Illustrative Mathematics

F-IF Identifying graphs of functions

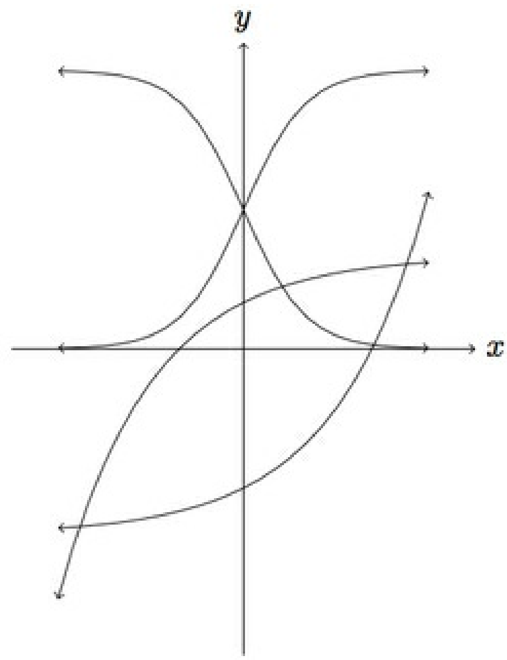
Alignments to Content Standards: F-IF.C.7

Task

Consider the following four functions

- $f(x) = \frac{3}{1+e^{-3x}}$
- $g(x) = 1 - \frac{e^{-x}}{2}$
- $h(x) = -2 + \frac{e^{x}}{2}$
- $k(x) = \frac{3}{1+e^{3x}}$

Below are four graphs of functions shown for $-2 \leq x \leq 2$. Match each function with its graph and explain your choice:



IM Commentary

This task can be used for assessment or instruction. In either case, a decision needs to be made by the instructor whether or not to label numbers on the x-y axes. The domain for the functions is $-2 \leq x \leq 2$ and the displayed range is $-3.3 \leq y \leq 3.3$. Labelling the axes might encourage students to plug in some numbers and evaluate the functions in order to pair them up with graphs. This is a good approach. A second approach is to determine the shape of the graphs from the structure of the expressions and for this marking numbers on the axes is not needed. The markings have been left off to give the teacher a choice.

The goal of this task is to get students to focus on the shape of the graph of the equation $y = e^x$ and how this changes depending on the sign of the exponent and on whether the exponential is in the numerator or denominator. It is also intended to develop familiarity, in the case of f and k, with the functions which are used in logistic growth models, further examined in ''Logistic Growth Model, Explicit Case'' and ''Logistic Growth Model, Abstract Verson.''

Solutions

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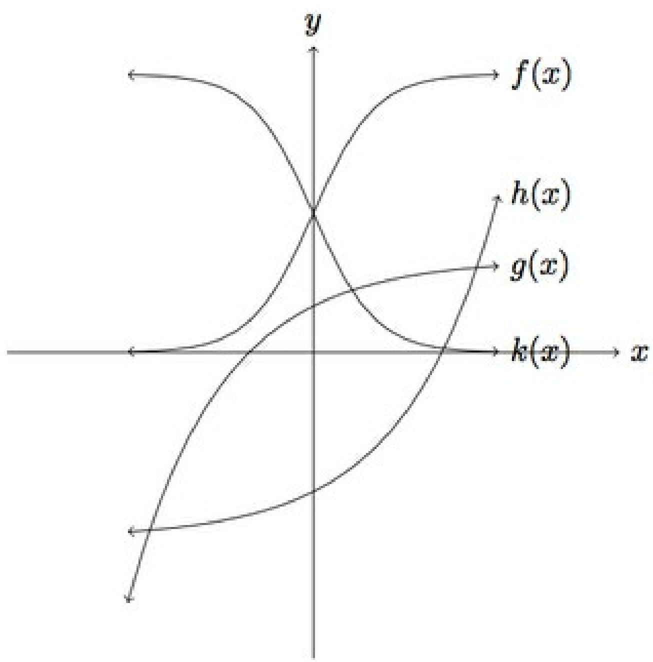
Solution: 1 Evaluating functions

The graphs of the four functions are identified below, followed by an explanation of how the identification can be made.



We may begin by observing that $f(x)$ and $k(x)$ always take positive values because the exponential function takes positive values. Evaluating these functions when $x = 1$ gives a value of about 2.86 for $f(1)$ and about 0.14 for $k(1)$. This information determines the graphs of f and k as indicated in the picture above.

As for g and h, we may determine this by evaluating at $x = 0$. Here we find $g(0) = 1 - \frac{e^0}{2} = \frac{1}{2}$ and $h(0) = -2 + \frac{e^0}{2} = \frac{-3}{2}$. Observing the y-intercepts of the remaining two graphs tells us which one is the graph of g and which one is the graph of h.

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Solution: 2. Abstract reasoning

As above, observation about the structure of the expressions shows that the graphs of f and k have to be the two which are above the x-axis. To determine which is which, note that e^x is an increasing function of x, that is as the value of x increases so does the value of e^x. Thus $1 + e^{3x}$ is an increasing positive function of x. Thus $\frac{1}{1+e^{3x}}$ is a decreasing positive function of x and this tells us which graph is the graph of k and, hence, which one is the graph of f. Alternatively, the same reasoning can be applied to determine that $1 + e^{-3x}$ is a decreasing function of x and so $\frac{3}{1+e^{-3x}}$ is an increasing function of x.

The reasoning of the previous paragraph shows that both g and h are increasing functions of x. On the other hand, h increases exponentially fast, like e^x, while g increases like much more slowly, taking values closer and closer to 1 as x increases.