Cramer's Rule for a 3 by 3 Matrix

Name _____

Date _____ Period _____

When given a system of three equations,

$$ax + by + cz = s$$
$$dx + ey + fz = r$$
$$gx + hy + iz = t$$

We can create a coefficient matrix from the coefficients of the variables.

$$\det \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = D$$

Then the system of equations,

$$ax + by + cz = s$$
$$dx + ey + fz = r$$
$$gx + hy + iz = t$$

has solutions,

$$x = \frac{\begin{vmatrix} s & b & c \\ r & e & f \\ t & h & i \end{vmatrix}}{D} \qquad y = \frac{\begin{vmatrix} a & s & c \\ d & r & f \\ g & t & i \end{vmatrix}}{D} \qquad z = \frac{\begin{vmatrix} a & b & s \\ d & e & r \\ g & h & t \end{vmatrix}}{D}$$

D = determinant of the coefficient matrix

This is knows as Cramer's Rule

Cramer's Rule

1. Put System of Equations in Form...

$ax + by + cz = s$
$dx + ey + fz = r$
$gx + hy + iz = t$

2. Find Coefficient Matrix

3. Find Determinant of Coefficient Matrix

4. Calculate value of x.

$$x = \frac{\begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}}{D}$$

5. Calculate value of y.

$$y = \frac{\begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}}{D}$$

6. Calculate value of z.

$$z = \frac{\begin{vmatrix} & & \\ & & \\ & & \end{vmatrix}}{D}$$

Use Cramer's Rule to solve the following system

$$\begin{aligned} x - y + z &= -4 \\ 2x - 3y + 4z &= -15 \\ 5x + y - 2z &= 12 \end{aligned}$$

Use Cramer's Rule to solve the following system

$$x - 2y + 3z = 7$$
$$2x + y + z = 4$$
$$-3x + 2y - 2z = -10$$

Cramer's Rule

1. Put System of Equations in Form...

$$ax + by + cz = s$$
$$dx + ey + fz = r$$
$$gx + hy + iz = t$$

2. Find Coefficient Matrix

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = \begin{matrix}(aei + bfg + cdh) \\ - \\ (ceg + bdi + afh)\end{matrix}$$

3. Find Determinant of Coefficient Matrix

Set = D

4. Calculate value of x.

$$x = \frac{\begin{vmatrix} s & b & c \\ r & e & f \\ t & h & i \end{vmatrix}}{D}$$

5. Calculate value of y.

$$y = \frac{\begin{vmatrix} a & s & c \\ d & r & f \\ g & t & i \end{vmatrix}}{D}$$

6. Calculate value of z.

$$z = \frac{\begin{vmatrix} a & b & s \\ d & e & r \\ g & h & t \end{vmatrix}}{D}$$