A model of endogenous credit creation and a credit crunch

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The financial crisis has emphatically confirmed that the dominant neoclassical models of macroeconomics and finance are seriously at fault. The hyper-rational, functional finance markets and stable macroeconomy implied by these models are in stark contrast to the actual behaviour of financial markets and the macroeconomy in the last two years. Policy makers who relied upon these models were seriously misled, and quite rightly the credibility of economic theory has been widely called into question.

If economics is to meet that challenge with anything other than “Black Swan” excuses, we need models of the economy that replicate the actual nature of the economy, rather than models imposing *a priori* abstractions of “rational agents” and “general equilibrium”.

In this paper I argue that such models are surprisingly close at hand, but constructing them will require abandoning many of the abstractions that economists have to date employed in their model-building.

Chief amongst these are the use of equilibrium modeling techniques, and the treatment of the economy as a barter system. While these originated largely to simplify economic analysis, they are now both superfluous and major impediments to realistic modeling of the economy.

Equilibrium analysis

Equilibrium modeling is so pervasive in economics—and not only in neoclassical thought—that its origin is often forgotten, and its use frequently unquestioned. But a simple perusal of the classic texts shows that equilibrium modeling was employed only because no other method was then known. Jevons (1888) gives an apposite statement of this:

> “We must carefully distinguish, at the same time, between the Statics and Dynamics of this subject. The real condition of industry is one of perpetual motion and change. Commodities are being continually manufactured and exchanged and consumed. If we wished to have a complete solution of the problem in all its natural complexity, we should have to treat it as a problem of motion—a problem of dynamics. But it would surely be absurd to attempt the more difficult question when the more easy one is yet so imperfectly within our power.” Jevons (1888, IV.25)

The proposition that avowedly imperfect equilibrium methods are simpler than dynamic analysis ceased being true in the late 20th century. Dynamic modeling techniques are now widely available, and substantially easier to use than the arcane equilibrium tools developed by economists.

The Barter Delusion

Joan Robinson once remarked that Milton Friedman was like a magician who put a rabbit into a hat in full view of the audience, and then expected applause when he pulled it out again.[1] There is no better instance of this than his influential admonition against analyzing the dynamics of the nominal money stock:

> IT IS A COMMONPLACE of monetary theory that nothing is so unimportant as the quantity of money expressed in terms of the nominal monetary unit— dollars, or pounds, or pesos. Let the unit of account be changed from dollars to cents; that will multiply the quantity of money by 100, but have no other effect. Similarly, let the number of dollars in existence be multiplied by 100; that, too, will have no other essential effect, *provided that all other nominal magnitudes (prices of goods and services, and quantities of other assets and liabilities that are expressed in nominal terms) are also multiplied by 100*. Friedman (1969, p. 1; emphasis added)

Indeed, if there were a planet on which all nominal magnitudes *including debts* were perfectly adjusted when inflation or deflation occurred, this statement—and the neutrality of money it supports—could be true. Since Earth is not such a planet, nominal monetary values can and do matter, because they are the link between financial commitments entered into in the past and our capacity to service them today.

Endogenous Money

There is now a substantial empirical literature (largely involving Post Keynesian rather than neoclassical economists, though Kydland and Prescott (1990) is an exception) that shows that the *a priori* hypothesis of the neutrality of the nominal money supply is empirically false (Moore (1979); Moore (1983)). In particular, the classic causal mechanism of the "money multiplier" model of money creation, in which base money must be created before credit money, is the reverse of what is found in the actual data. Kydland and Prescott's conclusion emphasizes this, and also throws down a challenge for economic theory that has been largely ignored by neoclassical economists:

> The fact that the transaction component of real cash balances (M_1) moves contemporaneously with the cycle while the much larger nontransaction component (M_2) leads the cycle suggests that credit arrangements could play a significant role in future business cycle theory. *Introducing money and credit into growth theory in a way that accounts for the cyclical behavior of monetary as well as real aggregates is an important open problem in economics.* Kydland and Prescott (1990, p. 15. Emphasis added.)

The Post Keynesian "Circuitist" school of thought has risen to this challenge, and has developed the theory of endogenous money creation (Moore (1988); Rochon (1999); Graziani (2003)) in which economic activity is explicitly driven by private credit. One essential aspect of this approach is a reversal of the causal mechanism of credit money creation given by the conventional "money multiplier" model.

The money multiplier model begins with an injection of government-created "fiat" money *$F* in a fractional banking system with a reserve requirement *RR*. The sum *$F* is deposited in a bank account, from which the bank makes a loan *$F.(1-RR)*, which is in turn deposited in another bank account. An iterative process then follows till the total

[1] This remark was made in a speech to Sydney University economics students in 1974.

sum of money created equals *$F/RR*. In this model, “deposits create loans (with a time lag)”.

The endogenous money approach instead begins with a bank making a loan *$L* to a firm, and that loan simultaneously creates an equivalent deposit (any need to meet reserve requirements is fulfilled later: Moore (1983, p. 539); Holmes (1969, p. 73)).[2] The endogenously created credit money then fuels economic activity. In this model, “loans create deposits (instantaneously)”.

While this perspective on money creation has been empirically validated, attempts by Post Keynesian economists to turn this Wicksellian vision of a pure credit economy into a viable model have to date been stymied by their reliance upon equilibrium methods—including simple diagrams and simultaneous equations (see Graziani (1995, pp. 526-530); Bellofiore, Davanzati and Realfonzo (2000, p. 413)). The use of such inappropriate modeling techniques has led to a number of logical conundrums, including an inability to explain the existence of profits (Rochon (2005, p. 125)) and a belief that constant economic activity requires an ever-increasing supply of money.

These conundrums are easily solved by working with modern explicitly dynamic modeling techniques. The result is a monetary model of production in which nominal quantities—in particular, the rate of turnover of the nominal money supply—determine real economic activity.

A simple approach to financial dynamics

The core of this approach is an extremely simple method of constructing a dynamic model of financial flows. All flows are recorded in a table, where each column is an aggregate bank account for a specific sector of the economy, and each row represents a particular economic activity: the model is thus explicitly monetary. The entries in the table are then summed by a symbolic algebra program to produce a system of ordinary differential equations, which can be easily analysed and simulated in the same software.[3]

The program code is extremely simple (see Figure 1):

$$
\begin{aligned}
\text{SystemODEs}(x) :=\ & \text{Functions} \leftarrow \text{submatrix}(x, 2, 2, 1, \text{cols}(x) - 1) \\
& \text{Equations} \leftarrow \text{submatrix}(x, 3, \text{rows}(x) - 1, 1, \text{cols}(x) - 1) \\
& \text{for } i \in 0 .. \text{cols}(\text{Functions}) - 1 \\
& \quad E_i \leftarrow \frac{d}{dt}\text{Functions}_i = \sum \text{Equations}^{\langle i \rangle} \\
& \text{return } E
\end{aligned}
$$

Figure 1

The program takes a matrix like that shown in Figure 2:

[2] “In the real world banks extend credit, creating deposits in the process, and look for the reserves later.” (Holmes, 1969, p. 73).

[3] The program used here is *Mathcad* (www.ptc.com/products/mathcad), but the same system could be replicated in any of a host of modern programs., such as *Mathematica* (www.wolfram.com), *Matlab* (www.matlab.com), *Scilab* (www.scilab.org) and *Scientific Workplace* (http://www.mackichan.com/)

$$S_1 := \begin{pmatrix} \text{"Type"} & 1 & -1 & 0 \\ \text{"Name"} & \text{"Loan 1"} & \text{"Deposit 1"} & \text{"Deposit 2"} \\ \text{"Symbol"} & L_1(t) & D_1(t) & D_2(t) \\ \text{"Activity 1"} & 0 & -A & A \\ \text{"Activity 2"} & 0 & B & -B \end{pmatrix}$$

Figure 2

and returns a set of differential equations (Figure 3):

$$\text{SystemODEs}(S_1) \rightarrow \begin{pmatrix} \frac{d}{dt}L_1(t) = 0 \\ \frac{d}{dt}D_1(t) = B - A \\ \frac{d}{dt}D_2(t) = A - B \end{pmatrix}$$

Figure 3

The placeholder symbols *A*, *B*, etc. are then replaced by relevant expressions for financial flows, and simulated (or, where applicable, symbolically solved) using differential equation routines in the relevant software package.

Base Model—a single loan in a constant output economy

The basic Circuitist model is of a pure credit economy—with only private banking and no government sector—in which an initial stock of endogenous money is created via a loan of $\$\Lambda$ from the banking sector to the firm sector. This loan simultaneously creates a deposit of the same magnitude that enables the firm sector to hire workers to produce output. The economic flows generated in this system are:

1. The loan contract allows the bank to compound the outstanding debt;
2. The deposit receives interest from the bank;
3. The firm sector hires workers from the household sector;
4. Households receive interest payments from the banks; and
5. Households and banks purchase goods from the firm sector.

These flows are shown in the table in Figure 4, with at the moment only placeholders used to indicate them.[4]

[4] Only the second element of the flows (B) requires some explanation: the payment of interest by the firm sector to the banking sector involves a transfer of money from the firm sector's deposit account F_D to the bank's income account B_I. In recognition of this payment, the banking sector records that the outstanding loan has been reduced by that amount in a book-keeping only operation on the loan account F_L.

$$\begin{pmatrix} \text{"Type"} & 1 & -1 & -1 & 0 \\ \text{"Account"} & \text{"Firm Loan"} & \text{"Firm Deposit"} & \text{"Worker Deposit"} & \text{"Bank Income"} \\ \text{"Account"} & F_L(t) & F_D(t) & W_D(t) & B_I(t) \\ \text{"Compound Interest"} & A & 0 & 0 & 0 \\ \text{"Pay Interest on Loan"} & -B & -B & 0 & B \\ \text{"Interest on Deposit"} & 0 & C & 0 & -C \\ \text{"Wages"} & 0 & -D & D & 0 \\ \text{"Interest on Deposit"} & 0 & 0 & E & -E \\ \text{"Consumption"} & 0 & F+G & -F & -G \end{pmatrix}$$

Figure 4

This table describes a system that can sustain a constant level of economic activity if:

1. Interest payments on the loan equal the rate of compounding ($A=B$);
2. Sales to workers and bankers plus deposit interest equal interest payments plus wages ($C+F+G=B+D$);
3. Wages plus deposit interest equals workers' consumption ($D+E=F$); and
4. Loan interest equals the sum of deposit interest payments plus bankers' consumption ($B=C+E+G$).

These are easily met conditions, as is shown by making sensible substitutions for the placeholders and attaching this model of financial flows to a simple model of production.[5] The substitutions are shown in Table 1.

Table 1

Term	Substitution	Explanation	Values in Simulation
A	$r_L \cdot F_L$	The rate of interest on loans r_L times the debt level F_L.	r_L=5% and Initial Loan Λ = \$100
B	$r_L \cdot F_L$	The rate of interest on loans r_L times the debt level F_L.	
C	$r_D \cdot F_D$	The rate of interest on deposits r_D times the deposit level F_D.	r_D = 2%
D	$\left((1-s)/\tau_S\right) \cdot F_D$	The share *(1-s)* in the surplus generated by production represented by wages (where the remainder of the share goes to firms as profit) divided by the time lag between the initial expenditure on production and sales receipts τ_s, measured in terms of fractions of a year, times the balance in F_D.	s = 30%; τ_S = ¼ of a Year
E	$r_D \cdot W_D$	The rate of interest on deposits r_D times the deposit level W_D.	
F	W_D/τ_W	Account balance W_D divided by the time lag in workers' consumption τ_W.	τ_W = 1/26th of a Year
G	B_I/τ_B	Account balance W_D divided by the time lag in bankers' consumption τ_B.	τ_B = 1 Year

[5] At this stage the model is deliberately skeletal, in that constant parameters are used. In subsequent models beyond the scope of this paper, these are replaced by nonlinear behavioural relations. Only one such relation is introduced in this paper—a nonlinear Phillips curve for nominal wage determination.

The substitutions and the symbolic generation of the system of ordinary differential equations (ODEs) in *Mathcad* is shown in Figure 5

$$A := r_L \cdot F_L(t) \quad B := A \quad C := r_D \cdot F_D(t) \quad D := \frac{1-s}{\tau_S} \cdot F_D(t) \quad E := r_D \cdot W_D(t) \quad F := \frac{W_D(t)}{\tau_W} \quad G := \frac{B_I(t)}{\tau_B}$$

$$S_2 := \begin{pmatrix} \text{"Type"} & 1 & -1 & -1 & 0 \\ \text{"Account"} & \text{"Firm Loan"} & \text{"Firm Deposit"} & \text{"Worker Deposit"} & \text{"Bank Income"} \\ \text{"Account"} & F_L(t) & F_D(t) & W_D(t) & B_I(t) \\ \text{"Compound Interest"} & A & 0 & 0 & 0 \\ \text{"Pay Interest on Loan"} & -B & -B & 0 & B \\ \text{"Interest on Deposit"} & 0 & C & 0 & -C \\ \text{"Wages"} & 0 & -D & D & 0 \\ \text{"Interest on Deposit"} & 0 & 0 & E & -E \\ \text{"Consumption"} & 0 & F+G & -F & -G \end{pmatrix}$$

$$\text{SystemODEs}(S_2) \rightarrow \begin{bmatrix} \frac{d}{dt}F_L(t) = 0 \\ \frac{d}{dt}F_D(t) = r_D \cdot F_D(t) - r_L \cdot F_L(t) + \frac{B_I(t)}{\tau_B} + \frac{W_D(t)}{\tau_W} + \frac{F_D(t)\cdot(s-1)}{\tau_S} \\ \frac{d}{dt}W_D(t) = r_D \cdot W_D(t) - \frac{W_D(t)}{\tau_W} - \frac{F_D(t)\cdot(s-1)}{\tau_S} \\ \frac{d}{dt}B_I(t) = r_L \cdot F_L(t) - r_D \cdot F_D(t) - r_D \cdot W_D(t) - \frac{B_I(t)}{\tau_B} \end{bmatrix}$$

Figure 5

In order to demonstrate the viability of this simple financial flows model, I attach it to an equally simple production model which assumes that physical output Q is equal to labour L times constant labour productivity ($a = 2$).[6] Labour employed equals the flow of wages (D in Table 1) divided by the wage rate ($W = 1$), while the price level can be worked out in either of two ways:

1. As a time-lagged response to the gap between monetary demand and the monetary value of output; or
2. As a time-lagged ($\tau_P = 1$) convergence (Blinder (1998, p. 84); Keen (2001, Chapter 4) Keen (2004); Keen (2005)) to an equilibrium price level.[7]

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[6] This model has no explicit intermediate goods, a fixed wage, and no population or productivity growth. All these simplifications are removed in the ultimate model.

[7] See Keen (2001, 2004 and 2005) for why I do reject marginal cost pricing, and Blinder (1998) for why most firms empirically do not and cannot set prices on the basis of marginal cost.

The latter form is used here for simplicity. The production component of the model is thus:

$$
\begin{aligned}
Q(t) &= a \cdot L(t) \\
L(t) &= \frac{1-s}{\tau_S} \cdot \frac{F_D(t)}{W} \\
\frac{d}{dt} P(t) &= -\frac{1}{\tau_P} \cdot \left(P(t) - \frac{W}{a \cdot (1-s)} \right)
\end{aligned}
\tag{1.1}
$$

With the values shown, this simple model converges to the equilibrium values shown in Table 2.

Table 2

Variable	**Equilibrium value**
F_L	\$100
F_D	\$87.508
W_D	\$9.431
B_I	\$3.061
Q (Output in units)	490.043
L (Labour)	245.021
P (Price level)	0.714
Y (output in \$ = $P.Q$ & F_D/τ_S)	\$350.03
W (Wages per annum = $W.L$ &$(1-s).\ F_D/\tau_S$)	\$245.021
Π (Profits per annum = $P.Q-W.L$ & $s.\ F_D/\tau_S$)	\$105.009
Gross Interest (per annum)	\$5

Intermediate model—A growing single sector economy

The previous section shows that this simple monetary model of production is internally consistent and describes a self-sustaining economy. This section extends the model to include debt repayment and both monetary and physical growth. The money wage is now shown as a variable determined by a Phillips curve relation, and rising labour productivity and population are introduced:

$$
\begin{aligned}
\frac{d}{dt} W(t) &= Ph(\lambda(t)) \cdot W(t) \\
\lambda(t) &= \frac{L(t)}{N(t)} \\
\frac{d}{dt} a(t) &= \alpha \cdot a(t) \\
\frac{d}{dt} N(t) &= \beta \cdot N(t)
\end{aligned}
\tag{1.2}
$$

The functions and parameter values are as shown in Table 3:

Table 3

Parameter or function	**Values**

$Ph(\lambda) = gen_{exp}(\lambda, x, y, s.m) = (y - m) \cdot e^{\frac{s}{y-m} \cdot (\lambda - x)} + m$ λ = actual employment rate at time t; (x,y) = Given pair of employment & wage change rate on the function; s = slope of function at (x,y); m = minimum value of function	$\begin{bmatrix} x \\ y \\ s \\ m \end{bmatrix} = \begin{bmatrix} 0.95 \\ 0 \\ 2 \\ -.04 \end{bmatrix}$
α (rate of productivity growth)	1.5%
β (rate of population growth)	1%

The Phillips Curve relation generates a *money wage* change function as shown in Figure 6.[8]

Figure 6

The financial flows matrix is also expanded to include loan repayment, relending of bank reserves, and the creation of new credit money. This necessitates the addition of one more account, to record the level of unlent reserves. This is not meant to replicate current institutional arrangements, but is in the spirit of Keynes's observations about how a "revolving fund of finance" could sustain constant economic activity, while growth in the economy was an additional source of the demand for money:

> If investment is proceeding at a steady rate, the finance (or the commitments to finance) required can be supplied from a revolving fund of a more or less constant amount, one

[8] This is the relation as first specified by Phillips. The allowance for more than 100% employment covers the real-world possibility of employment exceeding the normally active workforce and enticing members of non-working population into employment.

entrepreneur having his finance replenished for the purpose of a projected investment as another exhausts his on paying for his completed investment… Credit, in the sense of 'finance,' looks after a flow of investment. It is a revolving fund which can be used over and over again. It does not absorb or exhaust any resources. The same 'finance' can tackle one investment after another… if decisions to invest are (e.g.) increasing, the extra finance involved will constitute an additional demand for money… Keynes (1937, p. 247)

The new "Bank Reserves" account B_R introduced in Figure 7 represents this "revolving fund".

$S_2 :=$

"Type"	0	1	–1	–1	0
"Account"	"Bank Reserves"	"Firm Loan"	"Firm Deposit"	"Worker Deposit"	"Bank Income"
"Account"	$B_R(t)$	$F_L(t)$	$F_D(t)$	$W_D(t)$	$B_I(t)$
"Compound Interest"	0	A	0	0	0
"Pay Interest on Loan"	0	–B	–B	0	B
"Interest on Deposit"	0	0	C	0	–C
"Wages"	0	0	–D	D	0
"Interest on Deposit"	0	0	0	E	–E
"Consumption"	0	0	F + G	–F	–G
"Repay Loan"	H	–H	–H	0	0
"Lend Reserves"	–I	I	I	0	0
"New Money"	0	J	J	0	0

Figure 7

Three additional operations are shown here in addition to those in Figure 4: repaying loans, relending reserves, and creating new money and debt:

1. Loan repayment involves a transfer from the firm sector's deposit accounts F_D to the banking sector's reserves B_R—replenishing the "revolving fund" in Keynes's words—which the banking sector has to acknowledge by showing that the outstanding debt has been reduced by the amount of the repayment. There are therefore 3 entries on that row: one transfer from F_D to B_R, and one accounting recording of the transfer in F_L. This operation also involves a change in the distribution of the banking sector's assets—loans fall, reserves rise—but no increase in assets as such;
2. The act of lending out existing reserves involves a transfer from B_R to F_D, which increases the debt of the Firm Sector F_L by the same amount, again resulting in 3 entries; and
3. The creation of new money—which is akin to banks increasing available lines of credit, and firms availing themselves of that increase Moore (1983, p. 544)—increases both F_D and F_L. There is thus an increase in both the asset and liability side of the banking sector's accounts, but no change in bank reserves.

The new operations are introduced as time-lagged functions of system accounts:

Table 4

Term	Substitution	Explanation	Standard Run Value

H	F_L/τ_L	Outstanding balance divided by time lag in debt repayment	τ_L = 7 years
I	B_R/τ_R	Bank reserves divided by time lag in relending existing reserves	τ_R = 1 year
J	F_D/τ_M	Current balance in debtors' accounts divided by time lag in money creation.	τ_M = 10 years

With these extensions, the model display interesting dynamics and can already be used as a testbed for policy alternatives in the event of a credit crunch. The first interesting result is a simple demonstration of the non-neutrality of money.

Money Non-neutrality

The model generates growing output and financial sector balances with the standard run parameters. With rising account balances and rising output, an equilibrium distribution of funds and the debt to GDP ratio (the ratio of F_L to $P.Q$—price times quantity) develop over time (see Figure 8).

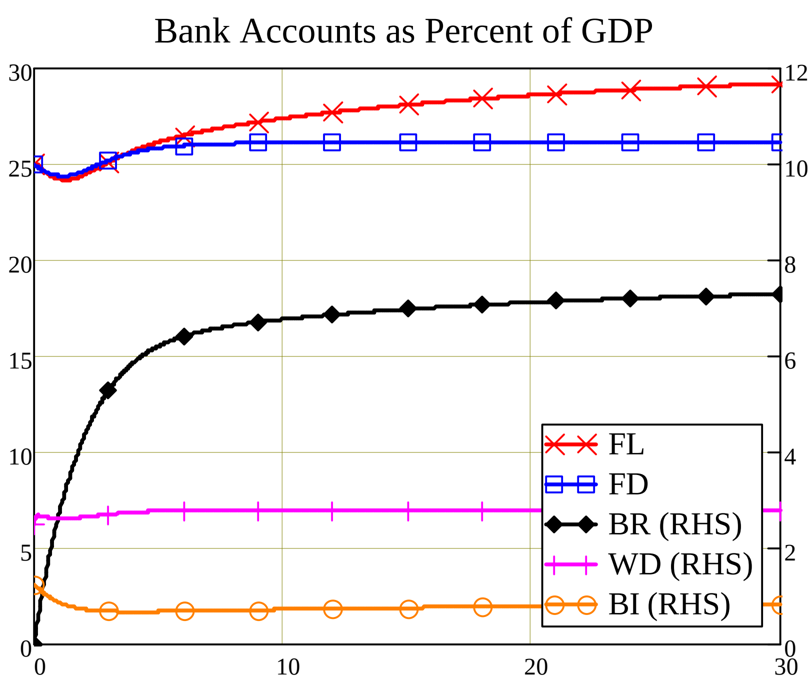

Figure 8

The levels of inflation, output and employment are also functions of the financial time lags introduced in Table 4. With its start at a zero level of inflation and moderate unemployment, a period of deflation and rising unemployment is followed by rising inflation and falling unemployment until the two stabilize at positive values (see Figure 9).

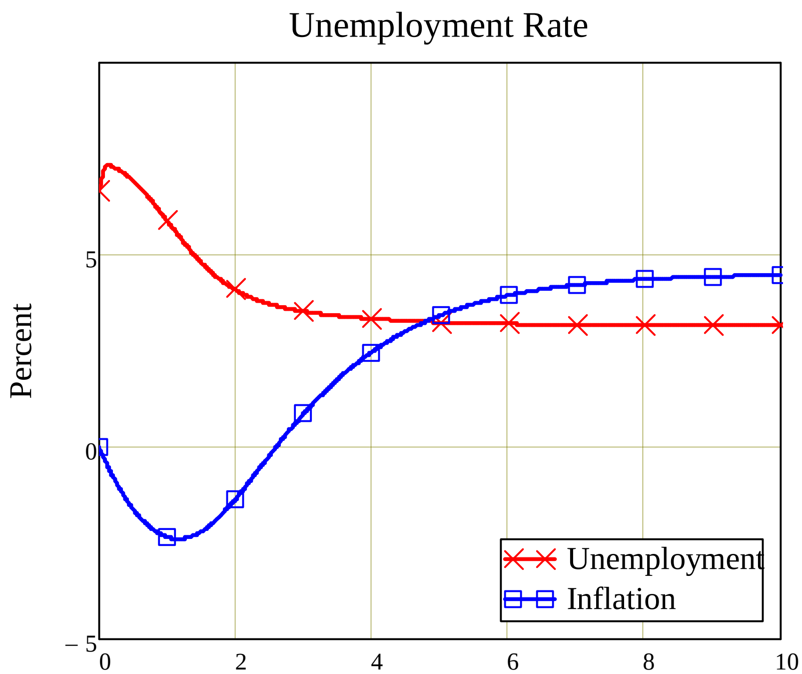

Figure 9

These levels are dependent on the financial flows parameters: a change that reduces the rate of growth of debt—decreasing the time lag in loan repayments by 2/3rds, and increasing the time lags in reserve relending and money creation by the same amount—produces a very different time path for employment and inflation (see Figure 10).

Phase plot of unemployment & inflation

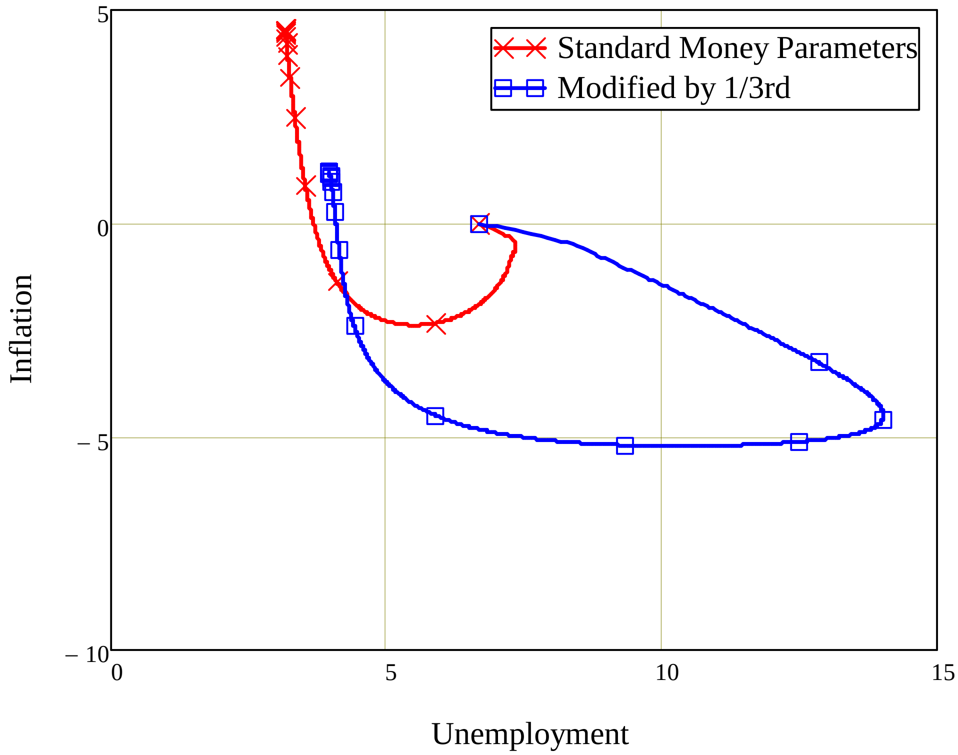

Figure 10

Though a government sector has not yet been formally incorporated, the model can also simulate the impact of an exogenous injection of money which is akin to the impact of the fiscal stimuli (if given to the debtors) and quantitative easing (if injected into bank reserves) being undertaken now to counter the financial crisis.

Fiscal Stimulus or Quantitative Easing?

A credit crunch is simulated by changing the three key financial parameters—τ_L, τ_R and τ_M—by 33% at the 25 year mark. The impact of this change on bank balances—and in particular the level of inactive reserves versus loans—is dramatic, as shown in Figure 11.

Bank Accounts with a Credit Crunch

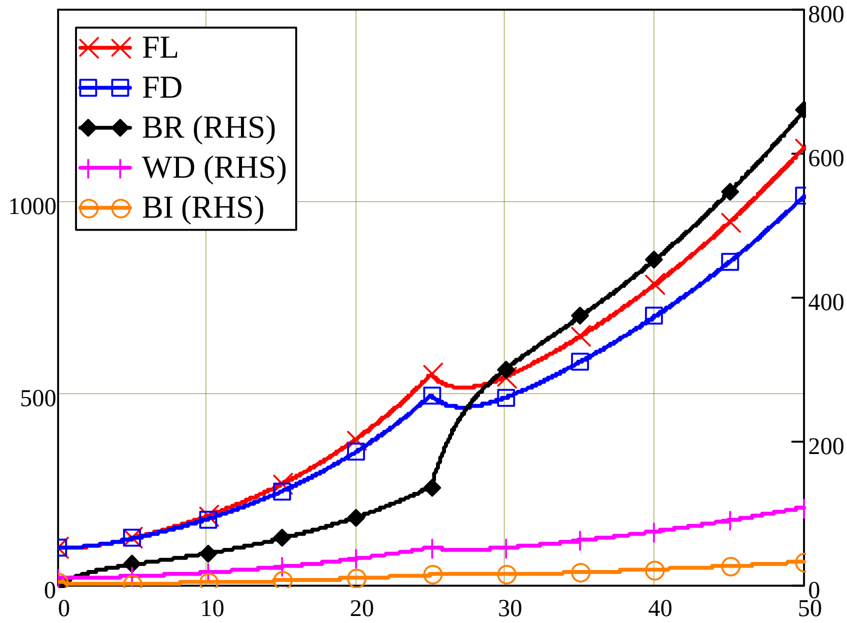

Figure 11

To simulate a government policy response to this crisis, two additional rows are added to the model, one which models an exogenous injection of funds into bank reserves, and the other an equivalent injection into firms' deposit account (see Figure 12). In both cases, an injection of $50 (billion) is made into the relevant fund over a year—equivalent to roughly 10% of active bank deposits at the time, or about 3% of annual GDP.

The former simulates the quantitative easing process that has been undertaken in the USA and UK; the latter simulates the fiscal stimulus that has been undertaken worldwide, but which is more characteristic of the Australian policy response (where as yet the banks have not had their solvency challenged by the crisis).

$$S_3 := \begin{pmatrix} \text{"Type"} & 0 & 1 & -1 & -1 & 0 \\ \text{"Account"} & \text{"Bank Reserves"} & \text{"Firm Loan"} & \text{"Firm Deposit"} & \text{"Worker Deposit"} & \text{"Bank Income"} \\ \text{"Account"} & B_R(t) & F_L(t) & F_D(t) & W_D(t) & B_I(t) \\ \text{"Compound Interest"} & 0 & A & 0 & 0 & 0 \\ \text{"Pay Interest on Loan"} & 0 & -B & -B & 0 & B \\ \text{"Interest on Deposit"} & 0 & 0 & C & 0 & -C \\ \text{"Wages"} & 0 & 0 & -D & D & 0 \\ \text{"Interest on Deposit"} & 0 & 0 & 0 & E & -E \\ \text{"Consumption"} & 0 & 0 & F+G & -F & -G \\ \text{"Repay Loan"} & H & -H & -H & 0 & 0 \\ \text{"Lend Reserves"} & -I & I & I & 0 & 0 \\ \text{"New Money"} & 0 & J & J & 0 & 0 \\ \text{"Fiscal Stimulus"} & 0 & 0 & K & 0 & 0 \\ \text{"Quantitative Easing"} & L & 0 & 0 & 0 & 0 \end{pmatrix}$$

Figure 12

The Obama Administrations reason for biasing their interventions towards funding the banking sector rather than the debtors was clearly spelt out in the President's speech of April 14 2009: he and his Administration were relying on the "multiplier effect" of credit creation to get more growth from refinancing banks than they expected from refinancing borrowers:

> "there are a lot of Americans who understandably think that government money would be better spent going directly to families and businesses instead of banks – 'where's our bailout?,' they ask… the truth is that a dollar of capital in a bank can actually result in eight or ten dollars of loans to families and businesses, a *multiplier effect* that can ultimately lead to a faster pace of economic growth." Obama (2009; emphasis added)

The simulation implies that this conventional economic guidance was misguided: the impact of the stimulus when given to the borrowers is far higher than that when given to the banks (see Figure 13).

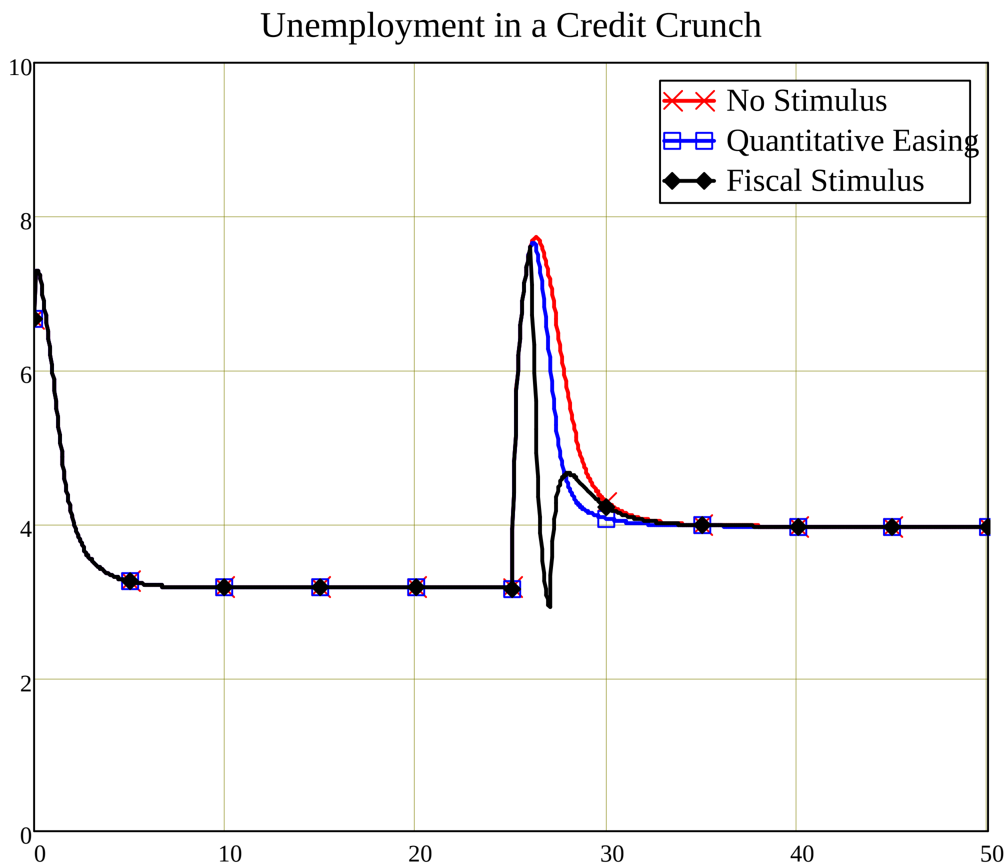

Figure 13

The phase plot of unemployment and inflation also indicates that the fiscal stimulus has a larger impact on prices—reducing the extent and depth of deflation (see Figure 13).

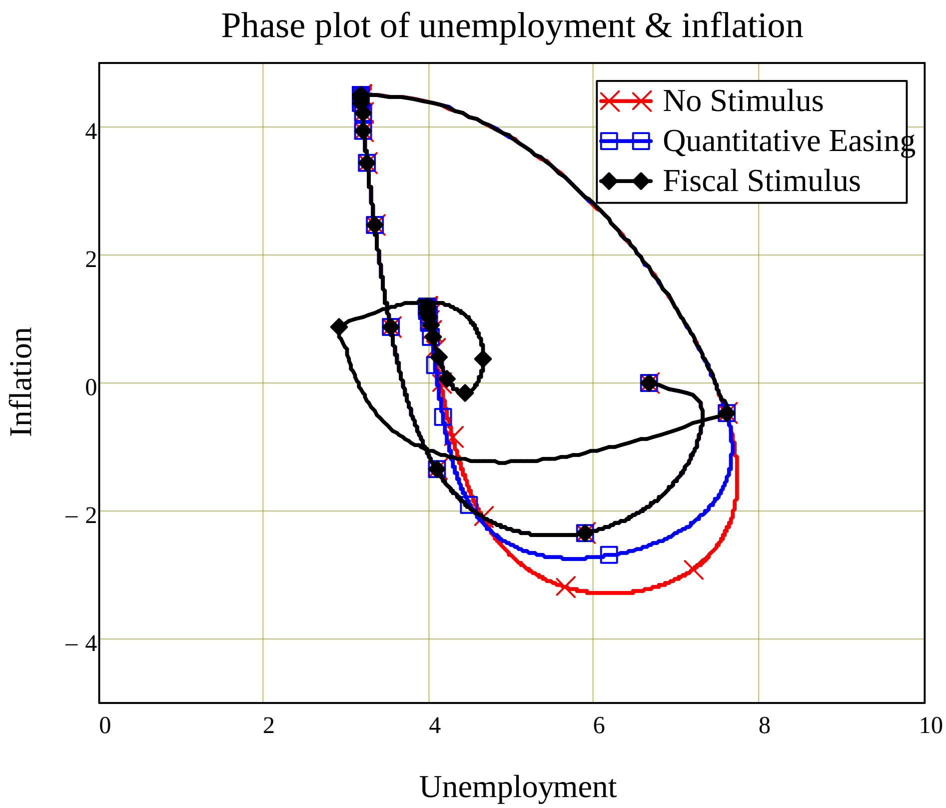

Figure 14

Lastly, the Debt to GDP ratio indicates that at least part of the stimulating effect of fiscal policy comes from a reduction in the debt ratio (Figure 14).

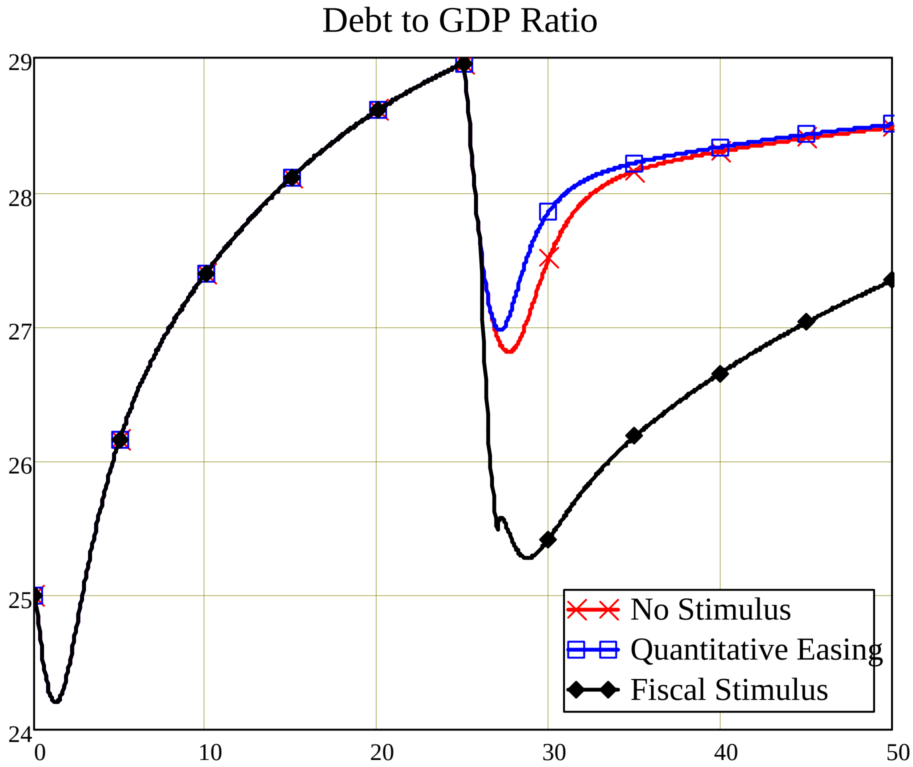

Figure 15

The above results are to some extent compromised by the abstraction of a single commodity system, which makes it impossible to consider the impact of a financial crisis on capital formation—the phenomenon that all three time paths for the economy converge to the same long run level of unemployment could be an artifact of the absence of fixed capital from the model. A first stage in introducing fixed capital is the modeling of multi-commodity production. The technology developed above easily scales to multiple sectors, and the model shown in the next section is, to my knowledge, the first ever strictly monetary continuous time multi-sectoral model of production.

Multi-sectoral production

As with the single-sectoral model developed above, starting from an explicitly monetary perspective in a disequilibrium environment simplifies the issues involved in multi-sectoral modeling. The established but poorly appreciated dilemmas with general equilibrium modeling that arise from the imposition of instantaneous equilibrium conditions on both prices and quantities—the so-called dual stability problem (see Blatt (1983, Chapter 7) for one of the few erudite discussions of this)— do not apply to this "disequilibrium" (or more strictly and simply, dynamic) model, while the monetary setting of the model simplifies inter-sectoral relations.

The only complications added by moving from a single-sectoral abstraction to multi-commodity production (but without fixed capital as yet) are inter-sectoral demand for intermediate goods inputs, and multi-sectoral consumption flows.

The sample model shown here has 3 sectors, each of which needs the others outputs as inputs to its own production (see Figure 16).

"Type"	0	1	1	1	–1	–1	–1	–1	0
"Name"	"Bank Reserve"	"Firm K Loan"	"Firm C Loan"	"Firm A Loan"	"Firm K Dep"	"Firm C Dep"	"Firm A Loan"	"Workers"	"Bank Income"
"Symbol"	$B_R(t)$	$F_{LK}(t)$	$F_{LC}(t)$	$F_{LA}(t)$	$F_{DK}(t)$	$F_{DC}(t)$	$F_{DA}(t)$	$W_D(t)$	$B_D(t)$
"Interest on Loans"	0	A	B	C	0	0	0	0	0
"Interest on Deposits"	0	0	0	0	D	E	F	0	–(D + E + F)
"Wages"	0	0	0	0	–G	–H	–I	G + H + I	0
"Intersectoral demand from K"	0	0	0	0	–(J + K)	J	K	0	0
"Intersectoral demand from C"	0	0	0	0	L	–(L + M)	M	0	0
"Intersectoral demand from A"	0	0	0	0	N	O	–(N + O)	0	0
"Consumption of C"	0	0	0	0	–P	P + Q + R + S	–Q	–R	–S
"Consumption of A"	0	0	0	0	–T	–U	T + U + V + W	–V	–W
"Pay Interest on Loans"	0	–X	–Y	–Z	–X	–Y	–Z	0	X + Y + Z
"Workers Interest"	0	0	0	0	0	0	0	AA	–AA
"Repay Loans"	AB + AC + AD	–AB	–AC	–AD	–AB	–AC	–AD	0	0
"Money Relend"	–(AE + AF + AG)	AE	AF	AG	AE	AF	AG	0	0
"New Money"	0	AH	AI	AJ	AH	AI	AJ	0	0

Figure 16

The intersectoral demands are proportional to the labour input as in the single commodity model (see Figure 17).[9]

$$\begin{pmatrix} J \\ K \\ L \\ M \\ N \\ O \end{pmatrix} := \overrightarrow{\left[\begin{pmatrix} \sigma_{KC} \\ \sigma_{KA} \\ \sigma_{CK} \\ \sigma_{CA} \\ \sigma_{AK} \\ \sigma_{AC} \end{pmatrix} \cdot \begin{pmatrix} G \\ G \\ H \\ H \\ I \\ I \end{pmatrix}\right]} \rightarrow \begin{bmatrix} -\dfrac{\sigma_{KC} \cdot F_{DK}(t) \cdot (s_K - 1)}{\tau_{SK}} \\ -\dfrac{\sigma_{KA} \cdot F_{DK}(t) \cdot (s_K - 1)}{\tau_{SK}} \\ -\dfrac{\sigma_{CK} \cdot F_{DC}(t) \cdot (s_C - 1)}{\tau_{SC}} \\ -\dfrac{\sigma_{CA} \cdot F_{DC}(t) \cdot (s_C - 1)}{\tau_{SC}} \\ -\dfrac{\sigma_{AK} \cdot F_{DA}(t) \cdot (s_A - 1)}{\tau_{SA}} \\ -\dfrac{\sigma_{AC} \cdot F_{DA}(t) \cdot (s_A - 1)}{\tau_{SA}} \end{bmatrix}$$

Figure 17

Consumption coefficients are proportional to the balances outstanding in the relevant deposit accounts, as before (see Figure 18).

[9] This is valid as a model of industrial production in which fixed capital is not a constraint. When fixed capital is introduced, this relation will remain between labour and intermediate goods.

$$\begin{pmatrix}P\\Q\\R\\S\\T\\U\\V\\W\end{pmatrix} := \overrightarrow{\left[\begin{pmatrix}F_{DK}(t)\\F_{DC}(t)\\W_{D}(t)\\B_{D}(t)\\F_{DK}(t)\\F_{DC}(t)\\W_{D}(t)\\B_{D}(t)\end{pmatrix} \div \begin{pmatrix}\tau_{KC}\\\tau_{CC}\\\tau_{WC}\\\tau_{BC}\\\tau_{KA}\\\tau_{KA}\\\tau_{WA}\\\tau_{WA}\end{pmatrix}\right]} \rightarrow \begin{pmatrix}\frac{F_{DK}(t)}{\tau_{KC}}\\\frac{F_{DC}(t)}{\tau_{CC}}\\\frac{W_{D}(t)}{\tau_{WC}}\\\frac{B_{D}(t)}{\tau_{BC}}\\\frac{F_{DK}(t)}{\tau_{KA}}\\\frac{F_{DC}(t)}{\tau_{KA}}\\\frac{W_{D}(t)}{\tau_{WA}}\\\frac{B_{D}(t)}{\tau_{WA}}\end{pmatrix}$$

Figure 18

The production equations for each sector are therefore identical to that for the single commodity model, but with industry-specific price-adjustment time lag, productivity and share-of-surplus coefficients, while the price equations are also generalizations of the price equation for the single commodity model:

$$\begin{aligned}\frac{d}{dt}P_K(t) &= -\frac{1}{\tau_{PK}}\cdot\left(P_K(t)-\frac{W(t)}{a_K(t)\cdot(1-s_K)}\right)\\ \frac{d}{dt}P_C(t) &= -\frac{1}{\tau_{PA}}\cdot\left(P_C(t)-\frac{W(t)}{a_C(t)\cdot(1-s_C)}\right)\\ \frac{d}{dt}P_A(t) &= -\frac{1}{\tau_{PA}}\cdot\left(P_A(t)-\frac{W(t)}{a_A(t)\cdot(1-s_A)}\right)\end{aligned} \tag{1.3}$$

The model now generates more complex but still coherent multi-sectoral financial dynamics (see Figure 19).

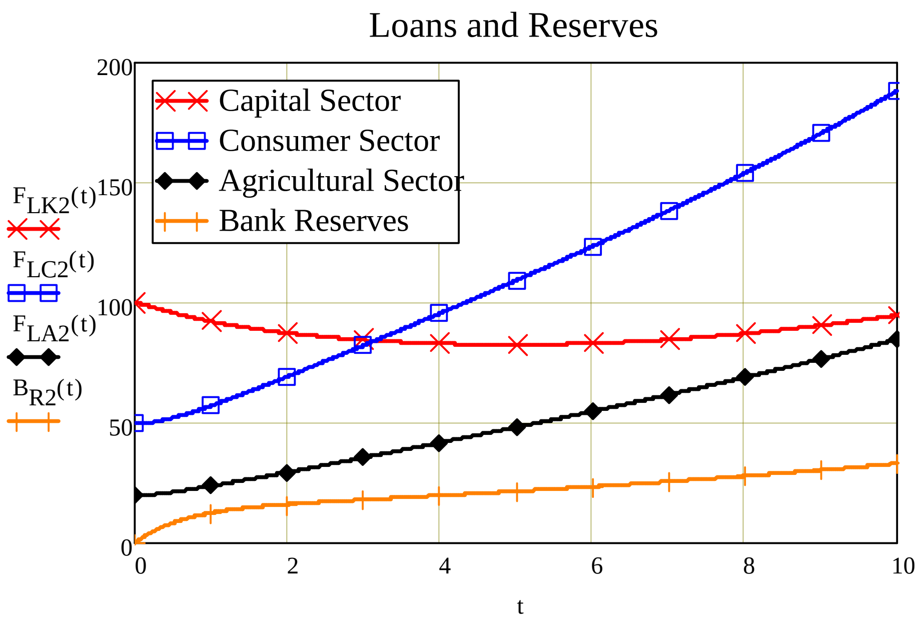

Figure 19

The price dynamics now differ between the sectors, while still generating overall inflation with the parameters used (Figure 20).

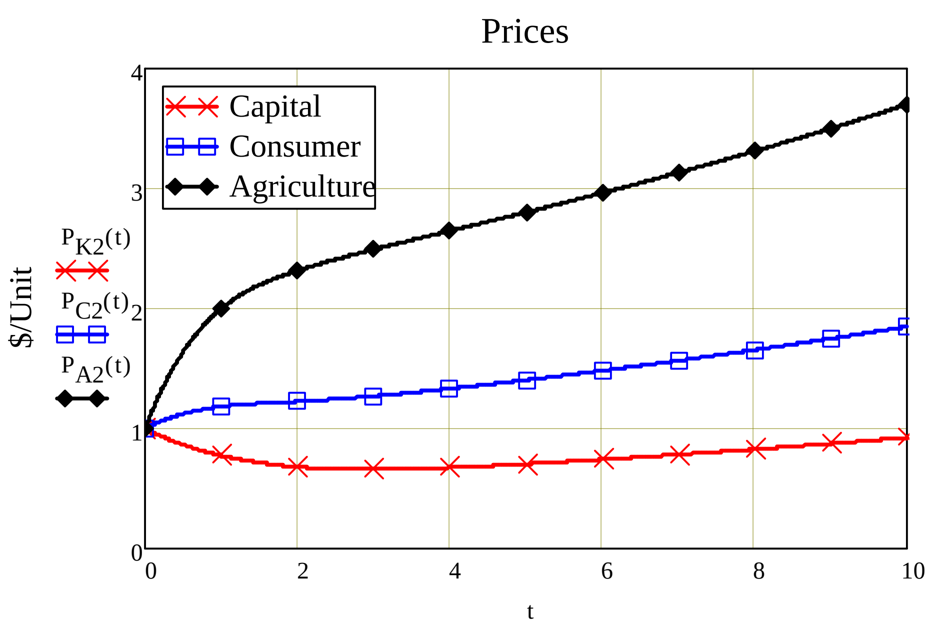

Figure 20

All sectors can make positive incomes after the addition of inter-sectoral demands (Figure 21 and Figure 22).

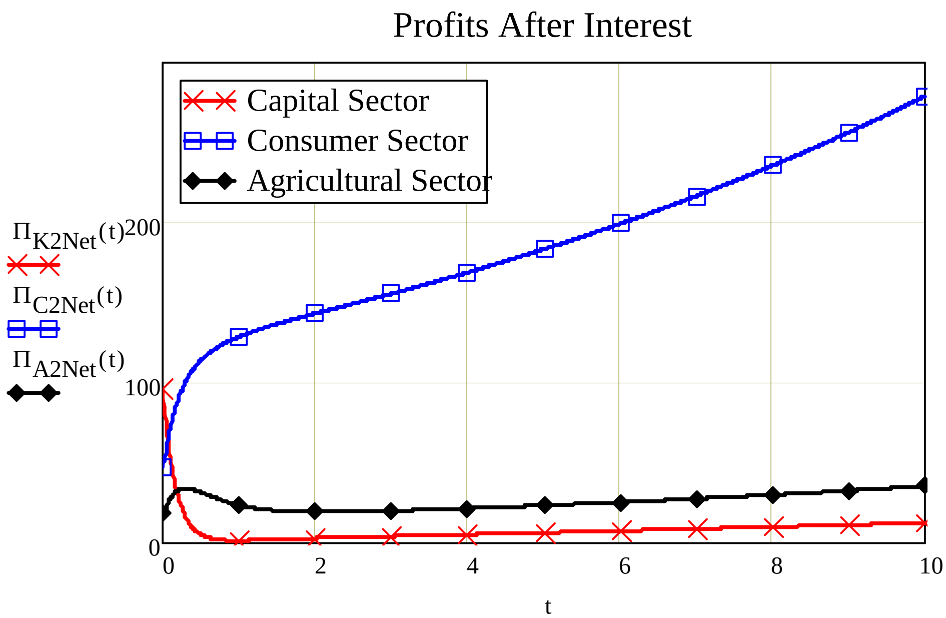

Figure 21

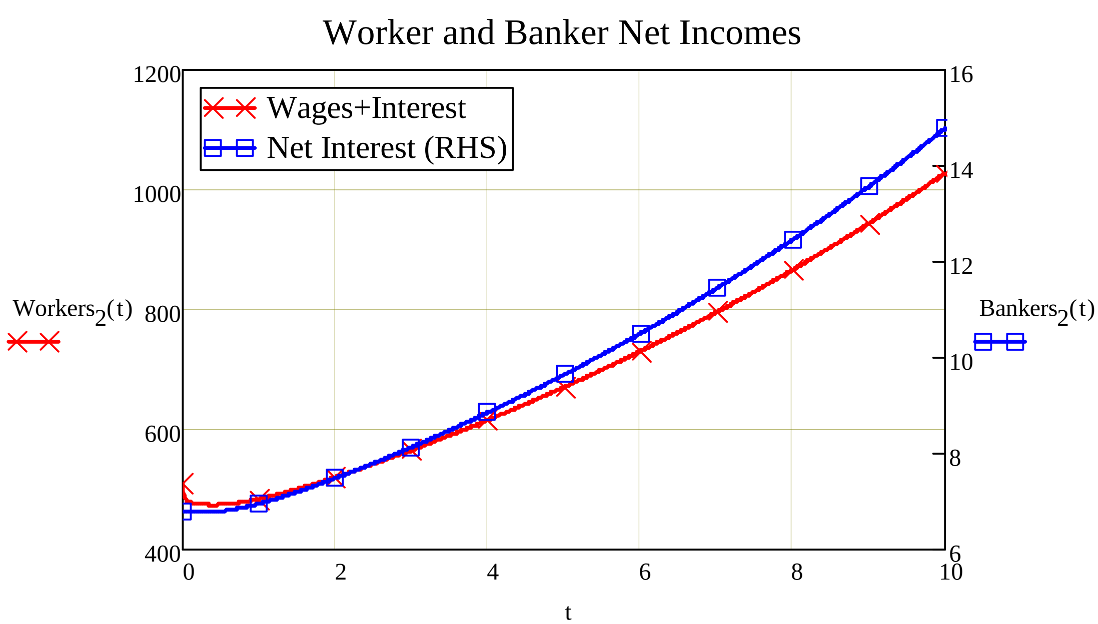

Figure 22

Conclusion

Several extensions are needed before the model will be capable of exploring the financial dynamics detailed in Minsky (1972) and modeled in an implicit money, single sectoral form in Keen (1995) and Keen (2009). These include the introduction of fixed capital, a tendency for profit equalization across sectors, and a properly specified government sector that can create "fiat" money, in addition to the creation of pure credit money shown here. The resulting model should allow a realistic simulation of the sometimes chaotic dynamics of the complex financial market economy in which we actually live.

Appendix

The full multi-sectoral model is shown in Figure 23.

$$
\begin{pmatrix}
\frac{d}{dt}B_R \\
\frac{d}{dt}F_{LK} \\
\frac{d}{dt}F_{LC} \\
\frac{d}{dt}F_{LA} \\
\frac{d}{dt}F_{DK} \\
\frac{d}{dt}F_{DC} \\
\frac{d}{dt}F_{DA} \\
\frac{d}{dt}W_D \\
\frac{d}{dt}B_D \\
\frac{d}{dt}W \\
\frac{d}{dt}N \\
\lambda \\
\frac{d}{dt}P_K \\
\frac{d}{dt}P_C \\
\frac{d}{dt}P_A \\
\frac{d}{dt}a_K \\
\frac{d}{dt}a_C \\
\frac{d}{dt}a_A \\
Q_K \\
Q_C \\
Q_A \\
L_K \\
L_C \\
L_A
\end{pmatrix}
=
\begin{bmatrix}
\frac{F_{LA}(t)}{\tau_{LA}} - \frac{B_R(t)}{\tau_{RC}} - \frac{B_R(t)}{\tau_{RK}} - \frac{B_R(t)}{\tau_{RA}} + \frac{F_{LC}(t)}{\tau_{LC}} + \frac{F_{LK}(t)}{\tau_{LK}} \\
\frac{B_R(t)}{\tau_{RK}} + \frac{F_{DK}(t)}{\tau_{MK}} - \frac{F_{LK}(t)}{\tau_{LK}} \\
\frac{B_R(t)}{\tau_{RC}} + \frac{F_{DC}(t)}{\tau_{MC}} - \frac{F_{LC}(t)}{\tau_{LC}} \\
\frac{B_R(t)}{\tau_{RA}} + \frac{F_{DA}(t)}{\tau_{MA}} - \frac{F_{LA}(t)}{\tau_{LA}} \\
r_D \cdot F_{DK}(t) - r_L \cdot F_{LK}(t) + \frac{B_R(t)}{\tau_{RK}} - \frac{F_{DK}(t)}{\tau_{KA}} - \frac{F_{DK}(t)}{\tau_{KC}} + \frac{F_{DK}(t)}{\tau_{MK}} - \frac{F_{LK}(t)}{\tau_{LK}} + \frac{F_{DK}(t)\cdot(s_K - 1)}{\tau_{SK}} - \frac{\sigma_{AK} \cdot F_{DA}(t)\cdot(s_A - 1)}{\tau_{SA}} - \frac{\sigma_{CK} \cdot F_{DC}(t)\cdot(s_C - 1)}{\tau_{SC}} + \frac{\sigma_{KA} \cdot F_{DK}(t)\cdot(s_K - 1)}{\tau_{SK}} + \frac{\sigma_{KC} \cdot F_{DK}(t)\cdot(s_K - 1)}{\tau_{SK}} \\
r_D \cdot F_{DC}(t) - r_L \cdot F_{LC}(t) + \frac{B_D(t)}{\tau_{BC}} + \frac{B_R(t)}{\tau_{RC}} - \frac{F_{DC}(t)}{\tau_{CA}} + \frac{F_{DC}(t)}{\tau_{CC}} + \frac{F_{DC}(t)}{\tau_{MC}} + \frac{F_{DK}(t)}{\tau_{KC}} - \frac{F_{LC}(t)}{\tau_{LC}} + \frac{W_D(t)}{\tau_{WC}} + \frac{F_{DC}(t)\cdot(s_C - 1)}{\tau_{SC}} - \frac{\sigma_{AC} \cdot F_{DA}(t)\cdot(s_A - 1)}{\tau_{SA}} + \frac{\sigma_{CA} \cdot F_{DC}(t)\cdot(s_C - 1)}{\tau_{SC}} + \frac{\sigma_{CK} \cdot F_{DC}(t)\cdot(s_C - 1)}{\tau_{SC}} - \frac{\sigma_{KC} \cdot F_{DK}(t)\cdot(s_K - 1)}{\tau_{SK}} \\
r_D \cdot F_{DA}(t) - r_L \cdot F_{LA}(t) + \frac{B_D(t)}{\tau_{BA}} + \frac{B_R(t)}{\tau_{RA}} + \frac{F_{DC}(t)}{\tau_{CA}} - \frac{F_{DC}(t)}{\tau_{CC}} + \frac{F_{DA}(t)}{\tau_{MA}} - \frac{F_{LA}(t)}{\tau_{LA}} + \frac{F_{DK}(t)}{\tau_{KA}} + \frac{W_D(t)}{\tau_{WA}} + \frac{F_{DA}(t)\cdot(s_A - 1)}{\tau_{SA}} + \frac{\sigma_{AC} \cdot F_{DA}(t)\cdot(s_A - 1)}{\tau_{SA}} - \frac{\sigma_{CA} \cdot F_{DC}(t)\cdot(s_C - 1)}{\tau_{SC}} + \frac{\sigma_{AK} \cdot F_{DA}(t)\cdot(s_A - 1)}{\tau_{SA}} - \frac{\sigma_{KA} \cdot F_{DK}(t)\cdot(s_K - 1)}{\tau_{SK}} \\
r_D \cdot W_D(t) - \frac{W_D(t)}{\tau_{WA}} - \frac{W_D(t)}{\tau_{WC}} - \frac{F_{DA}(t)\cdot(s_A - 1)}{\tau_{SA}} - \frac{F_{DC}(t)\cdot(s_C - 1)}{\tau_{SC}} - \frac{F_{DK}(t)\cdot(s_K - 1)}{\tau_{SK}} \\
r_L \cdot F_{LA}(t) - r_D \cdot F_{DC}(t) - r_D \cdot F_{DK}(t) - r_D \cdot F_{DA}(t) + r_L \cdot F_{LC}(t) + r_L \cdot F_{LK}(t) - r_D \cdot W_D(t) - \frac{B_D(t)}{\tau_{BA}} - \frac{B_D(t)}{\tau_{BC}} \\
Ph(\lambda(t)) \cdot W(t) \\
\beta \cdot N(t) \\
\frac{L_K(t) + L_C(t) + L_A(t)}{N(t)} \\
\frac{-1}{\tau_{PK}} \left[P_K(t) - \frac{W(t)}{a_K(t)\cdot(1 - s_K)} \right] \\
\frac{-1}{\tau_{PC}} \left[P_C(t) - \frac{W(t)}{a_C(t)\cdot(1 - s_C)} \right] \\
\frac{-1}{\tau_{PA}} \left[P_A(t) - \frac{W(t)}{a_A(t)\cdot(1 - s_A)} \right] \\
\alpha_K \cdot a_K(t) \\
\alpha_C \cdot a_C(t) \\
\alpha_A \cdot a_A(t) \\
a_K(t) \cdot L_K(t) \\
a_C(t) \cdot L_C(t) \\
a_A(t) \cdot L_A(t) \\
\frac{(1 - s_K)}{\tau_{SK}} \cdot \frac{F_{DK}(t)}{W(t)} \\
\frac{(1 - s_C)}{\tau_{SC}} \cdot \frac{F_{DC}(t)}{W(t)} \\
\frac{(1 - s_A)}{\tau_{SA}} \cdot \frac{F_{DA}(t)}{W(t)}
\end{bmatrix}
$$

Figure 23

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