

Dear Professor

When I had to deal with these kinds of problems, I used to go handbooks. For this one ROARK'S formulas for stress & strain.

Calculating area geometrically is straight forward, but by calculus :

$$y = \sqrt{R^2 - x^2} = R \sin \alpha$$

$$x = R \cos \alpha$$

$$\text{Area} = 2 \times \int_0^x \sqrt{R^2 - x^2} dx = 2 \left(\frac{x \sqrt{R^2 - x^2}}{2} + \frac{R^2}{2} \sin^{-1} \frac{x}{R} \right)$$

If substitute x and y with trigonometric form, you will get the same formula as ROARK.

$$\text{Static moment} = 2 \times \int_0^x x \sqrt{R^2 - x^2} dx = 2 \frac{\sqrt{(R^2 - x^2)^3}}{3}$$

The same for moment of inertia.