Name ______________________

Approximating Zeros of a Function Using Newton's Method

Date ______________ Period ______

Let function f be continuous on $[a,b]$ and differentiable on (a,b). If $f(a)$ and $f(b)$ have different signs, then, by the Intermediate Value Theorem, f must have a zero between a and b.

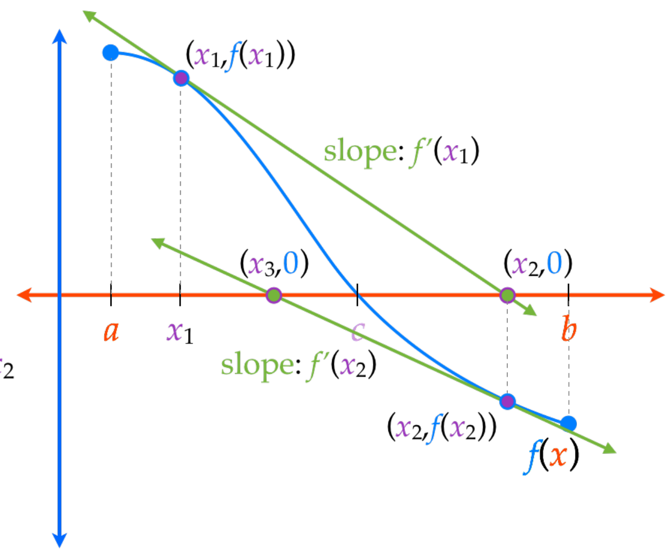
Estimate the zero: $x = x_1$

$y - f(x_1) = f'(x_1)(x - x_1)$ <= set $y = 0$, $x = x_2$ solve for x_2

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$ <= next estimate = x_2

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

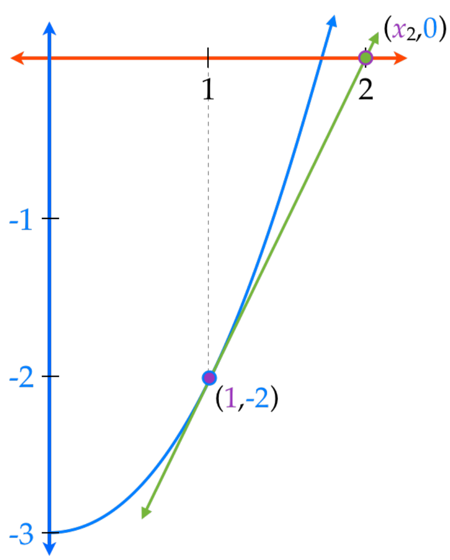
Repeated Application is called Newton's Method



Calculate three iterations of Newton's Method to approximate the zero of $f(x) = x^2 - 3$. Use $x_1 = 1$.

$f'(x) = 2x$

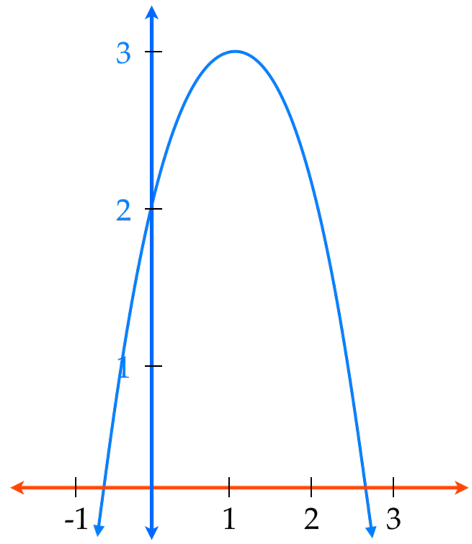
n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1					
2					
3					
4					



Calculate three iterations of Newton's Method to approximate the zeros of $f(x) = -x^2 + 2x + 2$.

$f'(x) = -2x + 2$

n	x_n	$f(x_n)$	$f'(x_n)$	$\frac{f(x_n)}{f'(x_n)}$	$x_n - \frac{f(x_n)}{f'(x_n)}$
1					
2					
3					
4					



Limitations to Newton's Method

If $f'(x_n) = 0$, method fails.

If f is non-differentiable at x-intercept (zero), method fails.

If each iteration moves further away from the x-intercept (zero), method fails.