LESSON 25

Weighted Averages and Expected Values

Key Concept

In this lesson, *weighted averages* are introduced as a shortcut for finding the mean when the frequencies of data values are known. However, students soon see that the idea can be generalized to situations in which each data value has a relative importance or weight. The idea also extends to probability. When a discrete random variable can take on a set of possible values, each with a known probability, the *expected value* of the random variable is simply the weighted average, where the probabilities are taken as the weights.

Key Question: Example 1

In this example, which numbers are the data values and which are the weights? Does it matter which numbers you take as the data values and which numbers you take as the weights?

The data values are the grades (4.0, 3.5, 3.7) and the weights are the credits (4, 3, 2). Choosing these correctly is important, since you divide by the sum of the weights (you get a different result if you divide by the sum of the data values).

Key Question: Example 2

Why does it make sense that Thomas's fluency score (6.45) is greater than the "unweighted" average of his three scores in grammar, vocabulary, and pronunciation $\left(\frac{8+6+5}{3} = 6.\overline{3}\right)$?

The weighted average gives greater weight to the grammar score than to the scores in the other categories. Thomas's highest score is in grammar so this pulls up his weighted average.

Differentiated Instruction

Expected Value Challenge students to explain the connection between the formula for the weighted average on page 120, the alternate formula on page 121, and the formula for the expected value on page 122. Help students see that when the sum of the weights is 1, the formula on page 120 becomes the formula on page 121. In the case of expected value, the probabilities are greater than or equal to 0 and less than or equal to 1, and the sum of all the probabilities is 1, so they may be used as the weights in the formula on page 121, which results in the formula for expected value on page 122.

Key Question: Example 3

If you spin the spinner in part b 1000 times, what do you expect the sum of the outcomes to be? Why?

4250; since the expected value of a spin is 4.25, the expected sum of 1000 spins is $4.25 \times 1000 = 4250$.

Key Question: Example 4

Why do you subtract $100 from each of the amounts of money that contestants can win by rolling the giant number cube?

You must subtract $100 from the prize amounts to find the net amount the contestants win.

Activity

- **Materials** number cube
- **Goal** Students roll a number cube to simulate buying a box of cereal and receiving one of six prizes. They use this simulation to estimate the number of boxes they would expect to have to buy in order to collect all six prizes.
- **Teaching Strategy** There are many interactive online simulations for this problem. These simulations make it possible to run hundreds of trials of the simulation and to change the parameters of the problem. Invite students to use these online tools and report their results.
- **Key Discovery** You expect to have to buy 14.7 boxes in order to collect all six prizes.

Closing the Lesson

Have students answer the following question: How do you find the expected value of a discrete random variable?

Multiply each possible value of the random variable by its probability and add these products.

Homework Help

Example 1: Exs. 1–3
Example 2: Exs. 4, 5
Example 3: Exs. 6, 7
Example 4: Ex. 8
Enrichment: Exs. 9–10

Homework Check

To quickly check student understanding of key concepts, go over the following exercises: 1, 4, 7, 8.

ANSWERS

Check Answers

1. 1.87
2. 10.2 inches
3. 5.5
4. about 6.83
5. lose $0.55

Activity Answers

1. Let each possible outcome (1, 2, 3, 4, 5, and 6) represent one of the prizes.
2. Check students' work.
3. Check students' work.
4. Answers will vary. The average number of boxes should be close to 14.7, which is the theoretical expected value.

Exercise Answers

1. about 6.1 ounces
2. 87
3. $1921.52
4. 82.75%
5. about 0.1%
6. 7
7. about 21.7
8. **a.** lose about $1.67
   b. about $167
9. **a.** 0
   b. Yes; the expected value improves to 0.0625.
10. Yes; the expected value of each roll is that 0 chips will be exchanged.
11. Disagree; although the total number of coupons given away increased from 600 to 650, the expected value of a coupon decreased from $9.17 to $7.85.