

N-Q Bus and Car

Alignments to Content Standards: N-Q.A.3

Task

Julio went to Germany to watch an international soccer tournament. He first watched Argentina play Germany in Berlin, Germany. The next day Julio went to Frankfurt, Germany to watch Brazil play France.

To get from Berlin to Frankfurt for this second game, Julio took a bus from Berlin to Erfurt (303 km); then he rented a car and drove from Erfurt to Frankfurt (254 km). Julio drove on German highways, called autobahns, which have no general speed limit for passenger vehicles; however, buses have an enforced speed limit of 80 km/hr.

- If the bus drove 80 km/hr from Berlin to Erfurt and Julio drove 130 km/hr from Erfurt to Frankfurt, what was the total amount of time it took Julio to travel from Berlin to Frankfurt (not counting the transfer time between the bus and the car)? Give your answer to a reasonable level of accuracy.
- What fraction of the time during the trip did Julio spend on the bus? In the car?
- What was Julio's average speed for the entire trip?
- If Julio had rented a car and driven from Berlin to Erfurt at 130 km/hr and then taken a bus from Erfurt to Frankfurt at 80 km/hr, would he have arrived sooner? Explain your answer.
- How long would Julio's trip have taken if he had ridden the bus the entire way?

IM Commentary

This task operates at two levels. In part it is a simple exploration of the relationship between speed, distance, and time. Part (c) requires understanding of the idea of average speed, and gives an opportunity to address the common confusion between average speed and the average of the speeds for the two segments of the trip.

At a higher level, the task addresses N-Q.3, since realistically neither the car nor the bus is going to travel at exactly the same speed from beginning to end of each segment; there is time traveling through traffic in cities, and even on the autobahn the speed is not constant. Thus students must make judgements about the level of accuracy with which to report the result.

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Solution

The distances are reported in whole kilometers, so it is likely that they are only accurate to the nearest kilometer, if that. A kilometer at 80 km/hr is 45 seconds, so it is unlikely that reporting the answer to an accuracy greater than the nearest minute will be meaningful. And, taking into account the possibility that the bus and car might drive at slower speeds getting to and from the autobahn, possibly compensating by exceeding the speed limit slightly on the autobahn, it seems reasonable to report times in a much broader range. Thus, rounding intermediate calculations to the hundredths place is more than sufficient.

a. We compute the time taken on each of the two legs by dividing the distance traveled by the speed on that leg of the trip, and then add those two times together:

$$\text{Time taken} = \frac{303 \text{ km}}{80 \text{ km/hr}} + \frac{254 \text{ km}}{130 \text{ km/hr}} \approx 5.74 \text{ hours.}$$

Thus the travel time is between 5.5 and 6 hours.

b. Of the approximately 5.74 total hours of travel time, $\frac{303}{80} = 3.79$ of it was spent on the bus. As a percent of the total travel time, this is

$$\frac{3.79}{5.74} \approx 0.66 = 66\%.$$

Thus, about two thirds of the time was spent in the bus, leaving about one third of the time in the car.

c. One way to solve this is to note that Julio traveled a total of $303 + 254 = 557$ miles over 5.74 hours, his average speed was

$$\frac{557 \text{ km}}{5.74 \text{ hr}} = 97 \frac{\text{km}}{\text{hr}} \approx 100 \frac{\text{km}}{\text{hr}}.$$

Alternatively, since Julio has spent 66% of the time traveling at 80 km/hr and 34% of the time traveling at 130 km/hr, his average speed over the whole trip is given by

$$0.66 \times 80 + 0.34 \times 130 = 97 \frac{\text{km}}{\text{hr}},$$

which we again report as approximately 100 km/hr.

d. Julio would have indeed reached his destination faster had he taken the car for the first leg and the bus for the second leg, since it is obviously helpful to take the fastest mode of transport on the longer of the two segments. We justify this numerically by switching the speeds from the solution to the first part of this problem:

$$\text{Time taken} = \frac{254 \text{ km}}{80 \text{ km/hr}} + \frac{303 \text{ km}}{130 \text{ km/hr}} \approx 5.51 \text{ hours},$$

a savings of about a quarter of an hour over the previous time of 5.74 hours.

e. Had Julio taken the bus, at a rate of 80 km/hr, for the entire $254 + 303 = 557$ kilometers, it would have taken him $\frac{557}{80} = 6.96$ hours, or about 7 hours, over an hour longer than the previous two approaches.

