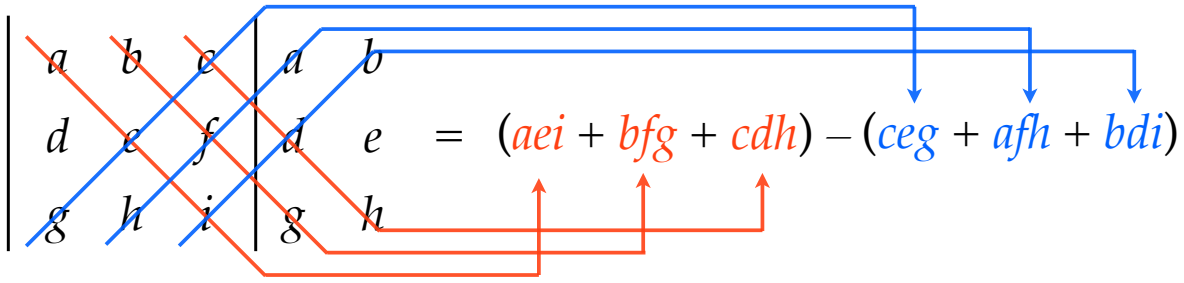
Cramer's Rule for Three Equations

Name _____

Date _____ Period _____

To find the determinant of a 3×3 matrix, find the sum of the products of the red diagonals, then subtract the sum of the products of the blue diagonals.



Given a system of three equations,

$$ax + by + cz = s$$
$$dx + ey + fz = r$$
$$gx + hy + iz = t$$

We can create a coefficient matrix from the coefficients of the variables.

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = D$$

Given a system of three equations,

$$ax + by + cz = s$$
$$dx + ey + fz = r$$
$$gx + hy + iz = t$$

has solutions,

$$x = \frac{\begin{vmatrix} s & b & c \\ r & e & f \\ t & h & i \end{vmatrix}}{D} \qquad y = \frac{\begin{vmatrix} a & s & c \\ d & r & f \\ g & t & i \end{vmatrix}}{D} \qquad z = \frac{\begin{vmatrix} a & b & s \\ d & e & r \\ g & h & t \end{vmatrix}}{D}$$

D = determinant of the coefficient matrix

This is knows as Cramer's Rule

Steps to use Cramer's Rule

1. Put System of Equations in Form...

$$ax + by + cz = s$$
$$dx + ey + fz = r$$
$$gx + hy + iz = t$$

2. Find Coefficient Matrix

$$\det\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} = D$$

3. Find Determinant of Coefficient Matrix

Set = D

4. Calculate value of x.

$$x = \frac{\begin{vmatrix} s & b & c \\ r & e & f \\ t & h & i \end{vmatrix}}{D}$$

5. Calculate value of y.

$$y = \frac{\begin{vmatrix} a & s & c \\ d & r & f \\ g & t & i \end{vmatrix}}{D}$$

6. Calculate value of z.

$$z = \frac{\begin{vmatrix} a & b & s \\ d & e & r \\ g & h & t \end{vmatrix}}{D}$$

Use Cramer's Rule to solve the following system

$$\begin{aligned} x - y + z &= -4 \\ 2x - 3y + 4z &= -15 \\ 5x + y - 2z &= 12 \end{aligned}$$

Use Cramer's Rule to solve the following system

$$\begin{aligned} x - 2y + 3z &= 7 \\ 2x + y + z &= 4 \\ -3x + 2y - 2z &= -10 \end{aligned}$$

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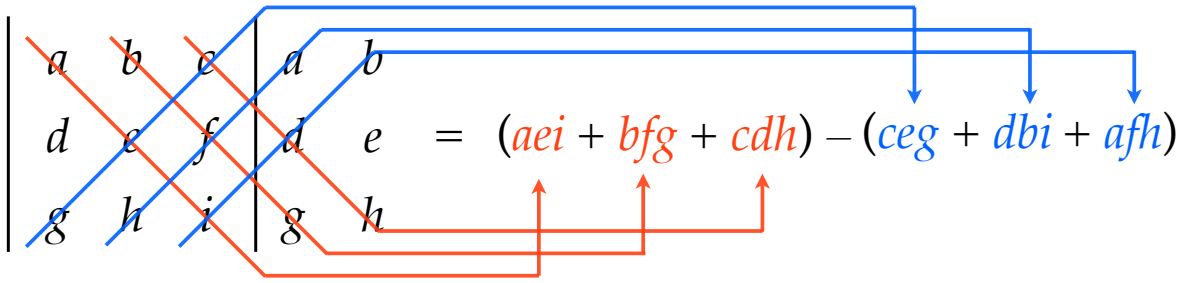

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