4th International FLAC Symposium on Numerical Modeling in Geomechanics – 2006 – Hart & Varona (eds.) Paper: 07-03

The performance of *FLAC* zones in bending

C. Pound
Mott MacDonald, Croydon, England

ABSTRACT: It is usually preferable to model structural members using beam or other structural elements. However there are certain situations where use of solid elements is necessary either because the physical thickness of the structural member needs to be modeled or because a more complex constitutive model needs to be considered for the structural material. The paper presents the results of parametric studies of the deflection of a cantilever and of a thin hollow cylinder to determine the minimum number of zones across the structural thickness necessary to predict accurately the deflection of the structural member. The influence of aspect ratio of the zones is also investigated. Results indicate that the deflection of a cantilever is predicted to within 3.5% of the theoretical result with six zones across the width and an aspect ratio of 1. Sensitivity studies indicate that it is often better to use a small number of zones across the structure width to permit a large number of elements in the longitudinal direction of the structure to be used. The most zone efficient arrangement is where the zones are three times larger in the direction perpendicular to the structure axis than in the direction parallel to the structure axis. Different methods to determine the bending moment in the structural member are also discussed and the optimum method identified. Use of zone stresses directly gives a poor assessment of the bending moments particularly where the number of zones across the width is small, but this can be improved by use of sub-zone stresses and more particularly by projection of the stresses to the boundary of the structural member. Using this latter approach bending moments can be predicted with the same accuracy as the deflection of the structural member.

1 INTRODUCTION

When carrying out ground-structure interaction analyses it is conventional practice to model the structure using beam or other structural element. The formulation of these elements is such that they accurately predict the displacements and structural forces due to the loads applied to them. They also have the advantage that structural forces can be directly output from the element without the need for complex post-processing. However in certain situations use of structural elements is not feasible or appropriate. For example when the physical thickness of the structural member is significant or needs to be modeled to ensure correct positioning of the ground or other structural members. Also it is sometimes necessary to consider structural materials such as concrete as non-linear elasto-plastic materials using stress-strain models such as that proposed in Eurocode 2. This is beyond the capability of most structural elements incorporated into numerical analysis programs. In both instances the structural member needs to be modeled using solid elements.

The solid elements or zones incorporated in *FLAC* comprise two pairs of superimposed constant strain triangles. This formulation ensures some intrinsic bending stiffness within the zones, but is of insufficient complexity to predict correctly the deformation or forces in a structural member without representing the structural member by a number of zones both in terms of length and thickness.

This paper describes a parametric study carried out to identify the accuracy with which *FLAC* is able to predict the bending of a cantilever and thin hollow cylinder and to provide guidance on the minimum number of zones necessary to model adequately structural elements. The cantilever was chosen because it replicates the behavior of slabs and walls and a hollow cylinder was chosen as it is analogous to many tunnel linings.

2 STRUCTURAL GEOMETRY

2.1 *Cantilever*

The cantilever was taken to be 10 m long and 0.5 m thick and of infinite length in the out-of-plane direction. The cantilever was assumed to be weightless, but a vertical force of 10 kN/m run was applied at the free extremity. The theoretical bending moment at the support therefore is 100 kNm/m. The cantilever was taken to be constructed from concrete with a Young's modulus of 25 GPa and a Poisson's ratio of 0.25. The theoretical deflection of the free end of the cantilever is 12.0 mm.

2.2 *Hollow Cylinder*

The hollow cylinder had a centerline radius of 5.0 m and a thickness of 0.25 m. The inner and outer radii were therefore 4.875 m and 5.125 m respectively. Equal and opposite inward-directed radial forces of 200 kN/m were applied across a diameter. The cylinder was taken to be constructed from the same weightless concrete as the cantilever. The theoretical bending moment across the section on the diameter at 90º to the applied force is 181.75 kNm/m and the theoretical radial outward deflection is 49.22 mm.

3 *FLAC* ANALYSES

All the analyses were run in small strain mode in which the model geometry is not updated to reflect the observed displacements. This is consistent with the theoretical solutions given above. An isotropic linear elastic constitutive model was used to replicate the behavior of the concrete.

Equilibrium was considered to have been achieved in the analyses when the out-of-balanced force was less than $1x10^{-6}$ kN or when the mechanical force ratio fell below 0.001%. Histories demonstrated that displacements had ceased when this degree of equilibrium had been achieved.

The cantilever was modeled in *FLAC* in plane strain. At the support the gridpoints had horizontal displacement fixity whereas the points were free to move vertically except for the gridpoint at mid-height which also had vertical displacement fixity. The vertical force was also applied to the gridpoint at mid-height at the free end. A typical view of a mesh for the cantilever is shown in Figure 1.

The hollow cylinder was also modeled in *FLAC* in plane strain. Advantage was taken of symmetry and therefore only one quarter of the cylinder was modeled. The force was applied on the vertical boundary, which also had horizontal displacement fixity. The horizontal boundary had vertical displacement fixity only. Because of symmetry a vertical force of only 100 kN/m was applied on the gridpoint on the centerline of the cylinder. A typical view of a mesh for the hollow cylinder is shown in Figure 1.

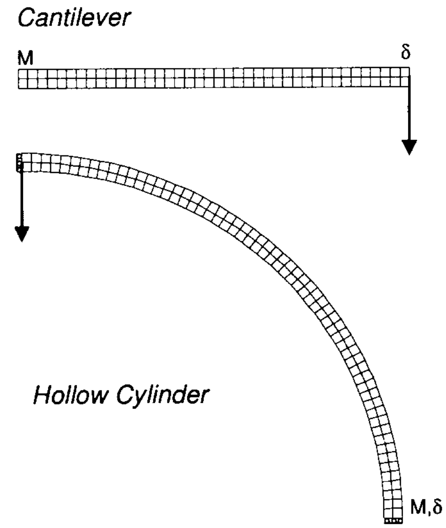

Figure 1. *FLAC* meshes, δ and M represent where deflection and bending moments are calculated.

4 RESULTS

4.1 *General*

The cantilever and hollow cylinder were first analyzed using beam elements. For the cantilever a single element was used, but for the hollow cylinder 360 elements were used to ensure that the curved shape of the structural member was replicated. The results of these analyses are given in Table 1.

Table 1. Results for structural elements

Setting	Theoretical		Beam Element	
	Disp (mm)	Moment (kNm/m)	Disp (mm)	Moment (kNm/m)
Cantilever	12.0	100.0	12.0	99.99
Cylinder	49.22	181.8	49.15	181.7

The fit to the theoretical results is excellent indicating that beam elements can be used to model accurately the bending of structural members.

4.2 *Cantilever analysis using FLAC zones*

A large suite of analyses were carried out to investigate the effect of:

- Number of zones in the thickness of the cantilever.
- Number of zones in the length of the cantilever.
- Aspect ratio of the zones.

4.2.1 *Displacements*

It was initially considered that an aspect ratio of unity would give the best response of the zones and therefore the best prediction of the bending of the cantilever. A series of analyses were therefore carried out where the number of zones across the thickness of the cantilever was increased from 1 to 20 and the corresponding number of zones along the length was increased from 20 to 400. Figure 2 shows the displacement of the end of the cantilever as a percentage of the theoretical value of 12.0 mm.

As the number of zones across the cantilever thickness is increased the predicted end displacement approaches the theoretical value from below. The overstiff response of individual *FLAC* zones in bending is represented by this data. The somewhat surprising result that bending could be obtained from a cantilever of single zone thickness can be explained by the subdivision of the quadrilateral zones into four triangular subzones.

Figure 2 suggested that a cantilever of 6 zones thickness would predict deflection to nearly 97% accuracy, but that more than 10 zones would be needed to obtain better than 99% accuracy.

To investigate the influence of zone aspect ratio on the accuracy of the bending prediction, a series of analyses was carried out with 6 zones across the cantilever width and between 6 and 360 zones along the length. This gave aspect ratios varying from 20 to 0.33 respectively. Figure 3 shows the results of these analyses.

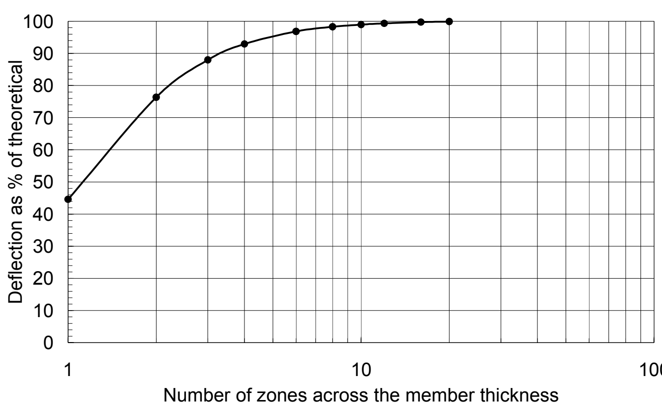

Figure 2. Results for an aspect ratio of unity.

As expected, as the aspect ratio of the zones increases above 1, the accuracy of the prediction of the deflection reduces rapidly. Where the aspect ratio of the zones exceeds 6, the deflection of the cantilever is less than 50% of theoretical. Surprisingly however, the accuracy of the prediction of the deflection continues to improve as the aspect ratio reduces below unity. For example, with an aspect ratio of 0.5, the deflection is predicted with an accuracy of better than 99%.

These results seem to imply that an increased number of elements along the length of the cantilever may be as important as the number of elements across the thickness. To investigate this possibility two further sets of analyses were carried out in which the number of zones was kept constant, but the number of zones across the cantilever thickness was varied. Obviously as the number of zones across the cantilever thickness was increased, the number of zones along the length reduced. One set of analyses was carried out with a total of 240 zones and the other with 1200 zones. In both cases the number of zones across the thickness was varied between 1 and 40. Figure 4 shows the results of the analyses.

Both sets of analyses show a distinct pattern of increasing accuracy as the number of zones across the thickness is increased from 1, to a peak followed by a reduction at higher numbers of zones thick. For the case with a total of 240 zones the most accurate result was achieved when the cantilever was 2 zones thick and 120 zones long. For the case with 1200 zones the most accurate result was achieved with 4 zones thick and 300 zones long. This suggests that for a cantilever with a thickness to length ratio of 1:20, the most accurate result is achieved with a zone ratio of between 1:60 and 1:80 for a finite total number of zones. This result can be seen more clearly by plotting the results against aspect ratio. Results are presented in Figure 5 for sets of analyses with total numbers of zones ranging between 240 and 1200. For any set of analyses the accuracy is greatest when the aspect ratio is about 0.3.

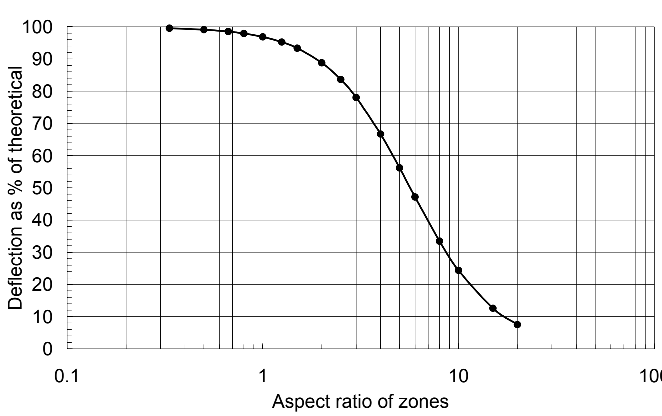

Figure 3. Results for 6 zones across the member thickness.

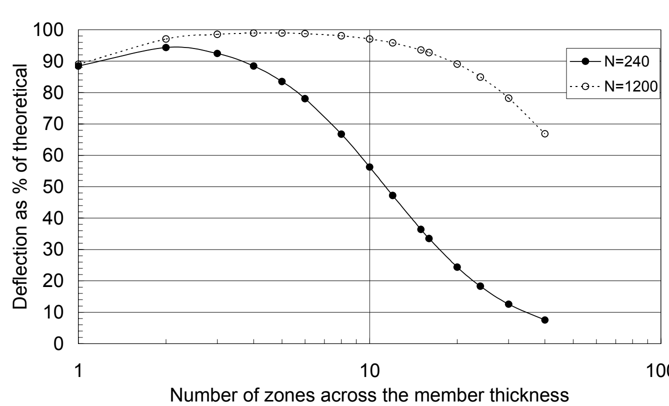

Figure 4. Results for varying total number of zones (N).

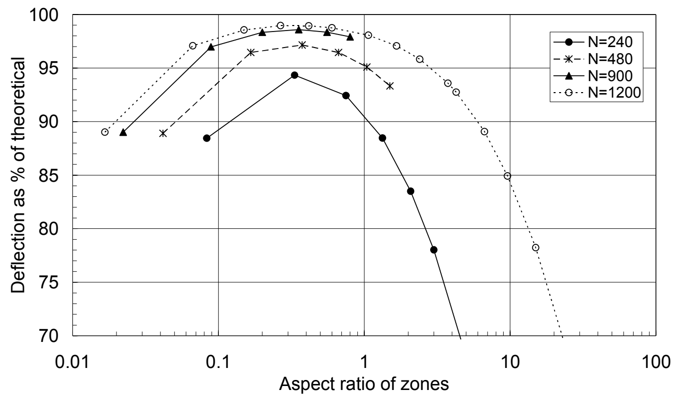

Figure 5. Results for varying total number of zones (N) and aspect ratio.

Finally, series of analyses were carried out with numbers of zones across the member thickness of 2, 4, 8 and 10 with zones along the length of between 20 and 400. The results of the analyses are shown in Figure 6 plotted along with the results already plotted for a zone width of 6. The results confirm that for a small number of zones it is more accurate to use fewer zones across the member thickness to allow more zones in the length. Figure 7 is a detail of the results, which indicates that only with larger numbers of zones does it become beneficial to utilize greater numbers of zones across the member thickness.

4.2.2 *Moments*

With solid elements such as *FLAC* zones, bending moments in the structural member need to be calculated from the internal axial stresses. The bending moments are defined by considering the distribution of stress across the section. Moments are calculated by summating the product of axial stress multiplied by offset from the member centerline. Because of the discretization of the zones, axial stress varies in a step-wise manner rather than continuously through the section and this adds difficulty to the accurate derivation of the bending moments.

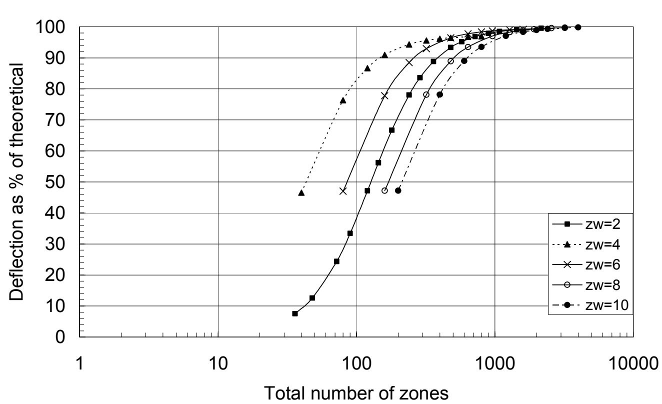

Figure 6. Results for varying numbers of zones (zw) across the member width.

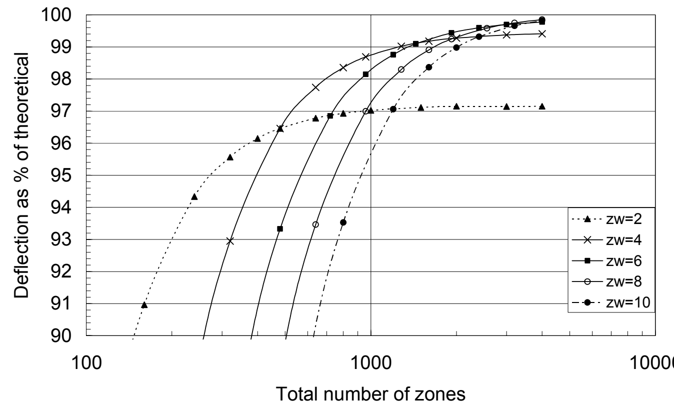

Figure 7. Detail of results for varying numbers of zones (zw) across the member width.

Figure 8 shows three ways to calculate the bending moments in the section. The first takes the axial stress for each zone and calculates a force from the stress by multiplying by the zone width and then calculates the moment by assuming the force acts at the mid-point of the zone. The second method assumes that the stress acts at the center of the zone but varies linearly between each zone centroid. An applied force is calculated by averaging the centroid stresses and multiplying by the distance between each centroid. The force is taken to act at the relevant point in the parallelogram.

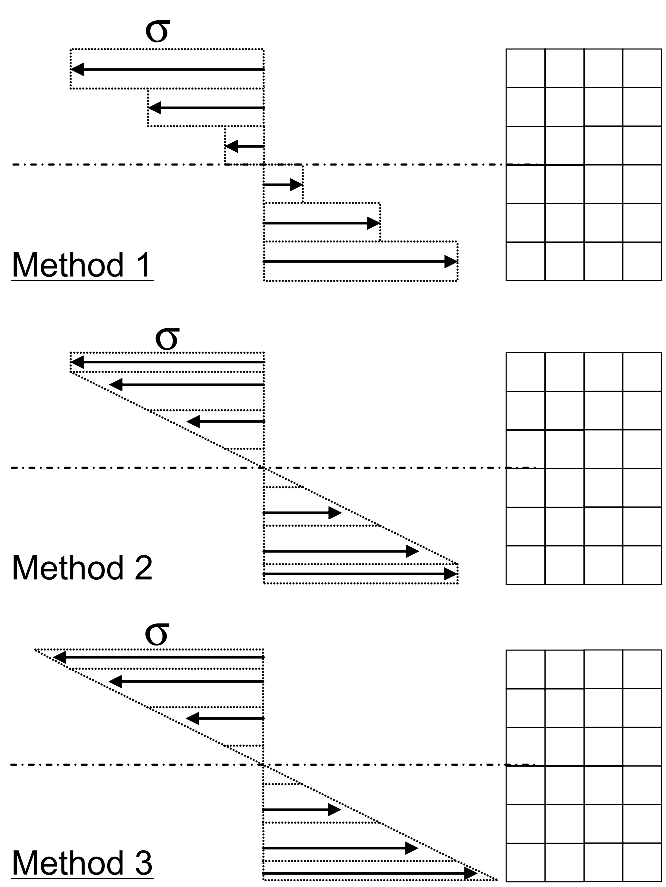

Figure 8. Different methods of calculating bending moments.

The stresses at the edges of the member are taken to be constant from the centroid of the zone to the edge of the structural member. The third method differs from the second only by assuming that the stress can be projected to the edge of the member in accordance with the variation of stress in the remainder of the section.

Figure 9 shows the calculated bending moment using the first method for the set of analyses with an aspect ratio of unity and a varying number of zones across the cantilever thickness. The accuracy of the bending moment prediction increases with the number of zones across the thickness of the member. Obviously using this method, where only one zone across the thickness is used there is no stress difference across the section and therefore no apparent bending moment. By comparison with Figure 2 it is apparent that using the first method, the bending moment is not predicted as accurately as the deflection.

The fact that a cantilever possessed bending stiffness even when only one zone thick is due to the subdivision of the quadrilateral zones into triangles. The subzone stresses cannot be output directly from *FLAC*, but a subroutine was written to extract the values (Itasca 2005). This gave a total of two stresses per zone and therefore twice as many values across the width of the member. The bending moments calculated from the same set of analyses and from the subzone stresses are also shown in Figure 9. The results derived from the subzone stresses are a significant improvement upon the results from the zone stresses.

To seek an improvement in the prediction of bending moments the second and third methods of deriving bending moments were used on the same set of analyses as those shown in Figure 9. The results are shown in Figure 10. The second method gave essentially identical results to that using the first method, but a significant improvement is obtained using the third method. This shows the importance of the distribution of stress at the extreme fibers of the member.

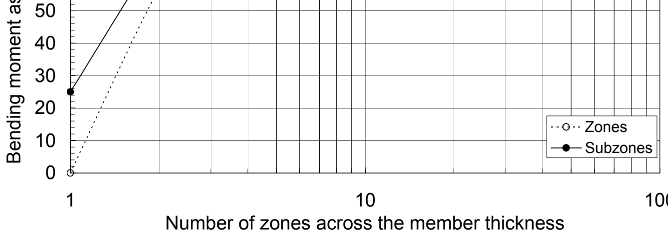

Figure 9. Bending moments derived from zone and subzone stresses.

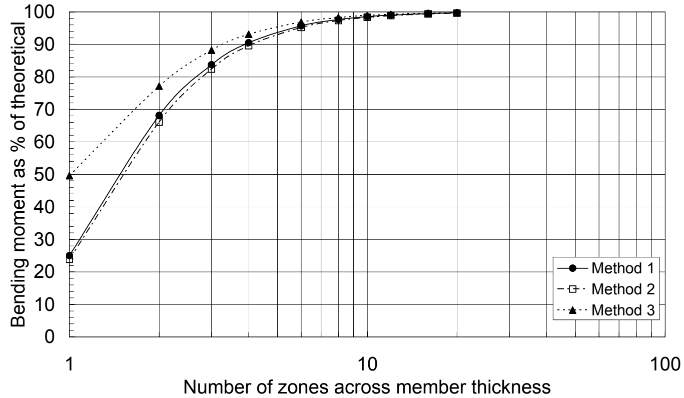

Figure 10. Alternative methods of deriving bending moments.

Figure 11 plots the accuracy of the deflection prediction plotted against the accuracy of the bending moment prediction using the third method. The correlation is excellent with the accuracy being the same for both. The few obvious deviations from perfect correlation occur when there is only one zone across the thickness of the member.

4.3 *Hollow Cylinder analysis using FLAC zones*

To investigate the performance of a ring in bending a similar but less comprehensive series of analyses were run as carried out for the cantilever. The geometry of the cylinder is such that the slenderness ratio of the section is approximately 31.4, which means that the cylinder is more slender than the cantilever.

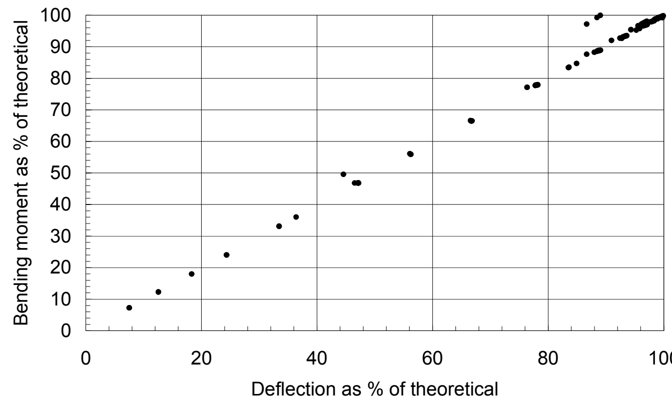

Figure 11. Comparison of predicted deflections and bending moments.

4.3.1 *Displacements*

Figure 12 shows the displacements of a point on the circumference of the hollow cylinder at 90º to the point of load application for a series of analyses with 6 zones across the cylinder thickness and with a varying number of zones around the circumference. The variation in predicted displacement shows a similar pattern to that for the cantilever with an increasing degree of accuracy as the number of zones increases. Also shown in Figure 12 are the equivalent results for the cantilever corrected for the difference in slenderness of the member. The fit of the cantilever data to the cylinder data is nearly perfect indicating that the bending performance of the zones is not dependent upon the shape of the member analyzed.

Figure 13 shows the results of a series of analyses with varying numbers of zones across the thickness of the hollow cylinder. Again the analyses show a similar pattern of results to that of the cantilever (cf Fig. 7) with a greater degree of accuracy achieved for small total numbers of zones by using a small number of zones across the cylinder thickness, whereas for a larger number of zones greater accuracy is achieved with more zones across the cylinder thickness.

Results again indicate that the optimum aspect ratio for the zones is one-third the length in the direction of bending to that across the thickness of the member.

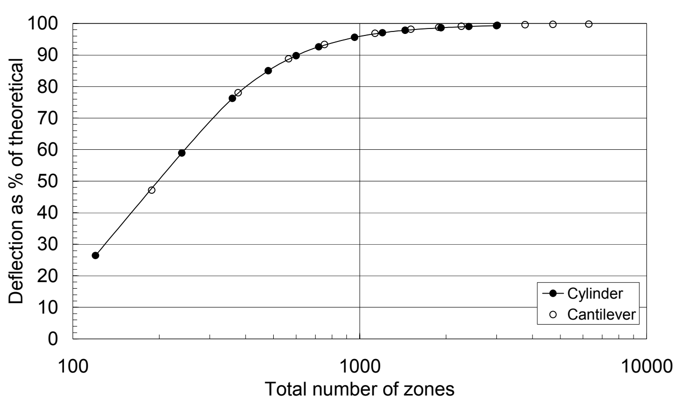

Figure 12. Results for the hollow cylinder with 6 zones across the member width.

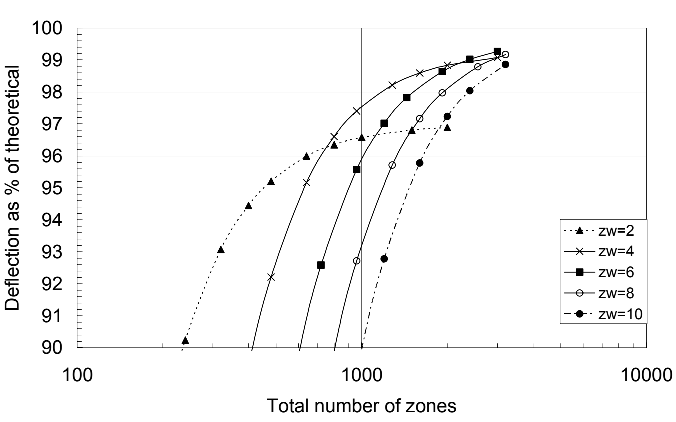

Figure 13. Results for the hollow cylinder with varying numbers of zones (zw) across the member width.

4.3.2 *Bending Moments*

The predicted bending moments for the hollow cylinder show a similar trend to those for the cantilever. A greater degree of accuracy is achieved by using subzone stresses and by projecting the axial stress variation across the member to the extreme fiber. By using this approach the bending moment can be predicted to the same degree of accuracy as the displacement. A nearly identical plot to Figure 11 can be generated for the results of the hollow cylinder analyses.

5 DISCUSSION

The results discussed above clearly show that the deflection of a beam can be accurately represented using *FLAC* zones. This is useful when it is desired to model the thickness of the structural member or when non-linear elastic or elastoplastic response of the structural material is to be modeled. A trade-off needs to be made between the practical limit to the total number of elements available for use in modeling the member and the accuracy that can be achieved. The analyses described above indicate that an accuracy of better than 90% can be achieved in predicting the deflection or the bending moment in a member with a few hundred zones.

The analyses also show that very accurate predictions of deflection and bending moment can be achieved with relatively few elements across the thickness of the structural member. All the analyses described above considered linear elasticity. It is suggested that where non-linear elastic or plastic behavior of the member is being considered it would be preferable to use a greater number of elements across the thickness of the element to ensure that the non-linear stress distribution is modeled accurately.

It is common practice in numerical modeling to combine use of solid elements with a beam element as a signal member. Typically the beam element would be given properties equal to 0.1% or less of the axial and bending stiffness of the actual member. This would introduce a negligible error to the prediction of the deflection and bending moment but would facilitate derivation of axial and shear forces and bending moments from the structural member.

6 CONCLUSIONS

The analyses described above indicate that solid elements can be used successfully to model the bending behavior of structural members. The deflection and bending moment in the section are always underestimated, but results within five percent of the theoretical answer can be achieved with a few hundred zones. Surprisingly, reasonably accurate results can be achieved with very few elements across the thickness of the structural member and more accurate results are often achieved with fewer numbers of elements across the section if the total number of elements is limited. The most accurate results are obtained where the aspect ratio of the zones is about 0.3; that is the dimension of the zone in the direction perpendicular to the axis of the member is approximately three times that parallel to the axis of the member.

Reasonably accurate predictions of bending moment can be achieved by using zone stresses except where there are very few zones across the width of the member. More accurate results can be achieved by using subzone stresses and projecting these stresses to the extreme fiber of the structural member. It is suggested that where non-linear elastic or elastoplastic behavior of the structural material is modeled a greater number of zones across the width of the member will probably be necessary to achieve an adequate prediction of the structural response.

REFERENCE

Itasca Consulting Group, Inc. 2005. *FLAC – Fast Lagrangian Analysis of Continua, Version 5.0 User's Manual*. Minneapolis: Itasca.