Dear Professor

When I had to deal with these kinds of problems, I used to go handbooks. For this one ROARK'S formulas for stress & strain version 6, 1989.

Calculating area geometrically is straight forward, but by calculus :

$\beta = 90 - \alpha$

$y = \sqrt[2]{R^2 - x^2} = R.\sin\alpha$

$x = R.\cos\alpha$

Area =

$$2 \times \int_0^x \sqrt[2]{R^2 - x^2}\, dx = 2(\frac{x.\sqrt[2]{R^2 - x^2}}{2} + \frac{R^2}{2}\sin^{-1}\frac{x}{R}) = x.y + R^2.\sin^{-1}\cos \propto\, = R^2.\sin \propto.\cos \propto + \propto$$

If substitute x and y with trigonometric form, you will get the same formula as ROARK.

Static moment = $2.\int x.y.dy = 2 \times \int_0^x y.\sqrt[2]{R^2 - y^2}\, dy = -2\frac{\sqrt[2]{(R^2-x^2)^3}}{3} = -\frac{2x^3}{3} = -\frac{2(R.\cos\propto)^3}{3}$

Negative sign shows that center of gravity is below x axes.

Center of gravity:

$$\frac{static\ moment}{area} = -\frac{2(R.\cos\propto)^3}{3(R^2.\sin\propto.\cos\propto - \propto)} = -\frac{2R.\cos\propto^3}{3(\sin\propto.\cos\propto - \propto)}$$

Which is negative. If you wish to get center of gravity relative to center of circle, then you will get ROARK formula. Fortunately, area and center of gravity formulas are correct.

The same concept for moment of inertia, but it is a little bit lengthy.