$$\bar{y}_1 = -\frac{1}{A_1}\int_{-R}^{R}\frac{1}{2}\left(\sqrt{R^2-x^2}\right)^2 dx = -\frac{1}{A_1}\int_{0}^{R}\left(R^2-x^2\right)dx =$$

$$-\frac{1}{A_1}\left(R^3-\frac{R^3}{3}\right) = -\frac{2R^3}{3A_1}$$

$$\bar{y}_2 = \frac{2}{A_2}\int_{R\sin\alpha}^{R}\frac{1}{2}\left(\sqrt{R^2-x^2}\right)^2 dx = \frac{1}{A_2}\int_{R\sin\alpha}^{R}\left(R^2-x^2\right)dx =$$

$$\frac{1}{A_2}\left(R^3-\frac{R^3}{3}-R^3\sin\alpha+\frac{R^3\sin^3\alpha}{3}\right) = \frac{1}{A_2}\left(\frac{2}{3}R^3+R^3\sin\alpha\left(\frac{\sin^2\alpha}{3}-1\right)\right)$$

$$\bar{y}_3 = \frac{2}{A_3}\int_{0}^{R\sin\alpha}\frac{1}{2}R^2\cos^2\alpha\, dx = \frac{1}{A_3}R^3\sin\alpha\cos^2\alpha$$

$$\bar{y} = \frac{A_1\bar{y}_1+A_2\bar{y}_2+A_3\bar{y}_3}{A} = \frac{-\frac{2R^3}{3}+\left(\frac{2}{3}R^3+R^3\sin\alpha\left(\frac{\sin^2\alpha}{3}-1\right)\right)+R^3\sin\alpha\cos^2\alpha}{A} =$$

$$\frac{R^3\sin\alpha\left(\frac{\sin^2\alpha}{3}-1\right)+R^3\sin\alpha\cos^2\alpha}{R^2\left(\pi-\alpha+\sin\alpha\cos\alpha\right)} = \frac{2R\sin\alpha\left(\frac{\sin^2\alpha}{3}-1\right)+R\sin\alpha\cos^2\alpha}{\pi-\alpha+\sin\alpha\cos\alpha}$$