

A rational function is a function that can be written in the form

$$R(x) = \frac{p(x)}{q(x)}$$

where $p(x)$ and $q(x)$ are polynomial functions and $q(x)$ is not the zero polynomial.

A rational function is Proper if the degree of $q(x)$ is greater than the degree of $p(x)$.

The **Horizontal Asymptote** of a Proper Rational Function is $y = 0$.

$$f(x) = \frac{1}{x^2 - 4} \quad \begin{array}{l} \text{deg} = 0 \\ \text{deg} = 2 \end{array}$$

$$g(x) = \frac{x}{2x^2 - 18} \quad \begin{array}{l} \text{deg} = 1 \\ \text{deg} = 2 \end{array}$$

$$h(x) = \frac{x^2 + 3}{x + 1} \quad \begin{array}{l} \text{deg} = 2 \\ \text{deg} = 1 \end{array}$$

A rational function is Proper if the degree of $q(x)$ is greater than the degree of $p(x)$.

The **Horizontal Asymptote** of a Proper Rational Function is $y = 0$.

To find the **horizontal/oblique asymptote** of an improper rational function.
use long division to express your improper rational function as...

$$R(x) = \frac{p(x)}{q(x)} = f(x) + \frac{r(x)}{q(x)}$$

1. If $f(x) = c$, a constant, the line $y = c$ is the **horizontal asymptote** of function R .
2. If $f(x) = ax + b$, then the line $y = ax + b$ is an **oblique asymptote** of function R .
3. All other cases, the graph of R approaches f , and there are no horizontal or oblique asymptotes.

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use long division to express your improper rational function as...

$$R(x) = \frac{p(x)}{q(x)} = f(x) + \frac{r(x)}{q(x)}$$

The difference between the **degree of $p(x)$** and **degree of $q(x)$** gives the **degree of asymptote**

$$R(x) = \frac{x + 3}{x + 1}$$

To find the **horizontal/oblique asymptote** of an improper rational function.
use long division to express your improper rational function as...

$$R(x) = \frac{p(x)}{q(x)} = f(x) + \frac{r(x)}{q(x)}$$

The difference between the **degree of $p(x)$** and **degree of $q(x)$** gives the **degree of asymptote**

$$R(x) = \frac{x^3 + 2x}{x^2 + 1}$$

To find the **horizontal/oblique asymptote** of an improper rational function.
use long division to express your improper rational function as...

$$R(x) = \frac{p(x)}{q(x)} = f(x) + \frac{r(x)}{q(x)}$$

The difference between the **degree of $p(x)$** and **degree of $q(x)$** gives the **degree of asymptote**

$$R(x) = \frac{2x^3 - x}{x + 5}$$

Find the horizontal/oblique asymptote of the improper rational function.

$$R(x) = \frac{3x^4 - x^2}{x^3 - x^2 + 1}$$

Find the horizontal/oblique asymptote of the improper rational function.

$$R(x) = \frac{2x^5 - x^3 + 2}{x^3 - 1}$$

Find the horizontal/oblique asymptote of the improper rational function.

$$R(x) = \frac{8x^2 - x + 2}{4x^2 - 1}$$