms and put a subtraction sign between them.
(x)^2 – (5)^2 = x^2 – 25

(3x + 4)(3x – 4): Recognize sum and difference. Square both and put subtraction sign between them.
(3x)^2 – (4)^2 = 9x^2 – 16

(2x – 6y)(2x + 6y): Recognize sum and difference. Square both and put subtraction sign between them.
(2x)^2 – (6y)^2: 4x^2 – 36y^2

Multiplying perfect squares:
(a + b)^2 = (a + b)(a + b): Distribute (a + b)
a(a + b) + b(a + b): Distribute a and b
a^2 + ab + ab + b^2: Combine like terms
a^2 + 2ab + b^2
When you square a binomial, do not make the mistake of distributing the exponent to each term inside the parenthesis.
(x – 5)^2 ≠ x^2 – 25 or x^2 + 25: As you can see, both of these are missing the middle term of -10x.
Write the expression in expanded form: \((x - 5)(x - 5)\) and simplify
\[x(x - 5) + (-5)(x - 5): \text{Distribute } x \text{ and } -5\]
\[x^2 - 5x - 5x + 25: \text{Combine like terms}\]
\[x^2 - 10x + 25\]

The shortcut for this special product is square the first term. Multiply the product of the two terms by 2. Square the last term.

\[(x + 7)^2: \text{Recognize it is a perfect square}\]
\[(x)^2: \text{Square the first term} = x^2\]
\[2(x)(7): \text{Twice the product of} = 14x\]
\[(7)^2: \text{Square the last term} = 49\]
\[x^2 + 14x + 49\]

\[(2y - 5)^2: \text{Recognize it is a perfect square}\]
\[(2y)^2 = 4y^2: \text{Square the first term}\]
\[2(2y)(-5) = -20y: \text{Two times the product of each term of the binomial}\]
\[(-5)^2 = 25: \text{Square the last term}\]
\[4y^2 - 20y + 25\]

\[(4a - 8b)^2: \text{Recognize it is a perfect square}\]
\[(4a)^2 = 16a^2: \text{Square the first term}\]
\[2(4a)(-8b) = -64ab: \text{Twice the product of each term of the binomial}\]
\[(-8b)^2 = 64b^2 \text{ Square the last term}\]
\[16a^2 - 64ab + 64b^2\]