8.NS Comparing Rational and Irrational Numbers

Task

For each pair of numbers, decide which is larger without using a calculator. Explain your choices.

a. π^2 or 9

b. $\sqrt{50}$ or $\sqrt{51}$

c. $\sqrt{50}$ or 8

d. -2π or -6

IM Commentary

This task can be used to either build or assess initial understandings related to rational approximations of irrational numbers.

The Standards for Mathematical Practice focus on the nature of the learning experiences by attending to the thinking processes and habits of mind that students need to develop in order to attain a deep and flexible understanding of mathematics. Certain tasks lend themselves to the demonstration of specific practices by students. The practices that are observable during exploration of a task depend on how instruction unfolds in the classroom. While it is possible that tasks may be connected to several practices, only one practice connection will be discussed in depth. Possible secondary practice connections may be discussed but not in the same degree of detail.

This task allows students to construct viable arguments by identifying and justifying the larger of two expressions in each part. As students reason through determining which amount is greater, they will use "previously established results," their knowledge of products, squares, and square roots of benchmark numbers, to reason about slightly larger or smaller numbers. In order to identify a larger expression without making an exact numerical calculation, students will build a logical progression of statements, starting with previous knowledge, and building from there. Students might reason, "I know that three squared is nine, and pi is slightly larger than three, therefore pi squared should be slightly larger than 9."

The teacher could have students individually solve each of the problems and then discuss their solutions in small group to promote collaboration and refinement of their solutions. They would then verbalize their final answers in large group defending or again refining their answers. The teacher could promote the development of number sense and further get to the meaning of the standard by posing questions such as "How would these comparisons look on a number line?" This approach of reasoning about quantities without making exact calculations ties to MP.7, Look for and make use of structure. This is also a prime opportunity for the teacher to make sure that students address MP.6 and use clear and precise language when justifying their conclusions.

Solution

a. $\pi > 3$ so $\pi^2 > 9$.

b. $\sqrt{50} < \sqrt{51}$ because $50 = (\sqrt{50})^2 < (\sqrt{51})^2 = 51$.

c. $7^2 = 49$ and $8^2 = 64$. Thus we have that $\sqrt{49} < \sqrt{50} < \sqrt{64}$. So $\sqrt{50} < 8$.

d. $\pi > 3$ so $2\pi > 2 \cdot 3$. If you look at these numbers on the number line, that means that 2π is farther to the right than 6. When you look at their opposites, -2π will be farther to the left than -6, so $-2\pi < -6$.