Product to Sum Formulas

Name ____________________

Date ____________ Period ______

$$\boxed{\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]} \qquad \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

Proof: Equation 1

$$\begin{array}{rl} & \cos(\alpha-\beta) = \cancel{\cos\alpha\cos\beta} + \sin\alpha\sin\beta \\ - & \cos(\alpha+\beta) = \cancel{\cos\alpha\cos\beta} - \sin\alpha\sin\beta \\ \hline \end{array}$$

$$\cos(\alpha-\beta) - \cos(\alpha+\beta) = 2\sin\alpha\sin\beta$$

$$\frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] = \sin\alpha\sin\beta$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] \qquad \boxed{\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]}$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

Proof: Equation 2

$$\begin{array}{rl} & \cos(\alpha-\beta) = \cos\alpha\cos\beta + \cancel{\sin\alpha\sin\beta} \\ + & \cos(\alpha+\beta) = \cos\alpha\cos\beta - \cancel{\sin\alpha\sin\beta} \\ \hline \end{array}$$

$$\cos(\alpha-\beta) + \cos(\alpha+\beta) = 2\cos\alpha\cos\beta$$

$$\frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)] = \cos\alpha\cos\beta$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] \qquad \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\boxed{\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]}$$

Proof: Equation 3

$$\begin{aligned} &\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cancel{\cos\alpha\sin\beta} \\ + \; &\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cancel{\cos\alpha\sin\beta} \end{aligned}$$

$$\sin(\alpha+\beta) + \sin(\alpha-\beta) = 2\sin\alpha\cos\beta$$

$$\frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)] = \sin\alpha\cos\beta$$

$$\boxed{\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]} \qquad \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

Evaluate the following

$$\sin(7\theta)\sin(4\theta)$$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] \qquad \boxed{\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]}$$

$$\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

Evaluate the following

$\cos(5\theta)\cos(2\theta)$

$$\sin\alpha\sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)] \qquad \cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\boxed{\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]}$$

Evaluate the following

$\sin(6\theta)\cos(\theta)$

$$\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\boxed{\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]}$$

$$\sin\alpha \cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]$$

Evaluate the following

$\cos(2\theta)\cos(7\theta)$

$$\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha-\beta) - \cos(\alpha+\beta)]$$

$$\cos\alpha \cos\beta = \frac{1}{2}[\cos(\alpha-\beta) + \cos(\alpha+\beta)]$$

$$\boxed{\sin\alpha \cos\beta = \frac{1}{2}[\sin(\alpha+\beta) + \sin(\alpha-\beta)]}$$

Evaluate the following

$\sin(3\theta)\cos(9\theta)$