Definition of Hyperbolic Trig Functions

Name ____________________

Date ____________ Period ______

Like our regular trig functions, we have six "hyperbolic" trig functions

The "h" represents a "hyperbolic" trig function

$\sinh x$	"sinch x"	$\operatorname{csch} x$	"cosinch x"
$\cosh x$	"cosh x"	$\operatorname{sech} x$	"sech x"
$\tanh x$	"tanch x"	$\coth x$	"cotanch x"

Like our regular trig functions, we have six "hyperbolic" trig functions

The "h" represents a "hyperbolic" trig function

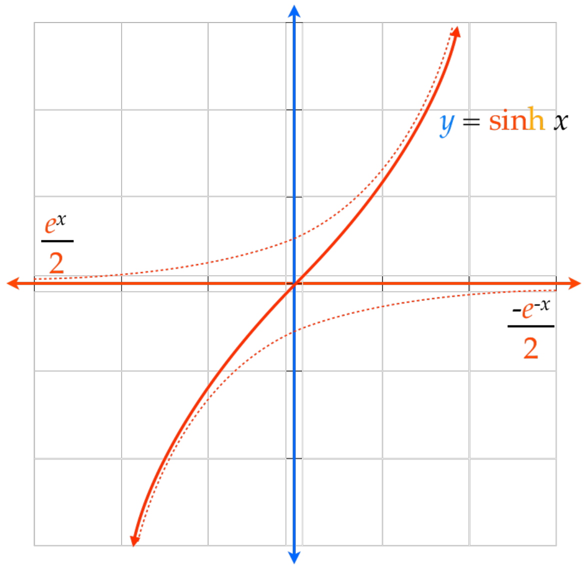
$$\sinh x = \frac{e^x - e^{-x}}{2} \qquad \operatorname{csch} x = \frac{1}{\sinh x}$$

$$\cosh x = \frac{e^x + e^{-x}}{2} \qquad \operatorname{sech} x = \frac{1}{\cosh x}$$

$$\tanh x = \frac{\sinh x}{\cosh x} \qquad \coth x = \frac{1}{\tanh x}$$

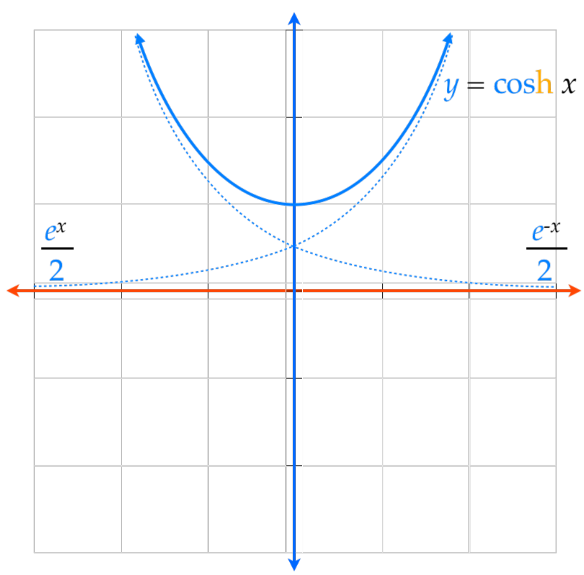
$$f(x) = \sinh x = \frac{e^x - e^{-x}}{2} = \frac{e^x}{2} + \frac{-e^{-x}}{2}$$

Domain: $(-\infty,\infty)$
Range: $(-\infty,\infty)$



$$f(x) = \cosh x = \frac{e^x + e^{-x}}{2} = \frac{e^x}{2} + \frac{e^{-x}}{2}$$

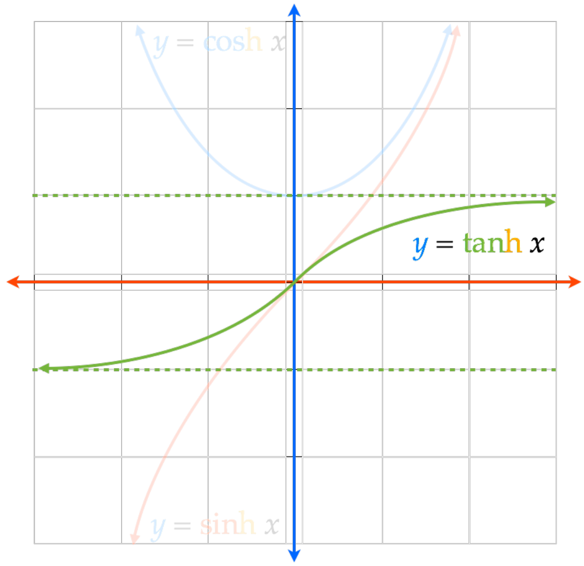
Domain: $(-\infty,\infty)$
Range: $[1,\infty)$



$f(x) = \tanh x = \dfrac{\sinh x}{\cosh x}$

Domain: $(-\infty,\infty)$

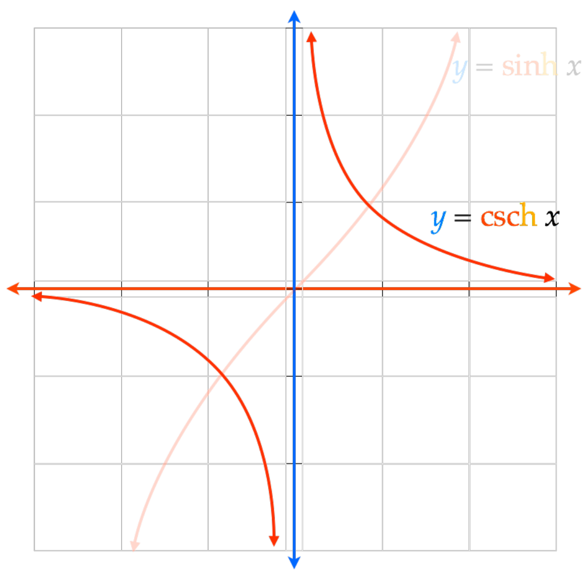
Range: $(-1,1)$



$f(x) = \operatorname{csch} x = \dfrac{1}{\sinh x}$

Domain: $(-\infty,0) \cup (0,\infty)$

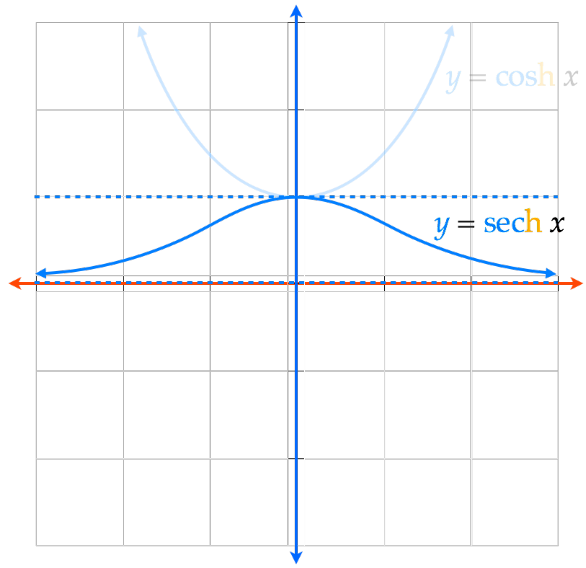
Range: $(-\infty,0) \cup (0,\infty)$



$$f(x) = \operatorname{sech} x = \frac{1}{\cosh x}$$

Domain: $(-\infty,\infty)$

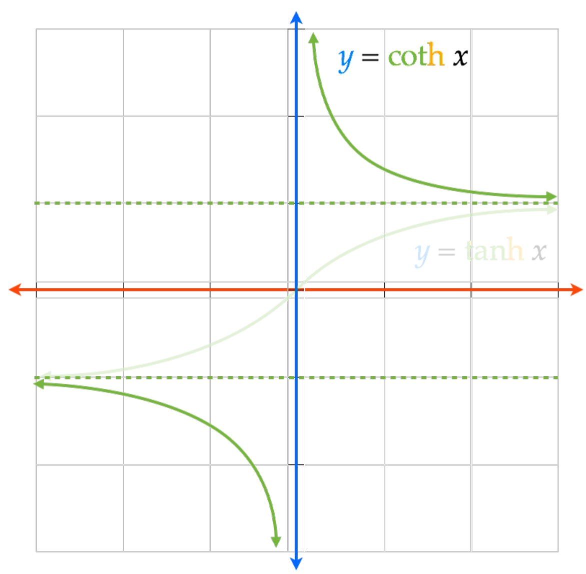
Range: $(0,1]$



$$f(x) = \coth x = \frac{1}{\tanh x}$$

Domain: $(-\infty,0) \cup (0,\infty)$

Range: $(-\infty,-1) \cup (1,\infty)$



Like our regular trig functions, hyperbolic trig functions have similar identities.

Pythagorean Identities

$$\cosh^2 x - \sinh^2 x = 1$$

$$\tanh^2 x + \operatorname{sech}^2 x = 1$$

$$\coth^2 x - \operatorname{csch}^2 x = 1$$

Sum/Difference Identities

$$\sinh(x \pm y) = \sinh x \cosh y \pm \cosh x \sinh y$$

$$\cosh(x \pm y) = \cosh x \cosh y \pm \sinh x \sinh y$$

Double Angle Identities

$$\sinh 2x = 2\sinh x \cosh x$$

$$\cosh 2x = \cosh^2 x + \sinh^2 x$$