Teaching Algebra with The Geometer’s Sketchpad®
UCB Extension X305.5 (2 semester units in Education)

COURSE DESCRIPTION
This is a professional level, moderated, online course in the use of The Geometer’s Sketchpad software for teaching mathematics, particularly beginning and advanced algebra. In addition to instruction in how to use the software, the course will offer participants pedagogical guidance on how to implement the use of Sketchpad® in their classrooms and promote a discussion of how dynamic mathematics software affects the teaching and learning of algebra. The course runs for six weeks with a scheduled start and end date and is structured into six weeklong units. While participants have flexibility within each week, the course is synchronous, meaning that participants are expected to begin and complete the activities for each week during the week they are assigned. This course is primarily intended for teachers.

COURSE OBJECTIVES
After participants complete this course, they will be comfortable using Sketchpad both as an investigation tool and as a demonstration tool. Participants will be able to:

- Illustrate slope and other graphing concepts in the coordinate plane
- Create dynamic graphs using a variety of techniques
- Explore functional relationships within geometric figures
- Use dynamic environments to explore properties of numbers and operations
- Construct alternative representations of functional relationships that can lead to deeper understanding
- Explore sequences and functions through iteration
- Create demonstrations that involve animation and action buttons
- Appreciate the pedagogical implications of exploring algebra in a dynamic environment

INTENDED AUDIENCE
This course is intended for secondary and middle school mathematics teachers and instructors of pre-service teachers. Although the course content focuses on algebra concepts, any current or prospective teacher can learn how to use Sketchpad to supplement the middle or high school mathematics curriculum.

PREREQUISITES
Participants should be familiar with middle school and high school algebra concepts. They should also be comfortable using computers and must have access to the Internet and The Geometer’s Sketchpad Version 5.
METHODS OF INSTRUCTION
Each week follows the same structure in which participants complete these activities:

- Interact with a dynamic sketch that introduces the week’s mathematical focus
- Watch three videos (approximately 5 to 10 minutes long each): an interview with a Sketchpad developer or teacher educator, an interview with a classroom teacher who uses Sketchpad, and a tutorial of Sketchpad features introduced that week
- Download six or seven PDF files of Sketchpad activities and any associated sketches
- Complete the activities offline using Sketchpad
- Participate in an asynchronous discussion forum
- Complete a project using Sketchpad
- Reflect by responding to specific prompts in an online journal

Some weeks include optional activities or videos for enrichment or additional support.

GRADE BREAKDOWN
In order to receive credit for the course, participants must complete all weekly activities, turn in all six projects, respond to all six journal prompts, and participate in the class discussion forum at least twice each week. Assessment is project-based and all six weeks are weighted equally.

The two units offered for this course are based upon the expectation that each week will require about five hours of work (at least two hours for online activities—visual media, discussion forums, and the journal—and at least three hours for offline Sketchpad activities and the project).

Participants will do a weekly activity check that determines their score for working on Sketchpad activities. They will also receive a score for participating in the discussion forum. The moderator will evaluate and provide feedback for projects and journal entries. Grades will be assigned on a standard percent scale based on the following breakdown:

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sketchpad activities</td>
<td>20%</td>
</tr>
<tr>
<td>Discussion forums</td>
<td>20%</td>
</tr>
<tr>
<td>Journal entries</td>
<td>20%</td>
</tr>
<tr>
<td>Weekly projects</td>
<td>40%</td>
</tr>
</tbody>
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REQUIRED TEXTS AND MATERIALS
All written materials are provided as PDF files. Participants are not required to purchase any books or materials other than The Geometer's Sketchpad software. Participants may reproduce any materials provided in this course to use with their own students. Source of PDF files is Sketchpad LessonLink online subscription service (2009, Key Curriculum Press).
SESSION-BY-SESSION SUMMARY

Week 1: Points and Lines
Using Sketchpad's tools and measurement capabilities, participants explore points, lines, and slope in the coordinate plane.

Visual Media:
- “A Formula for Slope” JavaSketch
- Interview with Nick Jackiw, “Moving between Shape and Number”
- Interview with Jennifer North Morris, “The Slope Game”
- Video demonstration with Janet Bowers, “Points and Lines”

Activities:
- Points Lining Up in the Plane
- Rate of Change
- The Slope of a Line
- The Slope Game
- The Slope-Intercept Form of a Line
- Solving Systems of Equations
- Slopes of Parallel and Perpendicular Lines

Slope Presentation Project:
Create a demonstration sketch that illustrates slope and the calculation of slope.

Journal Prompt:
Participants describe how to construct lines with given characteristics, and they reflect on how the activities and discussions impacted their thinking about teaching slope.

Week 2: Functional Relationships
Participants learn to create robust constructions of geometric objects and use these constructions to explore functional relationships with geometric figures.

Visual Media:
- “Fixed Perimeter” JavaSketch
- Interview with Nick Jackiw, “Functional Dependency”
- Interview with Rachel Chou, “Graphing for a Purpose”
- Video demonstration with Janet Bowers, “Functional Relationships”

Activities:
- Old-Style Plotting
- Circumference Function
- Functions in a Triangle
- Direct Variation
- Inverse Variation
- Functional Geometry

Functional Geometry Project:
Construct a rectangle with a fixed perimeter and create several representations of the functional relationship between a side length and the area.

Journal Prompt:
Participants choose one of the functional geometry activities and describe the prerequisite knowledge needed to do the activity and the algebraic concepts addressed in the activity.
Week 3: Families of Functions
Participants learn several techniques for constructing dynamic families of functions, and explore quadratic, exponential, and linear functions through dynamic constructions.

Visual Media:
- “Sine Transformations” JavaSketch
- Interview with Nick Jackiw, “Building Models”
- Interview with Nathalie Sinclair, “Teaching Sketchpad to Calculate Slope”
- Video demonstration with Janet Bowers, “Families of Graphs”

Activities:
- Graphing Quadratic Functions
- Exponential Functions
- The Point-Slope Form of a Line
- Exploring Parabolas in Vertex Form
- Quadratic Intercepts
- Translating Functions

Reading:
“Use of Letters in the Language of Algebra,” by Dick Stanley

Sine Challenge Project:
Create a sine function whose amplitude and period are controlled by an independent point.

Journal Prompt:
Discuss common student misconceptions about the use of letters in algebra, such as confusing coefficients and variables. Describe how varying a coefficient (to show a family of functions) is different from varying a variable.

Week 4: Number Properties
Participants use dynamic environments to explore properties of numbers and operations.

Visual Media:
- “Sum and Product” JavaSketch
- Interview with Nathalie Sinclair, “Taking Manipulatives Further”
- Interview with Steve Rasmussen and Nathalie Sinclair, “Raz’s Four Function Machine”
- Video demonstration with Janet Bowers, “Number Properties”

Activities:
- Squares and Square Roots
- Exponents
- Zero and Negative Exponents
- Raz’s Magic Multiplying Machine
- Mystery Machines
- Equivalent Expressions: The Border Problem
- The Product of Two Binomials

Four Operations Machine Project:
Construct a “machine” in Sketchpad that visually represents the sum, difference, product, and quotient of two values on the number line.

Journal Prompt:
Participants reflect on how the activities, videos, and discussions have influenced their thinking about how to teach properties of numbers and operations.
Week 5: Multiple Representations

Visual Media:
- “Dynagraphs” JavaSketch
- Interview with Nick Jackiw, “Objects in Motion”
- Interview with Steve Rasmussen and Nathalie Sinclair, “Mystery Machine”
- Video demonstration with Janet Bowers, “Multiple Representations”

Activities:
- Introducing Dynagraphs
- From Dynagraphs to Cartesian Graphs
- Approximating Solutions to Equations
- Solving Linear Equations by Jumping
- Solving Linear Equations by Balancing
- Trigonometry Tracers

Dynagraph Project:
Build dynagraphs to represent three or more functions.

Journal Prompt:
Participants are asked to reflect on the benefits and drawbacks of constructing their own sketches or manipulating prepared models.

Week 6: Dynamic Algebra: Iteration and More

Participants use iteration to explore ratios, functions, and sequences, and see how dynamic algebra can enrich the study of other algebra topics, such as parametric equations and linear programming.

Visual Media:
- “Sequences” JavaSketch
- Interview with Nick Jackiw, “How I Use Sketchpad”
- Interview with David Hallas, “Variables that Vary”
- Video demonstration with Janet Bowers, “Iteration”

Activities:
- Fractals and Ratios
- Functions Again and Again
- Generating Arithmetic and Geometric Sequences Numerically
- Quadratic Quandary: Finding the Vertex of a Parabola
- Modeling Linear Motion: An Ant’s Progress
- Linear Programming: Swans and Giraffes
- Box and Whiskers (optional)

Final Project:
Create a dynamic sketch with animation that can be used as a demonstration in an algebra class.

Journal Prompt:
Participants are asked to reflect on what dynamic algebra means to them and how the dynamic nature of Sketchpad can change the way students can learn and understand algebra.