The Relationship Between Differentiability and Continuity

Name ______________________

Date ____________ Period ____

An Alternative Form of the Derivative

The derivative of $f(x)$ at c is…

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

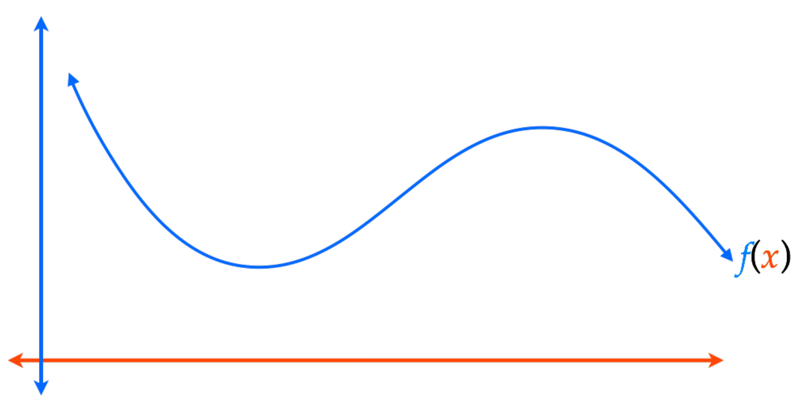
The slope of the tangent line of $f(x)$ at c.

In order for a limit to exist

$$\lim_{x \to c^-} \frac{f(x) - f(c)}{x - c} = \lim_{x \to c^+} \frac{f(x) - f(c)}{x - c}$$

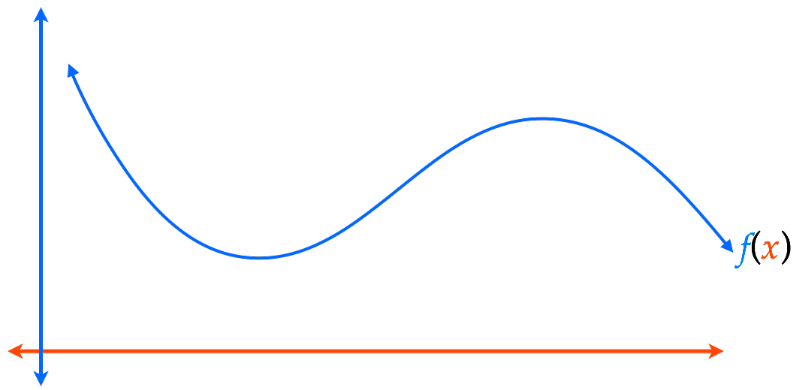
Differentiability Implies Continuity

If $f(x)$ is differentiable at c, then $f(x)$ is continuous at c.



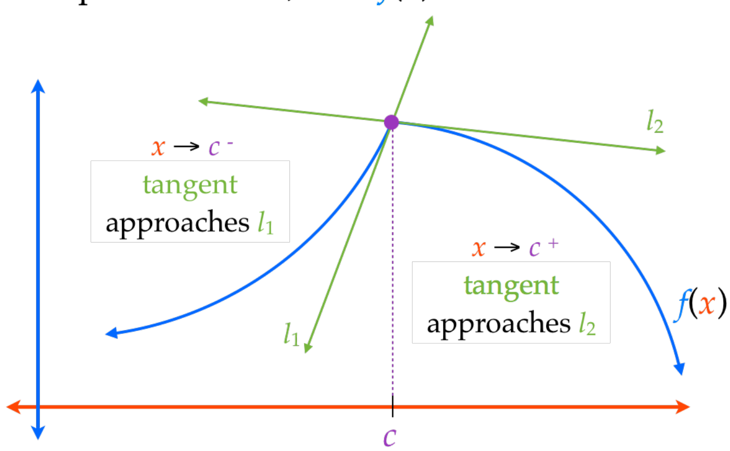
Continuity Does Not Imply Differentiability

If $f(x)$ is continuous at c, then $f(x)$ may or may not be differentiable at c.



Continuity Does Not Imply Differentiability

If $f(x)$ has a sharp turn at $x = c$, then $f(x)$ is not differentiable at $x = c$.



$l_1 \neq l_2$ implies

$\lim_{x \to c^-} \neq \lim_{x \to c^+}$

$\lim_{x \to c}$ DNE

$f'(c)$ does not exist

$f(x)$ is not differentiable at c

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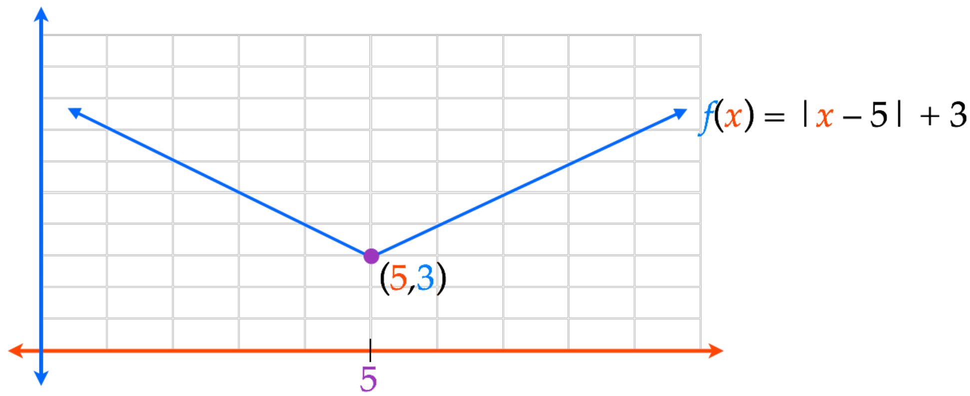

$f(x)$ is not differentiable at 5

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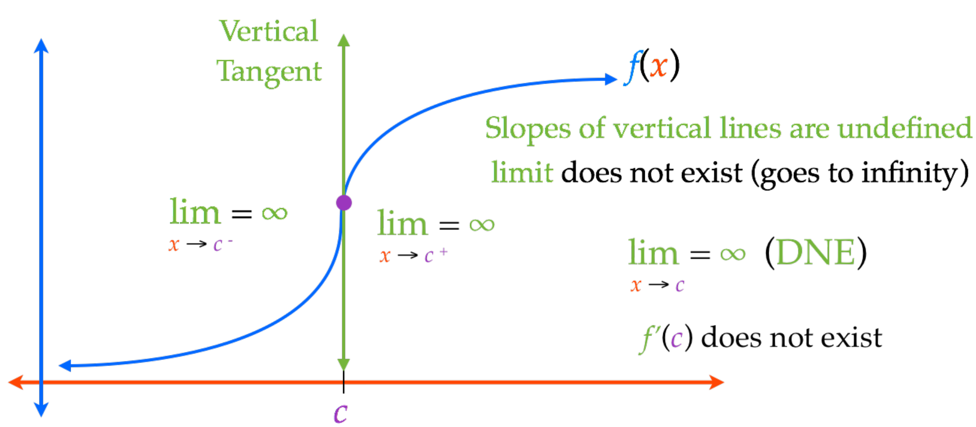
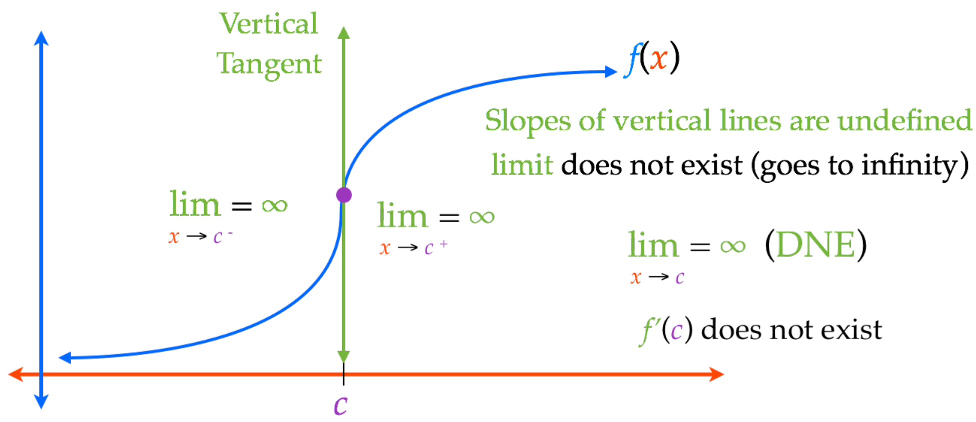


$f(x)$ is not differentiable at c

Continuity Does Not Imply Differentiability

If $f(x)$ has a vertical tangent line at $x = c$, then $f(x)$ is not differentiable at $x = c$.



$f(x)$ is not differentiable at c

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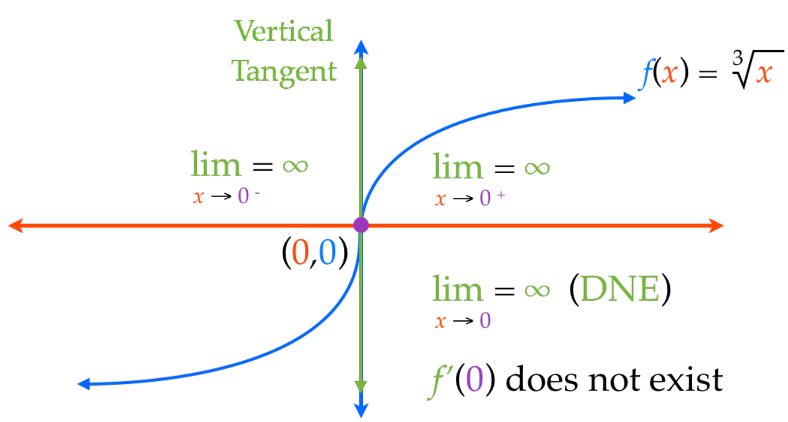

$f(x)$ is not differentiable at 0

Differentiable implies continuous

If $f(x)$ is differentiable at $x = c$, then $f(x)$ is continuous at $x = c$.

Continuous does not necessarily imply differentiable

Functions that have sharp turns at $x = c$ or vertical tangent lines at $x = c$ are not differentiable at $x = c$.