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A CENTURY OF LEADERSHIP IN
MATHEMATICS AND ITS TEACHING

The *Journal of Mathematics Education at Teachers College* is a publication of the
Program in Mathematics and Education at Teachers College
Columbia University in the City of New York.

Dr. Robert Taylor was selected by the Teachers College sponsored Teachers for East Africa program to teach mathematics of Uganda's Makerere University. He returned to TC as an instructor in the Department of Mathematics, Statistics, and Computing in Education where he developed an innovative programming language (FPL) intended to introduce educators to the then-new field of computer programming. His seminal work entitled *Computers: Tutor, Tool, Tutee* led to leadership in the new field of computers in education. Dr. Taylor completed 33 years as a member of the Teachers College faculty in 2009.

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Aims and Scope

The *JMETC* is a re-creation of an earlier publication by the Teachers College Columbia University Program in Mathematics. As a peer-reviewed, semi-annual journal, it is intended to provide dissemination opportunities for writers of practice-based or research contributions to the general field of mathematics education. Each issue of the *JMETC* will focus upon an educational theme. The themes planned for the 2012 Spring-Summer and 2012 Fall-Winter issues are: *Evaluation* and *Equity*, respectively.

JMETC readers are educators from pre K-12 through college and university levels, and from many different disciplines and job positions—teachers, principals, superintendents, professors of education, and other leaders in education. Articles to appear in the *JMETC* include research reports, commentaries on practice, historical analyses and responses to issues and recommendations of professional interest.

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TABLE OF CONTENTS

Foreword

Preface

Articles

TABLE OF CONTENTS, continued

Other

Journal of Mathematics Education at Teachers College
Fall–Winter 2011, Volume 2

Randomized Control Trials on the Dynamic Geometry Approach*

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The project reported here is conducting repeated randomized control trials of an approach to high school geometry that utilizes Dynamic Geometry (DG) software to supplement ordinary instructional practices. It compares effects of that intervention with standard instruction that does not make use of computer drawing/exploration tools. The basic hypothesis of the study is that use of DG software to engage students in constructing mathematical ideas through active investigations results in better geometry learning for most students. The study tests that hypothesis by assessing student learning in classrooms randomly assigned to treatment and control groups. The project is currently in its second year, and has just completed its first implementation of the DG approach, related data collection, and some initial data analysis. HLM models showed that the treatment group significantly outperformed the control group in geometry achievement. While the effect of the DG treatment is of moderate size for all participating students the largest effect size occurs with students in Regular Geometry classes.

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Keywords: Dynamic Geometry, Assessment, HLM models.

Dynamic Geometry (DG) represents geometry explorations performed with interactive computer software such as the Geometer's Sketchpad (Jackiw, 2009) or Cabri Geometry (Texas Instruments, 1994), which has been available since the early 1990's. As Goldenberg & Cuoco (1998) point out, "The term *dynamic geometry*...has quickly entered the literature as a generic term due to its aptness at characterizing the feature that distinguished DG from other geometry software: the continuous real-time transformation often called 'dragging.' This feature allows users, after a construction is made, to move certain elements of a drawing freely and to observe other elements respond dynamically to the altered conditions" (p. 351). Teachers use DG software to help students construct mathematical ideas through active explorations and investigations such as dragging, measuring, observing, conjecturing, conjecture testing, reasoning, and proving. A four-year research project has been funded by the National Science Foundation to conduct repeated randomized control trials of this instructional approach to high school geometry, which is referred to as the DG approach here. This article describes the project and reports the initial findings from its year-2 study.

Dynamic Geometry Related Research

Along with the widespread use of DG software, many related research studies have been conducted. A relatively small group of researchers (e.g., Dixon, 1997; Gerretson, 2004; Myers, 2009) used experimental or quasi-experimental designs in their studies. Results obtained from the statistical analyses of these researchers suggested that students experiencing the dynamic geometry instructional environment significantly outperformed students experiencing a traditional environment on content measures of the concepts and skills taught during the experiments.

Most of the studies used qualitative research methods. Choi-Koh (1999) used a clinical interview procedure to examine a secondary school student's development of geometric thought. She identified four learning stages—intuitive, analytical, inductive, and deductive learning in terms of symbol, signal, and "implicatory" properties; and found that the use of active visualization with the dynamic software facilitated the progress from symbol to signal and then to implicatory character. In an exploratory, phenomenological study, Hannafin, Burruss, & Little (2001) noted two overarching themes: issues of power and learning. The teacher had difficulty relinquishing control of the learning environment, while students enjoyed their new freedom, worked hard, and expressed greater interest in the mathematics content. Through a constructivist teaching experiement, Jiang (2002) discovered that as they explored geometry problems with DG software, preservice teachers developed a new learning style—exploring problem situations through a learning process characterized by initial conjecture—investigation—more thoughtful conjecture—verification (or proof)—proof (or verification).

Vincent (2005) found that the motivating context and the dynamic visualization afforded by the DG learning environment fostered conjecturing and intense argumentation; and that the teacher's intervention was important in fostering the students' augmentations. Accascina and Rogora (2006) claimed that the use of DG software facilitated students' understanding by helping them create good concept images of three-dimensional objects. Hollebrands (2007) identified different purposes for which high school Honor Geometry students used dragging and measures, and found that these purposes seemed to be influenced by students' mathematical understandings, the types of abstractions they made, and the different strategies they used. Sinclair et al (2009) argued that as the very nature of dynamic mathematical representations, continuity and continuous change occurring over time offer different opportunities for students' narrative thinking, which the static diagrams and pictures are unable to provide. Building on the work of Arzarello, Olivero and other researchers, and as part of a more general qualitative study aimed at investigating cognitive processes during conjecture-generation in a DG environment, Baccaglini-Frank and Mariotti (2010) presented "a model describing some cognitive processes that can occur during the production of conjectures in dynamic geometry and that seem to be related to the use of specific dragging modalities" (p. 225) and used it to analyze students' explorations of open problems.

According to the research studies described above, if DG software is used effectively, it can make a significant difference to students' learning; when used as a cognitive tool, it can facilitate students' exploration and investigation activities, promote their conjecturing, verifying, explaining, and logical reasoning spirits and abilities, and enhance their conceptual understanding of important geometric ideas. However, almost all of the studies were either exploratory phenomenological studies that involved a small number of participants, or comparative studies that were conducted during a relatively short period of time (ranging from a week to less than one semester).

The qualitative studies are important since they can reveal students' actual, detailed learning processes. "In fact, good quantitative studies generally require a qualitative rationale" (American Statistical Association, 2007, p. 43). However, there is a need for education research designs using modern statistical methodologies "if the quality of education research is to meet the requirements that government policies and societal expectations are placing upon it" (American Statistical Association, p. 44). If one wants to find the convincing efficacy of DG, quantitative comparative studies are necessary (Schneider et al, 2007). To make sure that the changes observed in a dependent variable are caused by the intervention rather than by some extraneous factors, true experimental designs with randomized assignment to treatment and comparison conditions should be used whenever and wherever possible.

Van Hiele (1986) mentioned that it took nearly two years of continual education to have the students experience the intrinsic value of deduction. Research found that the use of DG software can save instructional time (Gray, 2008), but to find significant development of students' geometric thought such as having moved from one geometric thinking level up to the next higher level, a relatively long term of instruction (such as a full school year) is very much needed.

Based on these considerations, most (if not all) of the studies mentioned above need careful replication and amplification (Jones, 2005). DG software, though very widely used, has not been rigorously evaluated. The need for achieving a more thorough understanding of the power of DG software is clear.

A Four-Year Research Project Funded by NSF

The primary goal of this project is to investigate the efficacy of the DG approach on students' geometry learning over the course of a full school year. Based on the idea that effects of innovations like dynamic geometry are often greater in the second year of use than in the initial getting-acquainted first year, data collection opportunities are provided in two consecutive implementations of the dynamic geometry treatment. Thus, the general plan for the four-year project is as follows: Year 1: Preparation (All research instruments, recruitment of participants, professional development training and resource materials, etc.); Year 2: The first implementation of the DG treatment, and related data collection and initial data analysis; Year 3: The second implementation of the DG treatment, and related data collection and continued data analysis; and Year 4: Careful and detailed data analysis and reporting. The project is presently in progress during its year 2.

Theoretical Framework

The theoretical foundation of the DG approach and the theoretical framework of this research project consist of the constructivist perspective and the van Hiele model.

The constructivist perspective suggests that knowledge cannot be passively transmitted from one individual to another but is actively constructed by the learners themselves (Steffe & Cobb, 1988). The traditional approach to geometry instruction is teacher centered and based on definitions, theorems, and proofs, with little attention to whether students understand teacher's lecture. In contrast, the DG approach to geometry instruction is based on students' experimentation, observation, data recording, conjecturing, and proving. As Olive (1998) indicates, "Such an approach would give students the opportunity to engage in mathematics as mathematicians, not merely as passive recipients of someone else's mathematics knowledge. From a constructivist point of view, this is the only way children can learn mathematics" (p. 399).

Van Hiele (1986) postulated that students progressed through a sequence of five discrete thought levels in geometric reasoning. The five levels are: *1. Recognition*, *2. Analysis*, *3. Order*, *4. Deduction*, and *5. Rigor*. According to the van Hiele theory, the main reason the traditional geometry curriculum fails is that it is presented at a higher level than those of the students (de Villiers, 1999). The DG learning environment is a suitable environment in which students can explore geometry at their geometric thinking levels. Teachers can prepare activities that match students' current van Hiele levels so that students can continue their explorations with little help from their teachers and make the transition to the next higher level.

Research Questions

The project seeks to answer the following research questions: 1) How do students taught in a DG oriented instructional environment perform in comparison with students in the control condition on measures of a geometry test and a conjecturing-proving test? 2) How does the DG intervention affect student beliefs about the nature of geometry and their beliefs about the nature of mathematics in general? 3) How does the DG intervention contribute to narrowing the achievement gap between students receiving free or reduced price lunch and other students? 4) How is students' learning related to the fidelity and intensity with which the teachers implement the DG approach in their classrooms? and 5) What characterizes the different learning communities in the experimental and control classes?

Sample

The population from which the participants of this efficacy trial were sampled is the geometry teachers and their students at high schools in Central Texas School Districts in which 50% or more of the students are eligible for free or reduced price lunch. The rationale for us to focus on this group of students is twofold: 1) It is very important to study how economically disadvantaged students learn mathematics, especially in Central Texas where there are high percentages of this group of students; 2) According to Riordan and Noyce (2001), the two strongest predictors of school performance are the baseline mean school score on the previous statewide test and percent of students receiving free or reduced price lunch.

For determining the sample size, a power analysis was conducted. We used the Optimal Design software (Spybrooke, Raudenbush, Liu, & Congdon, 2006) in order to determine the optimal number of classrooms to be included in the study. Having set appropriate parameters we ran the Optimal Design algorithm, which indicated that the optimal number of classrooms would be 30 for one treatment group, and 60 in total. Taking a 20% attrition rate into consideration, 76 classes (76 – 76*20% = 60) were used for the study. With help from the school districts in Central Texas, 76 geometry teachers were selected from those who applied to the project with support from their principals.

Research Design

The research study follows a mixed methods, multi-site randomized cluster design. Random assignment was used, with teachers as the unit of randomization. The 76 teachers selected were randomly assigned to two groups—the experimental group and the control group. For schools where the selected teachers teach more than one class, only one class per teacher was randomly selected to participate in the study. Therefore each teacher is represented in the study with measurements from only one classroom of students, and the classroom and teacher unit of analysis overlap, yielding the design where the students are nested within teachers/classrooms, which are nested within schools.

In this project, the DG software used is mainly the Geometer's Sketchpad (GSP). For some 3-D activities, the Cabri 3-D software is used with GSP. To help users take full advantage of the power of GSP, we provided them with many well-designed GSP-based learning activities. We chose the activities that most fit Texas geometry curriculum from sources such as GSP curriculum modules published by the Key Curriculum Press and learning activities that the project staff members have created during their long careers in using GSP with high school mathematics teachers and students.

The teachers randomly assigned to the experimental group have been participating in a GSP workshop, of which the main part was conducted in the summer of project year 1. In the workshop, these teachers have opportunities to become familiar with GSP, use GSP to investigate problem situations, and explore ways in which they could use GSP with their students. The follow-up sessions of the summer institute consist of six Saturday half-day sessions during the 2010-2011 school year, a whole-day session in summer 2011, and three more Saturday half-day sessions during the 2011-2012 school year.

The control group is a "business-as-usual" group. The teachers in this group teach as before. Specifically they teach in the paper-and-pencil manner, involving use of manipulatives, compass, protractor, and ruler. They also participate in a workshop, in which the same mathematical content taught in the GSP workshop is introduced to them, in a non-GSP environment.

Measures and Data Collection

Student-level Measures

The instruments used to assess students' geometry learning during project year 2 included a pretest—Entering Geometry Test (ENT) used by Usiskin (1982) and his research team at University of Chicago, and a geometry posttest (XGT), which was developed by the project team through selecting questions from released items of the California Standards Test: Geometry (CSTG).

The reason of using ENT for the pretest is that this test has been used by numerous studies on students' geometry learning over the past 29 years, and has been considered as a good and easy-to-administer multiple-choice geometry test to assess students' geometric background before entering a full-year high school geometry course. ENT consists of 20 multiple choice items each with 4 possible responses and has a reliability of α=.77.

Released items from CSTG were chosen for XGT because all CSTG questions have been evaluated by committees of content experts, including teachers and administrators, to ensure their appropriateness for measuring the California academic content standards in Geometry. In addition to their content validity, all items were reviewed and approved to ensure their adherence to the principles of fairness and to ensure no bias exists with respect to characteristics such as gender, ethnicity, and language (California Department of Education, 2009).

When developing XGT, the project team worked carefully in selecting items from CSTG that are closely aligned with Texas geometry standards and the geometry curricula of the participating school districts. Thirty items were chosen and pilot-tested in the non-project classes at a participating high school. Based on the pilot-test results and the feedback of the master teachers (who are high school geometry curriculum and instruction experts working for the project), five items were removed. The final version for XGT has 25 multiple-choice items. Based upon the pilot results, the instrument has high reliability (α= .875). Factor analysis provided strong evidence that XGT corresponded to uni-dimensional scale. Item Response Theory (IRT) scoring routines were applied to the scored posttest to generate examinee 'abilities' and item parameters, which allowed us to determine that collectively the items included on the posttest provided a range of performance that holistically represented a well-functioning instrument. The adherence of the data to the three-parameter logistic IRT model provided some evidence for the assessment's construct validity.

For all other measures to be described below, psychometric properties were also examined, and their Cronbach's Alpha statistical values are within the acceptable ranges for reliability. More psychometric analyses were examined for some of the instruments and provided evidence supporting the validity of each.

Student-level measures also included a Conjecturing-Proving Test and a student belief questionnaire. Both were developed by the project team. The Conjecturing-Proving Test was used as a pilot test at the end of 2010-2011 school year, and the data will be analyzed mainly for establishing the validity and reliability of the measure. The student belief questionnaire was developed to measure student beliefs about the nature of geometry and their beliefs about the nature of mathematics in general, and administered with the pretest and the posttest. It was adapted from the mathematics version of the Views about Sciences Survey (VAMS) (Halloun, 1996). The purpose of VAMS is to assess student views about knowing and learning mathematics and assess their relation to student understanding. Because VAMS measures students' views of mathematics more generally, the project team paid close attention to incorporating the critical features of the DG approach in the adaptation process.

Teacher-level Measures

To determine how to capture the critical features of the DG approach, we have designed measures of fidelity of implementation—both a DG implementation questionnaire and a classroom observation instrument. The DG implementation questionnaire was adapted from a teacher questionnaire developed by the University of Chicago researchers (Dr. Jeanne Century and her colleagues) in an NSF funded project. We made significant changes and additions to address the extent to which the DG approach is implemented by the teachers. The current version of the questionnaire consists of items designed to gather information on aspects such as how many times per week the students worked in a computer lab with GSP installed on each computer; what features of GSP were used over the past month; and how the use of GSP has influenced the way the teacher plan and implement instruction. This questionnaire has been administered with the teachers in the experimental group six times during project year 2. An equivalent but different version of the questionnaire has been administered with the control group teachers to examine the degree to which they faithfully implement the business-as-usual approach.

The classroom observation instrument—the Geometry Teaching Observation Protocol (GTOP) has been developed by adapting the Reformed Teaching Observation Protocol (Piburn et al, 2000) based on the critical features of the DG approach. GTOP has been used for both groups of teachers. The scores provide data to compare the teachers' teaching styles and strategies. During project year 2, we have observed classes of eight teachers in the experimental group and eight in the control group. These teachers were chosen to ensure diversity in their gender, level of class (Regular, Pre-AP, or Middle School) and years of teaching experience. There are at least two observers for observing each class, and each teacher has been observed four or five times. The four or five observations were evenly distributed throughout the entire school year. The purpose is to get as comprehensive a picture of the teacher's teaching practice as possible.

To probe more deeply into the teachers' and students' thinking processes, and to gather evidence about the range and variability of students' development of the most important abilities that the DG approach fosters, this study uses in-depth interviews of selected students and teachers to collect qualitative data. Interview protocols have been designed and used for the interviews. During project year 2, eight students and six teachers were selected for the interviews, from a diverse set of most engaged students and teachers involved in the project. We have completed three interviews for each of the interviewees except for one student (two interviews) and one teacher (one interview). All interviews were videotaped and the taped footage will be transcribed.

All instruments mentioned above (with necessary changes informed by the implementation feedback) will be used again during project year 3.

Data Analysis

The project team has completed the first implementation of the DG approach and related data collection. Some initial data analysis (the analysis on the geometry pretest and posttest data and the psychometric analysis on the project developed instruments) has been conducted. More thorough analysis of the collected data is still on going and will be conducted during project year 3. The analysis of the geometry pretest and posttest data is reported below.

Two-level hierarchical linear modeling (HLM) was employed to model the impact of the use of the Dynamic Geometry approach on student achievement while taking into account the nested structure of the data (i.e. students nested within teachers' classrooms). The models were analyzed once with student pretest (ENT) scores included as a covariate and once without the pretest scores. The rationale for excluding the ENT scores is twofold. First of all, teachers were randomly assigned to each of the treatment groups, so pretest control is not required to determine accurate estimates of the treatment effect. In fact, a separate analysis of the pretest, not presented here, showed no significant difference (p = .724) between the two treatment groups. Secondly, the subsample of students with matched pretest and posttest scores is smaller than the sample of students taking just the posttest. This is due to a variety of factors including the fact that some students were absent the day of the administration of either pretest or posttest and that some students changed classes during the school year. Additionally, in order to maintain privacy, each student was issued a special project ID only known to the student and their teacher. In some instances, students and/or teachers made errors in using the ID making the match impossible. The reduction of the sample was not uniform across the different types of students and classes and may introduce bias into the results.

Results

The sample of classrooms studied included three different levels of Geometry: Regular, Pre-AP and Middle School (middle school students taking Pre-AP Geometry). Since the classroom expectations and quality of the students in each of these levels are very different, the factor *Class Level* was included in each model. Additionally, the years of classroom experience of the teachers in the sample varied a lot, ranging from 0 years all the way up to 35 years. For this reason, the covariate *Years Exp* (number of years of classroom experience) was included in the models.

During the project year 1 professional development workshop, the participating teachers completed a demographic survey that included information about years of teaching experience, the level of the class chosen and gender. From our initial teacher sample (76 participants), six teachers did not compete project year 2 mainly due to either

Table 1. Summary Statistics for Years of Experience of Teachers by Treatment and Level

	DG			Control		
	n	*M*	*SD*	*n*	*M*	*SD*
Overall	33	7.00	7.18	31	6.48	8.29
Class Level						
Regular	20	6.44	7.89	19	5.63	6.72
Pre-AP	12	8.30	6.25	8	10.75	11.94
Middle School	1	4.00	NA	4	2.00	2.16

Table 2. Model 1: HLM Results without Pretest as a Covariate

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx d.f.	p-value
Intercept	90.60	6.323	14.328	88	.000
DG Effect	7.435	1.763	4.217	51	.000
Level					
Regular	-35.10	6.282	-5.587	88	.000
Pre-AP	-21.15	6.566	-3.221	83	.002
Level*Years Exp					
Regular*Years Exp	-.3241	.1587	-2.042	57	.046
Pre-AP * Years Exp	.4785	.1763	2.714	46	.009
M. School*Years Exp	.6477	1.947	0.333	91	.740

Note. XGT is the response variable.

family/health or job displacement reasons. An additional six teachers submitted incomplete posttest data or failed to submit the data. Therefore, 64 teachers submitted complete posttest data for analysis in the study. Among them, 33 are in the experimental group (DG group), and 31 are in the control group.

Table 1 shows the summary statistics for years of experience of these teachers by *Treatment* and *Class Level*. For each of the two HLM models described in Table 1, full factorial designs were explored and insignificant interactions were discarded. The final models are discussed below.

HLM Results without Pretest as a Covariate

Model 1, shown in Table 2, examines the differences in student outcomes between the DG group and the control group while accounting for years of teaching experience and level of the class. The DG group significantly outperformed the control group (p =.000, ES = .3327). As expected, level of the class was highly significant as well (p = .000). Due to the coding of variables, the intercept reflects the Middle School group performance. Examining the coefficients in Table 2 and the mean values in Table 3, we see that the Middle School group substantially outperformed the high school Pre-AP students, who in turn outperformed the students in Regular Geometry. In particular, controlling for experience of the teacher and treatment group, the Pre-AP students scored 21.15 points lower than the Middle School students, while the Regular students scored a full 35.1 points lower than the Middle School group. Though not indicated in Table 2, the difference between the Pre-AP and Regular students is statistically significant (p = .000). Interestingly, the effect of years of experience differed by level of the class as well. Experience had a positive effect on the two higher performing groups, but had a negative effect on the achievement of the students in Regular Geometry classes. However, the effect of experience in the middle school group was not significant and the size of the coefficients in all groups is somewhat small. For the Regular group an increase of 10 years of experience corresponded to a drop of 3.2 points on the XGT, while for the Pre-AP group a similar change in experience was associated with a 4.8 point increase. Compare this to the 7.4 point increase due to the DG effect. Note the main effect for *Years Exp* is not shown in Table 2. A model including this effect was considered, but it was insignificant.

Table 3 shows the summary statistics for each level of

Table 3. Summary Statistics for XGT by Treatment and Level

	DG			Control			
	n	*M*	*SD*	*n*	*M*	*SD*	*ES*
Overall	617	62.44	19.06	604	55.91	20.19	.33
Level of Class							
Regular	357	54.38	17.64	366	45.54	15.10	.55
Pre-AP	244	72.56	16.09	191	68.27	15.50	.27
Middle School	16	88.25	7.01	47	86.38	10.10	.19

Note. Sample includes all Post Test data.

class separately. The DG group outperformed the control group in each level, but the effect size was largest for the students in the Regular Geometry classes.

HLM Results with Pretest as a Covariate

Model 2, shown in Table 4, examines the effect of the DG intervention when taking into account Entering Geometry Test (*ENT)* as well as *Class Level* and *Years Exp*. To simplify interpretation of the other coefficients, *ENT* was centered by subtracting the overall mean. Due to complications with matching student codes, including *ENT* in the model reduces the student sample. Comparing the sample sizes listed in Tables 3 and 5, we see that loss in sample was largest for the Regular Geometry classes. At that level the sample dropped 30% from 723 to 508. Comparing the treatment groups, DG dropped 19% and the control group dropped by 27%. Given the results shown in Table 3 above, this sample reduction would be expected to diminish the size of the DG effect, since the DG had the largest effect for the students in the Regular Geometry group. Table 5 shows the summary statistics for each level of class in the smaller matched subsample. Comparing the means, we see the following pattern again: the DG group outperformed the control group in each level of Geometry and the effect was substantially larger at the Regular Geometry level. However, the effect sizes were smaller for each of the levels of Geometry than in the full sample reported in Table 3. As with Model 1, the results for Model 2 indicate the DG effect was strongly significant (p = .002). As expected, including *ENT* in the model reduced the size of the *Class Level* effect on student performance on *XGT*. However, even controlling for the pretest, compared with Middle School students, on average Pre-AP students scored 13.2 points lower (p =.049) and Regular students scored 20.1 points lower (p =.004). Consistent with the results from Model 1, teaching experience had a positive effect on the two higher performing groups, but had a negative effect on the achievement of the students in Regular Geometry classes. Once again, the effect of experience in the Middle School group was not significant. The size of the coefficient in the Middle School group was much larger in Model 2 than Model 1, but in neither case was it significant. For the other two groups, the coefficients were similar in value between the two models. In Model 2, an increase in 10 years of experience raised the scores 4.5 points for the Pre-AP group and decreased the scores by 4.1 points for the Regular group.

Discussion

Effect of the DG Treatment

Both data analysis models (HLM without pretest as a covariate and HLM with pretest as a covariate) showed that the Dynamic Geometry group significantly outperformed the Control group in geometry achievement. This project used random assignment to form the treatment and control groups. The teachers in the control group also attended a professional development workshop. The amount of instructional time spent on this regular workshop was the same as that for the GSP workshop offered to the DG group teachers. The purpose of holding this workshop was to address a confounding variable. With this comparable amount of professional development, if differences appear on the project's measures between the treatment and control groups, we are able to rule out the possibility that the professional development activities can account for them rather than the DG learning environment. This true control group, in addition to random assignment, provides strong evidence to support the finding that the DG approach did make a difference—it did cause the improved geometry achievement observed in the study. In the first efficacy study on the DG approach at a moderately large scale in the nation, this finding is a noteworthy contribution to the field of mathematics education.

While the effect of the DG treatment is significant and of moderate size for all participating students, the largest effect size occurs with Regular Geometry students. There are many factors that could contribute to this. First, dynamic constructions offer stronger visualization than static drawings (Laborde, 1998). As bringing a strong visual component to mathematics is key to understanding for all students (Archavi, 2003; Clements & Battista, 1992), it is indispensible for students challenged by language learning and cognitive issues (Reimer & Moyer, 2005; Key Curriculum Press, 2009). Students in Regular classes are those of low to average academic abilities and are more likely (than Pre-AP students) from the group of special education (learning disabilities), at risk, or economically disadvantaged students. They require more visual and more innovative activities to help them develop their conceptual understanding and mathematical reasoning ability. Therefore, they could benefit more by the stronger visualization brought by DG tools than those in Pre-AP classes. Secondly, in comparison to Pre-AP students who are better prepared and more motivated, Regular students need more motivation and engagement to spark their learning interest. DG software is helpful to teachers as they design environments and contexts that address the motivational needs of the students. For instance, the transformations available in GSP and its animation feature, as well as the ease of using buttons make the software a wonderful tool to design and implement various engaging environments for learning mathematics. The DG engagement effect would be greater to Regular students than to Pre-AP students. In addition, from a state curriculum standpoint, Pre-AP classes work on some logic and proofs whereas Regular classes do not. Since the DG approach focuses on conjecturing, reasoning, and proving, Regular DG students are exposed

Table 4. Model 2: HLM Results with Pretest as a Covariate

Fixed Effect	Coefficient	Standard Error	T-ratio	Approx d.f.	p-value
Intercept	79.05	6.119	12.919	28	.000
DG Effect	5.62	1.678	3.352	43	.002
Level					
Regular	-20.10	6.119	-3.284	21	.004
Pre-AP	-13.23	6.293	-2.102	20	.049
Level*Years Exp					
Regular*Years Exp	-.4137	.1516	-2.729	53	.009
Pre-AP * Years Exp	.4451	.1617	2.753	22	.012
M. School*Years Exp	1.811	1.868	0.969	20	.344
ENT (Mean Centered)	.4114	.0366	11.237	47	.000

Note. XGT is the response variable.

to these important activities and abilities, and would do much better than Regular "control" students who have not seen these. In Pre-AP classes, the students see proofs whether they are in the DG group or not, so the difference would be smaller.

Effect of Teaching Experience

One unusual result of the HLM analysis of the data is the effect of the teaching experience on student achievement on the geometry posttest. In both models discussed above, greater experience of the teacher had a positive impact on achievement for the Pre-AP level and a negative impact for the Regular level. Further research is necessary to fully understand why this occurred. One possible explanation is the expectations of teachers for the two different levels. Based on years of interaction with middle school and high school teachers, the project researchers have noticed the tendency for experienced teachers to have very different and very rigid beliefs about the ability of students to achieve. Pre-AP classes are composed of mostly middle to high achieving students, and hence teachers have high expectations of what those students can learn. Meanwhile, since students in Regular classes have a record of low to middle achievement, teachers have very low expectations of what students can learn. It is not uncommon to hear an experienced teacher react to some innovative teaching activity by saying, "Well this might work with my Pre-AP students, but my Regular kids won't get it. With the Regular kids we need to focus on the basics." Our experience has been that more novice teachers are willing to believe that all students can learn.

Future Work

With the results described above, we have partially answered the first research question of the project. As many researchers (e.g., Artigue, 2000) have pointed out, the issue is not only which is best, but also how is the DG approach different—what are the epistemic differences? Because of this, the project has included a strong qualitative component. We will further analyze the data (both quantitative and qualitative) that have been collected. During the second implementation of the DG approach

Table 5. Summary Statistics for XGT by Treatment and Level

	DG			Control			
	n	*M*	*SD*	*n*	*M*	*SD*	*ES*
Overall	501	62.36	19.26	438	59.12	20.40	.16
Level of Class							
Regular	276	54.19	17.64	232	46.81	15.10	.45
Pre-AP	210	71.26	16.09	163	69.28	15.50	.13
Middle School	15	88.27	7.01	43	87.07	10.10	.13

Note. Includes only posttest data for subsample with matching pretest results that will be conducted in project year 3, we will continue to focus on collecting high-quality quantitative and qualitative data and analyzing the data.

To thoroughly address the first research question and answer the second research question, the principal method of data analysis will continue to involve fitting two- and three-level Hierarchical Linear Models to the data. This multilevel approach also enables us to address research question 3 and examine the potential treatment effect with respect to the ethnic, socio-economic, and linguistic characteristics of the students and the demographic composition of schools.

Qualitative data analysis will use the constant comparative approach (Glaser & Strauss, 1967; Grove, 1988) to answer research question 5. Constant comparison involves analyzing and interpreting data during and after data collection. By systematically analyzing data during its collection, the researchers can make appropriate adjustments to look for evidence that conflicts with emerging theories, as well as evidence that might support those theories. This process reduces the likelihood that the researchers' theories are based on personal biases.

The quantitative data analysis and the qualitative data analysis reported above, as a whole, will answer research question 4 that relates to implementation fidelity.

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