

Title: Frequency Transformation

Size: 96" W x 72" H x 1.5" D (two 48" x 72" x 1.5" canvases side-by-side, connected)

Medium: Acrylic on Canvas

Description/Artist Statement:

The painting *Frequency Transformation* is a mathematical visualization of the Fast Fourier transform (FFT). Called "the most important numerical algorithm of our lifetime" by mathematician Gilbert Strang, the FFT is an essential algorithm in science, engineering, and mathematics, with many applications in art and communications.

The Fourier transform is a mathematical method to represent a signal, like a sound, picture, or electrical signal, as the sum of sinusoids of different frequencies. Each frequency sinusoid is scaled by some magnitude and shifted by some phase. The Fourier transform tells you the magnitude and phase of each frequency component.

The FFT is an algorithm for efficiently computing the Fourier transform of a discrete signal. A discrete signal is a sequence of data samples, like an audio recording sampled at 44.1 kHz. The input signal in this painting is a measurement of elevation of soil and root formations in Bridle Trails State Park. In this painting, the top row depicts the discrete input signal to our example FFT, both visually (through shape, color, and texture) and numerically (using symbols).

FFT is a recursive divide-and-conquer algorithm. At each step, the input signal is split into two parts based on even and odd indices. FFT is called recursively on the even and odd parts. The results of FFT(odd) are multiplied by specific roots of unity and then added to the results of FFT(even). The crisscross patterns created by this data flow are called "butterfly diagrams."

The "root" structure of the painting is the signal-flow graph of the FFT. The signal flows from the top row (input), through bit reverse index reordering (in binary), through each stage of the FFT butterfly network, and finally to the bottom two rows, where the signal is represented in the frequency domain with magnitude and phase.

The painting depicts a signal with 128 data points and $\log_2 128 = 7$ rows, a visualization of the $O(N \log N)$ computational complexity of the FFT algorithm.

The number system constructed for this painting is based on a visual vocabulary inspired by lush life and transformation. It is base-20 (vigesimal), a common base used for world number systems (e.g. Maya and Kaktovik numerals). According to scientist Neil Gershenfeld, 20 basic building blocks can be used to construct many complex systems (e.g. all proteins are made of 20 amino acids). The numerical glyphs are based on transformative symbols from the natural world, such as shapes found in butterfly wings (a nod to the "butterflies" that appear in the algorithmic structure of the FFT).

Other visual symbols in this painting include abstracted tree roots and ripples on the surface of the water. The base of the trees (trunks, stumps, and root tops) combine to form the visual shape of the input signal. The root structure visually represents the structure of the FFT algorithm. A Fourier transform can be described as a projection onto a set of sinusoids of all different frequencies, represented abstractly as reflections on the ripples in the water.

The title "Frequency Transformation" refers not only to the transformation of a signal into the frequency domain, but also to the shift in mood and brainwave patterns that occur when people are immersed in nature. As a scientist, the process of discovery about how the natural world works brings me similar feelings of wonder and connection.