LESSON 12 – SIGNED NUMBERS

INTRODUCTION

In the first few lessons of this course, we learned about different number types including whole numbers and fractions. In this lesson, we will revisit those numbers but with a twist…a twist called “signs”. We will add, subtract, multiply, divide, and otherwise combine signed numbers. The knowledge gained in this lesson will open a whole new world of applications and number situations and is perhaps the most important foundation for success in an algebra course.

The table below shows the specific objectives that are the achievement goal for this lesson. Read through them carefully now to gain initial exposure to the terms and concept names for the lesson. Refer back to the list at the end of the lesson to see if you can perform each objective.

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KEY TERMS

The key terms listed below will help you keep track of important mathematical words and phrases that are part of this lesson. Look for these words and circle or highlight them along with their definition or explanation as you work through the MiniLesson.

- Signed Numbers
- Integers
- Absolute Value
- Opposite
- Order of Operations
- Three Signs of a Fraction
- Combining Signs
LESSON CHECKLIST

Use this page to track required components for your class and your progress on each one.

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MINILESSON

SIGNED NUMBERS

If Fred has $200 in his checking account and he writes a check for $250, how would we represent his account balance? We could say that:

Fred’s Balance = $200 - $250

But how can we subtract a number that is larger from one that is smaller? Common sense tells us that Fred’s account is at a deficit status of $50. We would say his account balance is -$50. Therefore,

Fred’s Balance = $200 - $250 = -$50

In order to deal with situations such as this one, we need to be able to work with both positive numbers and negative numbers. These numbers together are called signed numbers.

**Example 1:** The counting numbers, 0, and the negative counting numbers comprise what are called integers. Label the following number line so that it includes 0 and the integers from -5 to 5:

![Number Line Example 1](image)

**Example 2:** Fractions can be signed as well. Label the following number line so that it includes numbers from -1 to 1 in increments of 1/5.

![Number Line Example 2](image)

When we compare signed numbers, we do so the same way we compare whole numbers. Numbers further to the right on the number line are greater than numbers on the left.
GIVE MEANING TO SIGNED NUMBERS

Similar to the bank example that we started with, these examples illustrate the power of signed numbers to describe a given situation with realistic numbers.

Example 3: Provide a numerical quantity that accurately represents each of the following situations.

a) Tom gambled in Vegas and lost $52.50. __________________

b) Larry added 25 songs to his playlist. _________________

c) The airplane descended 500 feet to avoid turbulence. _________________

APPLICATIONS WITH SIGNED NUMBERS

Example 4: Camden, SC had a record low temperature of -19°F on Jan 21, 1985, and Monahans, TX had a record low temperature of -23°F on Feb 8, 1933. Which of the two temperatures was the lowest? (Data Source Wikipedia: http://en.wikipedia.org/wiki/U.S._state_temperature_extremes)

ABSOLUTE VALUE

• The absolute value of a number is the distance from the number to 0 on the number line.
• The notation | | is used to represent absolute value.
• The absolute value of a number is always positive.

Example 5: a. \[ \left| \frac{-2}{3} \right| = \]

Example 6: a. \[ \left| \frac{-7}{5} \right| = \]

Example 5: b. \[ |3| = \]

Example 6: b. \[ |-1| = \]
YOU TRY

7. Determine the signed number that best describes the statements below.

<table>
<thead>
<tr>
<th>Statement</th>
<th>Signed Number</th>
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<tbody>
<tr>
<td>A balloon dropped 59 feet.</td>
<td></td>
</tr>
<tr>
<td>Lori deleted 324 songs from her iPod.</td>
<td></td>
</tr>
<tr>
<td>A credit card has a balance of $235.34.</td>
<td></td>
</tr>
<tr>
<td>A checking account has a balance of $235.34.</td>
<td></td>
</tr>
</tbody>
</table>

8. a. |-3| = ___________  b. |3| = ___________
      c. -|-3| = ___________  d. -|3| = ___________

9. Below the tick marks on the graph, place the numbers -5 to 5, in order from left to right. Place a dot on the graph for each of the numbers in the list and label above the dot with the number (exactly as it appears on the list).

-3, 2.5, |-4|, -1.5, -5

10. Two rides at the amusement park claim to be the greatest thrill. One ride, the Chaotic Coaster drops 100 feet from its highest point. The other ride, the Fearless Flyer, drops 89 feet from its highest point. Which ride drops the most and by how much?
Now that we know a little bit about signed numbers, let’s see how to correctly combine them using mathematical operations or exponents. The following example will get us started:

When the Badwater Ultra-Marathon in California first began, the race started at Death Valley and ended at the top of Mt. Whitney (see elevation graph below). What was the total elevation gain for the race?

To compute total elevation, we subtract the lower elevation from the higher one.

$$14505 - (-280)$$

We can see from the graph that what we really need to do is add $14505 + 280$ to get a total elevation of 14,785 feet. How does that work with the symbols we have in the first expression? Subtracting a negative number is the same as adding that number so we can rewrite as follows:

$$14505 - (-280) = 14505 + 280 = 14785$$

The total elevation gain for the race, then, was 14,785 feet.
In addition to PEMDAS, the following information will help when working with signed numbers.

When working with signed numbers:
• In application problems, use ( ) to separate numbers with negative signs
• Use PEMDAS for order of operations
• Fraction rules and rules for exponents also apply
• Combine signs in a series of simplification steps to simplify your expression
• When combining two signs given together, use the following rules:
  \((-)(-) = +, \quad (-)(+) = -, \quad (+)(-) = -, \quad (+)(+) = +\)
• Use your calculator carefully to help you check results

**Example 11:** Combine each of the following signed numbers. Use a number line to help you visualize. Show steps if possible. Start by combining signs if possible.

a. \(4 + (-3)\)  

b. \(-5 + 8\)

c. \(-35 + (-20)\)

d. \(-3.5 - 2.1\)

e. \(\frac{1}{4} + (-2)\)

f. \(-2\frac{3}{4} - \left(-\frac{1}{4}\right)\)

**YOU TRY**

12. Combine each of the following signed numbers. Use a number line to help you visualize. Show steps if possible. Write improper fractions as mixed numbers.

a. \(-3 - (-4)\)  

b. \(\frac{1}{5} - 3\)

c. \(12 + (-1)\)
**Example 13:** Combine each of the following using correct order of operations. Start by combining the signs if possible.

a. \((-20) - 20 + (-10)\)  
b. \(8 - (-3) + 4 + (-3) - 2\)

**YOU TRY**

14. Combine each of the following using correct order of operations. Start by combining signs if possible.

\[7 - (-2) + (-1) - 5\]

**Example 15:** Multiply or divide each of the following. Show steps if possible.

a. \((-8) \cdot (1)\)  
b. \((-8) \cdot (-1)\)  
c. \((-2) \cdot (-1)\)

d. \((-0.4)^2\)  
e. \(\left(-\frac{1}{3}\right)^2\)  
f. \((0) \cdot (-3) \cdot (2)\)

g. \((-4) \cdot (3) \cdot (-1)\)  
h. \(8 \div (-4)\)  
i. \(-3 + \left(-\frac{1}{8}\right)\)
THREE SIGNS OF A FRACTION
The following fractions are all equivalent (meaning they have the same value):
\[
\frac{-1}{2} = \frac{1}{-2} = -\frac{1}{2}
\]
Notice that only the placement of the negative sign is different.
HOWEVER, only the last one, \(-\frac{1}{2}\), is considered to be in simplest form.

YOU TRY

16. Multiply or divide each of the following. Show steps if possible.
   a. \((-12) \cdot (-1)
   b. \(-\left(\frac{1}{4}\right)^3
   c. \ (-12 \div (-3)

Example 17: Simplify each of the following using correct order of operations and showing all possible steps.
   a. \((-10) \left(\frac{1}{5}\right) (8) - 3
   b. \frac{2^3 + (-10)}{-4}

YOU TRY

18. Simplify each of the following using correct order of operations and showing all possible steps. Write improper fractions as mixed numbers.
   a. \(-6 + (-4)^2 + 2(-3)
   b. \frac{6 - (2 - 3)^2}{-3 + (-5)}
APPLICATIONS WITH SIGNED NUMBERS

Example 19: On the first five rolls of your FARKLE game in Facebook, you earned 400, 1250, 0, 0, -500 points. What is your total after the 5 rolls? Write your answer in a complete sentence. Start by circling the GIVENS and underlining the GOAL.

YOU TRY

20. Ryan has an outstanding balance of $2,312.43 on his credit card. If he incurs charges totaling $324.56, makes a payment of $425, and incurs an interest charge of $43.12 what is his new balance? Write your answer in a complete sentence. Start by circling the GIVENS and underlining the GOAL.