Deriving the Fundamental Theorem of Calculus

Name ____________________

Date ____________ Period ______

Define an Area Function

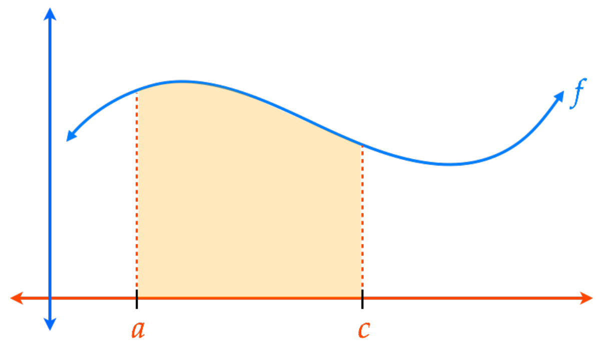
Let f be continuous for $t \geq a$.

The Area function for f with left endpoint a is defined as

$$A(x) = \int_a^x f(t)\, dt$$

$$A(c) = \int_a^c f(t)\, dt$$

$A(c)$ will give the area under f on $[a,c]$



Actual Area vs. Approximate Area

$[x, x + \Delta x]$

$$A(x + \Delta x) - A(x) \approx f(x) \cdot \Delta x$$

$$\frac{A(x + \Delta x) - A(x)}{\Delta x} \approx f(x)$$

$$\boxed{\lim_{\Delta x \to 0} \frac{A(x + \Delta x) - A(x)}{\Delta x}} = f(x)$$

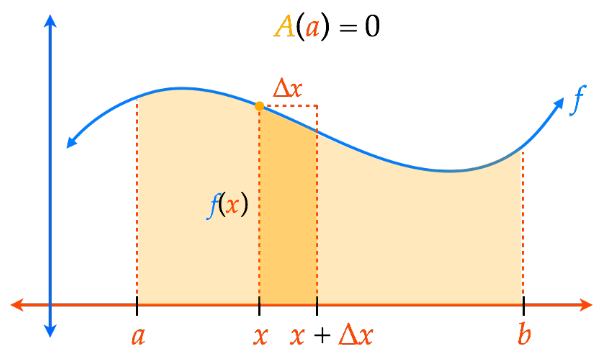
$$A'(x) = f(x)$$

$A(x)$ must be an antiderivative of $f(x)$

$$A(x) = \int_a^x f(t)\, dt$$

$A(b)$ = area under f on $[a,b]$

$A(a) = 0$



$$A(x) = F(x) + C$$

Use $A(a) = 0$

$$A(a) = F(a) + C = 0 \Rightarrow C = -F(a)$$

$$A(x) = F(x) - F(a)$$

Therefore $A(b) = F(b) - F(a)$

$$A(b) = \int_a^b f(t)\,dt = F(b) - F(a)$$

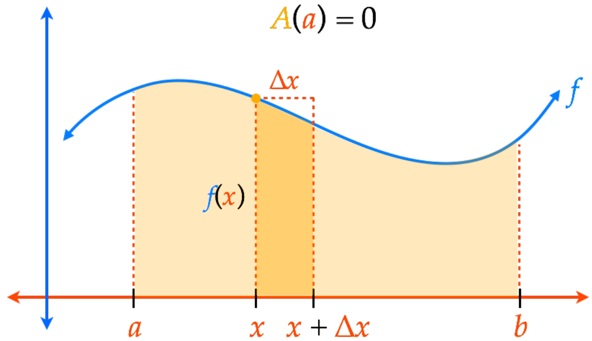
F is the antiderivative of f

$A(x)$ must be an antiderivative of $f(x)$

$$A(x) = \int_a^x f(t)\,dt$$

$A(b)$ = area under f on $[a,b]$

$A(a) = 0$

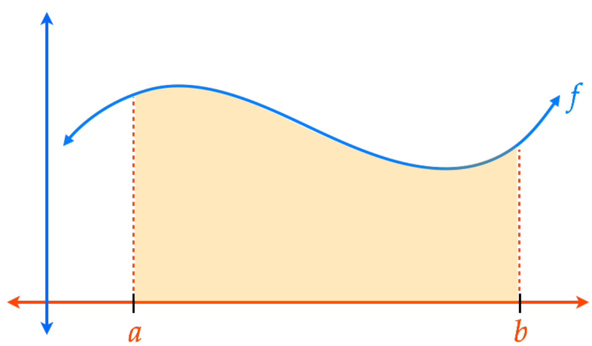


The Fundamental Theorem of Calculus

Let f be continuous on $[a,b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where $F(x)$ is the antiderivative of $f(x)$



Before, to calculate a definite integral (area under a curve)…

$$\int_a^b f(x)\,dx \quad \Rightarrow \quad \underbrace{\sum_{i=1}^{n} f(x_i)\cdot\Delta x}_{\text{Riemann Sum} \;\Rightarrow\; S(n)} \quad \Rightarrow \quad \lim_{n\to\infty} S(n) \quad = \text{Area}$$

Now, to calculate a definite integral (area under a curve)…

$$\int_a^b f(x)\,dx = F(b) - F(a)$$