Illustrative Mathematics

F-BF,IF The Canoe Trip, Variation 2

Alignments to Content Standards: F-IF.B.4 F-IF.B.5 F-BF.A.1.a

Task

Mike likes to canoe. He can paddle 150 feet per minute. He is planning a river trip that will take him to a destination about 30,000 feet upstream (that is, against the current). The speed of the current will work against the speed that he can paddle.

a. Mike guesses that the current is flowing at a speed of 50 feet per minute. Assuming this is correct, how long will it take for Mike to reach his destination?

b. Mike does not really know the speed of the current. Make a table showing the time it will take him to reach his destination for different speeds:

Speed of Current (feet per minute)	Mike's Speed (feet per minute)	Time for Mike to travel 30,000 feet (minutes)
0		
50		
100		
140		
149		
s		

c. The time T taken by the trip, in minutes, as a function of the speed of the current is s feet/minute. Write an equation expressing T in terms of s. Explain why $s = 150$ does not make sense for this function, both in terms of the canoe trip and in terms of the equation.

d. Sketch a graph of the equation in part (c). Explain why it makes sense that the graph has a vertical asymptote at $s = 150$.

IM Commentary

The primary purpose of this task is to lead students to a numerical and graphical understanding of the behavior of a rational function near a vertical asymptote, in terms of the expression defining the function. The canoe context focuses attention on the variables as numbers, rather than as abstract symbols. Variation 1 of this task uses function notation and expects students to derive the formula for the function directly, without the aid of a table.

The task also provides an opportunity to discuss mathematical models, their interpretation, and their limits. For example, teachers could ask if it makes sense for s to be negative. This might correspond to a flow of water moving in the same direction as Mike, and indeed the equation in the solution gives the correct answer in that case.

More fanciful, and requiring a longer discussion, is the question of whether it makes sense to consider values of s larger than 150. If $s = 300$, for example, a naive application of the formula predicts that Mike will arrive at his destination in -200 minutes! It is reasonable to say that negative times do not make sense and to exclude values of s greater than 150. However, value -200 could also be interpreted as referring to an event that takes place 200 minutes *before* the trip starts. If Mike had been at his destination 200 minutes ago, then a river which was flowing at 300 feet per minute against his direction of travel would push him precisely the 30,000 feet from his destination that the problem began with.

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Solution

a. The current is working against Mike's paddling, so he'll move 150 - 50 = 100 feet per minute. Thus, it will take him 30000/100 = 300 minutes = 5 hours of paddling to reach his destination.

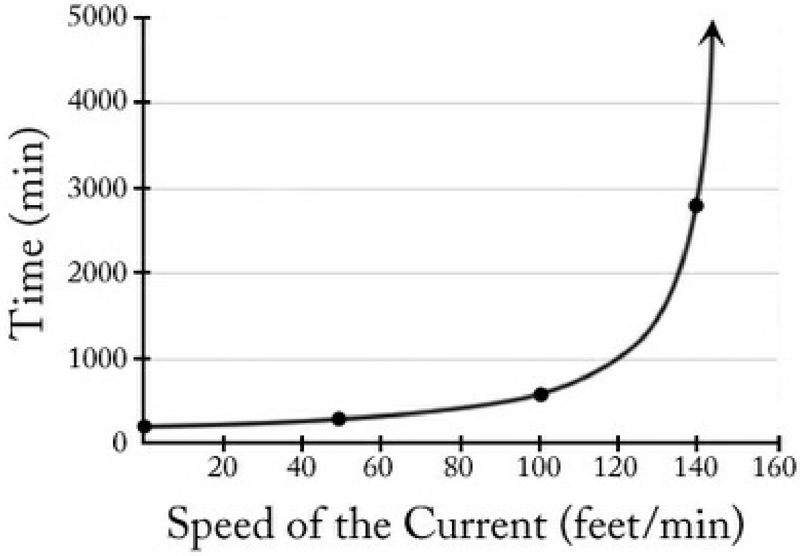
b.

Speed of Current (feet per minute)	Mike's Speed (feet per minute)	Time for Mike to travel 30,000 feet (minutes)
0	150	200
50	100	300
100	50	600
140	10	3,000
149	1	30,000
s	$150 - s$	$\frac{30{,}000}{150-s}$

c. From the last row of the table we see that

$$T = \frac{30,000}{150 - s}.$$

If the speed of the current is 150 feet per minute, then Mike isn't strong enough to paddle against it. Paddling his hardest, he would just sit still so he would never reach his destination; that is, there is no "time required for Mike to travel 30,000 feet" because he can't travel. If you put $s = 150$ into the equation you will find yourself trying to divide by zero, which also tells you that the answer is undefined.



d. The graph shows the values of T getting larger and larger as s gets close to 150. The graph can never touch the vertical line at $s = 150$ because that would imply that there is a "time it takes Mike to travel 30000 ft" when the current is 150, but there isn't. When s is close to 150 the speed of the current is close to Mike's paddling speed. That means that Mike is traveling very slowly. Thus, when we calculate the time it takes him to travel 30,000 ft we are dividing by a very small number, which gives a very large answer. The closer s is to 150, the closer Mike's actual speed is to zero, and the larger the time it takes.