

A Calculus Challenge

It has been quite a while since I last posted anything here, but an interesting problem has come to mind that I would like to share with you. If you really know calculus, this should be straight forward; if you don't know calculus, don't even try!

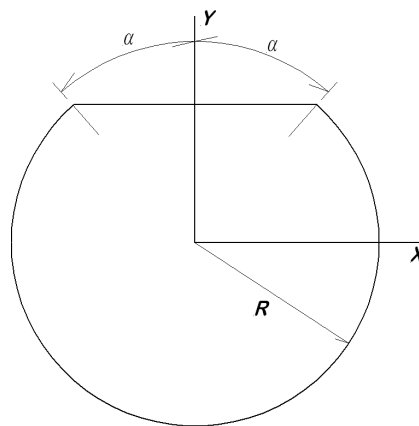


Figure 1: Circular Disk With A Flat

Consider the simple circular disk with a flat side, as shown in the Figure. The radius R and the angle α are known. This might be a machine shaft with a mounting flat for a set screw, or perhaps it is a horizontal cylinder with flat ends that is partially filled with a liquid. No doubt there are other engineering situations where this geometry arises.

The problem is simply this: Can any of you find general expressions in terms of R and α for these items?

- (a) area enclosed?
- (b) centroid location for the enclosed area?
- (c) area moments of inertia I_{xx} and I_{yy} for the enclosed area with respect to the axes shown?
- (d) area product of inertia I_{xy} for the enclosed area with respect to the axes shown?
- (e) area polar moment of inertia with respect to the origin of coordinates?

I would be very pleased to see the results that any of you obtain.