Building Procedural Fluency through Conceptual Understanding

LATRENDAL KNIGHTEN
LDOE TEACHER LEADER SUMMIT
JUNE 1, 2016
ERNEST MORIAL CONVENTION CENTER, NEW ORLEANS, LA
Session Agenda

- Opening Activity
- Introduction/Workshop Overview
- Game of Nine Cards Activity
- Making Connections
  - Fluency Expectations
  - Strategies, problem types...
  - Translation Task
  - Activity Session
- Reflection/Questions
Louisiana Student Standards

- Operations and Algebraic Thinking
  - 2.OA.1
  - 2.OA.2

- Number and Operations in Base Ten
  - 2.NBT.5

- Math Practice Standards
  - 1. Make sense of problems and persevere in solving them
  - 2. Reason abstractly and quantitatively
  - 3. Construct viable arguments and critique the reasoning of others
  - 4. Model with mathematics
  - 6. Attend to precision
  - 7. Look for and make use of structure
  - 8. Look for and express regularity in repeated reasoning.
Where Are You?

On a scale of 1 to 5 rate your use of activities that build fluency from conceptual understanding.
Warm Up Activity
Commit and Toss

- You’ll need an index card and a pen or marker.
- You have 30 seconds to write down your thoughts on fluency. What does it mean to be fluent? What are strategies you could use to help your students develop fluency?
- When you hear the signal, toss your card as far as you can.
- Pick up a card (not your own) and share and discuss with a partner.
What is Fluency?

Basic Fact Fluency – “the efficient, appropriate, and flexible application of single-digit calculation skills and ... an essential aspect of mathematical proficiency”. (Baroody, 2006)

Computational Fluency – “the efficient, appropriate and flexible application of single-digit and multi-digit calculation skills” (Baroody, 2006)

Procedural Fluency – “skill in carrying out procedures flexibly, accurately, efficiently, and appropriately”. (CCSS 2010, p. 6)
Game of Nine Cards


Object: Be the first person to identify three cards in your hand that add up to 15.

- Take turns selecting cards.

- Note: You may have more than 3 cards in your hand, but you must use exactly 3 cards to make a sum of 15. (For instance, if you have 2, 3, 5 and 7 in your hand, you would win because $3 + 5 + 7 = 15$. You don’t need to use the 2.)

- Play the game with a partner. HAVE FUN!!!!

http://illuminations.nctm.org/DeepSeaDuelLP/
Play to Win?

- If you play first, should you choose an **even or an odd** number?
- If your opponent plays first and picks an even number, what number should you choose to avoid a loss?
- Who is more likely to win — the first player or the second player? Why?
- Will someone always win? Lose?
- Are there ways to make sure that you don’t lose?
- Is there a “best” card to choose?
Think-Pair-Share

- Where’s the math? (Louisiana Student Standards)
- How would you modify, extend, and/or differentiate the game to address varying ability levels, grade levels, student needs, etc.?
- What mathematical practices did you observe?
- What role (if any) does fluency play in being successful at this game?
## Fluency Requirements

<table>
<thead>
<tr>
<th>Grade and standard</th>
<th>Addition</th>
<th>Subtraction</th>
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</thead>
<tbody>
<tr>
<td><strong>K</strong> Fluency K.OA.5</td>
<td>Numbers within 1 to 5</td>
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<tr>
<td><strong>1</strong> Fluency 1.OA.6 and 1.NBT.5</td>
<td>Numbers within 10 Mentally find 10 more or 10 less than a two-digit number.</td>
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<tr>
<td><strong>2</strong> Fluency 2.NBT.5 and 2.OA.2</td>
<td>Use strategies of place value and properties to add and subtract numbers within 100. Mentally add and subtract numbers within 20.</td>
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<td>Grade and standard</td>
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<td>Fluency K.OA.5</td>
<td>Numbers within 1 to 5</td>
<td></td>
</tr>
<tr>
<td>Solving K.OA.2 and K.OA.3</td>
<td>Solve word problems by using objects or drawings and decompose numbers less than or equal to 10.</td>
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<tr>
<td>Modeling ideas K.NBT.1</td>
<td>Use objects or drawings to compose and decompose numbers from 11 to 19 into tens and ones.</td>
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<tr>
<td>Fluency 1.OA.6 and 1.NBT.5</td>
<td>Numbers within 10</td>
<td>Mentally find 10 more or 10 less than a two-digit number.</td>
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<tr>
<td>Solving 1.OA.1</td>
<td>Solve word problems using various strategies for numbers within 20, including problems with three whole numbers, by using objects, drawings, and equations.</td>
<td>Solve word problems using various strategies for numbers within 20, by using objects, drawings, and equations.</td>
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<tr>
<td>Modeling ideas 1.OA.2</td>
<td>Use models, properties, and place-value strategies to add a two-digit number to a one-digit number or add a two-digit number to a multiple of 10.</td>
<td>Use models, properties, and strategies to subtract one multiple of 10 from another multiple of 10 in the range of 10 to 90.</td>
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<td><strong>2</strong></td>
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<tr>
<td>Fluency 2.NBT.5 and 2.OA.2</td>
<td>Use strategies of place value and properties to add and subtract numbers within 100.</td>
<td>Mentally add and subtract numbers within 20; know from memory all sums of two one-digit numbers.</td>
</tr>
<tr>
<td>Solving 2.OA.1</td>
<td>Solve word problems involving one or two steps for numbers within 100 by using drawings and equations.</td>
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<tr>
<td>Modeling ideas 2.NBT.7 and 2.NBT.8 and 2.OA.4</td>
<td>Use models, properties, and place value to add and subtract numbers within 1000.</td>
<td>Mentally add or subtract 10 or 100 to a number between 100 and 900.</td>
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</tbody>
</table>
Four Components of Fluency with Basic Facts

Appropriate Strategy Selection

Flexibility –

Efficiency –

Accuracy –
How much is 1006 – 98?

- Record your solution and reasoning.
- Share your reasoning with your table group.
- Share reasoning and strategies with the group.
Three Phases

- Children typically progress through 3 phases in mastering the basic number combinations (single-digit addition and multiplication combinations and their complementary subtraction and division combinations.)

- Counting strategies – including object or verbal counting to derive an answer

- Reasoning strategies – using known facts and relationships to solve an unknown combination

- Mastery – achieving automaticity in basic fact combinations
Strategies Anyone?
Strategies Anyone? (Addition)

Count On
- Students count on from the higher number regardless of its placement in the expression.

Sum to 10 OR Make Ten
- Applies to addition facts for sums of 10.
- Provides an opportunity to engage students in the Mathematical Practice Standards 7 & 8
  - Look for and make use of structure (7)
  - Look for and express regularity in repeated reasoning (8)
Strategies Anyone? (Addition)

Doubles and associated strategies

- Includes all the facts that have two addends that are the same quantity, such as $2 + 2 = 4$, $3 + 3 = 6$, etc.
- Doubles facts provide an anchor for other facts. Students derive facts about “near doubles” from known doubles.
Strategies Anyone? (Subtraction)

Think Addition
- Focus on the relationship between the quantities
- Encourages students to compare the two values
- How can you use addition to solve $12 - 9$?
- Sample Questions to prompt the “think addition” strategy:
  - How many more is 12 than 9?
  - How many less is 9 than 12?
  - How could you use addition to find the answer?
  - How could you use the “count on” strategy to find the difference?
Strategies Anyone? (Subtraction)

Down Over 10

15 – 5 =  16 – 6 =  12 – 2 =  14 – 4 =  13 – 3 =

15 – 6 =  16 – 7 =  12 – 3 =  14 – 5 =  13 – 4 =

Questions to pose:

◦ Could you use the pattern that you noticed in the first set of problems to solve the problems in this set? How?
◦ How does thinking about 10 help you subtract?
◦ Could you use 10 as a benchmark to help you subtract? How?
Strategies Anyone? (Subtraction)

Take from 10

How could you use the sums of 10 to solve the following subtraction problems?

13 – 8 =
15 – 8 =
17 – 9 =
12 – 8 =
Get Your Game On!

Games allow students to participate in engaging, efficient, and self-motivating activities to reinforce math fluency.
Focus Questions

Answer the following questions as you examine each fluency activity:

• Is there evidence of one or more of the four components of fluency? Which one (s) are best aligned to the game/activity?

• How would you modify, extend, and/or differentiate the activity to address student needs (varying ability levels, grade levels, etc.)?
What’s the Question?

Write an addition or subtraction word problem that has the solution 24 alligators.
Problem – Solving Task
Let’s look back at the problems we wrote for 24 alligators. Which problem solving structure does your problem represent?

Where does your problem fit?

24 alligators
Key Instructional Shift

From emphasis on:
How to get answers

To emphasis on:
Understanding mathematics
<table>
<thead>
<tr>
<th>Result Unknown</th>
<th>Change Unknown</th>
<th>Start Unknown</th>
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<tr>
<td><strong>Add to</strong></td>
<td>Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? 3 + 5 = ?</td>
<td>Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? 5 + 3 = 8</td>
</tr>
<tr>
<td><strong>Take from</strong></td>
<td>Five apples were on the table. I ate two apples. How many apples are on the table now? 5 - 2 = ?</td>
<td>Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? 5 - 3 = 2</td>
</tr>
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<table>
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<tr>
<th>Total Unknown</th>
<th>Addend Unknown</th>
<th>Both Addends Unknown</th>
</tr>
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<tr>
<td><strong>Put Together/Take Apart</strong></td>
<td>Three red apples and two green apples are on the table. How many apples are on the table? 3 + 2 = ?</td>
<td>Five apples are on the table. How many apples are green? 5 + 3 = 8</td>
</tr>
<tr>
<td><strong>Difference Unknown</strong></td>
<td>Bigger Unknown</td>
<td>Smaller Unknown</td>
</tr>
<tr>
<td></td>
<td>(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy? 2 + ? = 5, 5 - 2 = ?</td>
<td>(“Version with ”more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have? 2 + 3 = 5, 3 + 2 = ?</td>
</tr>
<tr>
<td></td>
<td>(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? 2 + ? = 5, 5 - 2 = ?</td>
<td>(“Version with “fewer”): Lucy has three fewer apples than Julie. Lucy has two apples. How many apples does Julie have? 2 + 3 = 5, 3 + 2 = ?</td>
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*These take apart situations can be used to show all the decompositions of a given number. The associated equations, which have the total on the left of the equal sign, help children understand that the = sign does not always mean makes or results in but always does mean the same number as.*

*Either addend can be unknown, so there are three variations of these problem situations. Both Addends Unknown is a productive extension of this basic situation, especially for small numbers less than or equal to 10.*

*For the Bigger Unknown or Smaller Unknown situations, one version directs the correct operation (the version using more for the bigger unknown and using less for the smaller unknown). The other versions are more difficult.*
Translation Task

Allows a teacher to assess whether students can use representations flexibly.

• In this translation task students are asked to demonstrate their understanding of an addition or subtraction problem by –
  • Creating a word problem;
  • Making a visual model of the problem;
  • Representing the problem in an equation; and
  • Giving an explanation of their thinking.
Think-Pair-Share

- How could tasks like the translation task help students apply fluency, problem solving & reasoning skills, and model strategies?
- Share your response with a partner.
- Share with your table group.
Build Procedural Fluency from Conceptual Understanding

Teacher Actions:

Provide students with opportunities to use their own reasoning strategies and methods for solving problems.

Ask students to discuss and explain why the procedures that they are using work to solve particular problems.

Connect student-generated strategies and methods to more efficient procedures as appropriate.

Use visual models to support students’ understanding of general methods.

Provide students with opportunities for distributed practice of procedures.
Build Procedural Fluency from Conceptual Understanding

Student Actions

Make sure that they understand and can explain the mathematical basis for the procedures that they are using.

Demonstrate flexible use of strategies and methods while reflecting on which procedures seem to work best for specific types of problems.

Determine whether specific approaches generalize to a broad class of procedures.

Strive to use procedures appropriately and efficiently.
Where Are You Now?

On a scale of 1 to 5 rate your use of activities that build fluency from conceptual understanding.
Questions