G.1 Constructions and Rigid Motions

- **Know and be able to use precise definitions of geometric terms.**
- **Make formal geometric constructions by hand and using geometry software.**
- **Given a geometric figure and a rotation, reflection, or translation draw the transformed figure.**
- **Develop definitions of rotation, reflection, and translation.**
- **Represent transformations in the plane; describe transformations as functions whose inputs and outputs are points in the plane.**
- **Describe the rotations and reflections that carry a given quadrilateral or regular polygon onto itself.**
- **Prove that the measures of the interior angles of a triangle have sum 180°.**

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

Students have worked with geometric shapes since kindergarten. In grade 4, they classified two-dimensional shapes by properties of their sides and angles, and in grade 5, they classified these shapes in a hierarchy based on properties. In grade 8, they were introduced to the concepts of rotation, reflection, and translation via

physical models, transparencies, or geometry software. They defined congruence of two-dimensional figures in terms of these rigid motions, understanding that two figures were congruent if one could be obtained from the other via a sequence of rigid motions. Given a two-dimensional figure, students described the effect of a rotation, reflection, or translation on the figure in terms of coordinates. Students also worked with determining distances in the coordinate plane using the Pythagorean Theorem.

In this unit, the focus is on more precise definitions for many of the geometric figures that students worked with before high school. These definitions are given in terms of a few undefined notions, in contrast to definitions that students may have used earlier (e.g., a square is a rectangle in which all sides have equal length). The unit begins by revisiting this prior knowledge. Next, students construct, by hand and using technology, perpendicular lines, parallel lines, equilateral triangles, squares, etc. and develop formal definitions of these objects. Students then continue to examine rigid motions in the plane. In the second part of the unit, students use coordinates to represent rigid motions as functions, that is, as mappings of points in the plane to points in the plane. The third part of the unit gives students an opportunity to attend to precision when describing shapes with various properties, e.g., a quadrilateral with exactly one rotation symmetry and no reflection symmetry.

In unit G2, students work with sequences of rigid motions, specifying sequences that will carry one figure onto another, thus addressing the second part of standard G-CO.A.5. Congruence is defined, as opposed to understood, in terms of such sequences. Students continue to see and use geometric constructions in units G2, G3, and G6, and work with geometric diagrams in all geometry units.

G.1.0 Pre-unit diagnostic assessment

Assess students' ability to

• draw geometric shapes that satisfy given conditions;
• given two congruent figures, describe a sequence that exhibits the congruence between them;
• describe the effects of rigid motions on two-dimensional figures using coordinates.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade)

perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

G.1.1 Definitions and Geometry

Recall and reconnect with the meanings of geometry terms that will be used in this unit.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

G.1.2 Introduction to Constructions

• **Use a compass and a straightedge to construct various geometric figures.**
• **Use geometric software to construct various geometric figures.**
• **Begin to understand the formal definitions of geometric figures through constructions.**

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

G.1.3 Representing transformations

• Draw the result of a rotation, reflection, or translation on given geometric figures by hand.
• Draw the result of a rotation, reflection, or translation on given geometric figures using geometric software.
• Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.
Tasks
G-CO Reflected Triangles

G.1.4 Mid-Unit Assessment

Assess students' ability to

• **make formal geometric constructions (copying a segment);**
• **construct a square;**
• **given a geometric figure and a rotation, reflection or translation draw the transformed figure;**
• **understand and explain the formal definition of rotation.**

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

G.1.5 Mappings of the plane

• **Represent transformations in the plane using various tools (transparencies, geometry software, etc.).**
• **Describe transformations as functions that take points in the plane as inputs and given other points as outputs.**
• **Compare transformations that preserve distance and angle and those that do not (translation vs. horizontal stretch).**

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

Tasks

G-CO Horizontal Stretch of the Plane

G.1.6 Symmetries

• **Understand the idea of reflection symmetry.**
• **Understand the idea of rotation symmetry.**
• **Be able to describe the rotations and reflections that carry a given rectangle, parallelogram, trapezoid, or regular polygon onto itself.**

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

Tasks

G-CO Seven Circles II

G-CO Symmetries of a quadrilateral I
G-CO Symmetries of a quadrilateral II
G-CO Symmetries of rectangles

G.1.7 Bringing it all together

Create a given design through construction and rigid motions.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

G.1.8 Summative Assessment

Assess students' ability to

• **give definitions of geometric terms;**
• **make formal geometric constructions (parallel and perpendicular lines);**
• **construct a square and explain why the construction yields a square;**
• **given a geometric figure and a rotation, reflection or translation draw the transformed figure;**
• **understand and explain the formal definition of rotation;**
• **describe transformations as functions;**
• **given a square or trapezoid, describe the rotations and reflections that map it onto itself.**

Students construct (with both compass and straight-edge and technology,

rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.