Illustrative Mathematics

Integrated Math 1

In Integrated Math 1, students build on the descriptive statistics, function, expressions and equations, and geometric work first encountered in the middle grades, taking the ideas further while using more formal reasoning and precise language. Specifically, students add to the statistical work from the middle grades by working with standard deviation, describing statistical distributions more precisely, and measuring goodness-of-fit with residuals and correlation coefficient. They formalize their concept of function and encounter exponential functions as well as other examples of non-linear functions. Explicit comparisons and contrasts are made between linear and exponential functions. They develop their abilities to see structure in expressions to show that expressions involving several operations are equivalent (for example, grasping that "substitution" works at various levels of complexity), and they solve equations and inequalities by writing a series of equivalent statements, justifying each step. In the Geometry units in this course, students develop precise definitions of shapes and relationships previously encountered. Instead of informal arguments, they create more rigorous proofs. Students use geometric transformations to define congruence and, once triangle congruence and criteria are established, they prove a variety of theorems and solve problems related to these ideas.

M1.1 One Variable Statistics

• Create dot plots, histograms, and box plots.
• Use available classroom technology to create histograms and box plots and calculate measures of center and spread.
• Use terms such as "flat," "skewed," "bell-shaped," and "symmetric" to describe data distributions.
• Analyze and compare data sets.
• Understand relationships between mean and median for symmetrical and skewed data distributions.
• Recognize outliers when they exist, and know to investigate their source.
• Know that outliers affect the mean, but not the median of a data set.
• Describe variability by calculating deviations from the mean.
• Compare two data sets with the same means but different variabilities, and contrast them by calculating the deviation of each data point from the mean.
• Understand that IQR is a description of variability better suited to a skewed distribution.
• Work with two-way tables.

Instead of creating representations of data, the emphasis in high school is on interpreting representations and judiciously interpreting measures of center and spread. Students describe the shape of a data distribution in more detail (symmetric, skewed, flat, or bell-shaped). Students develop a more precise understanding of measures of center and understand relationships between mean and median for symmetrical and skewed data distributions. They learn that outliers affect the mean of a data set but not the median. They recognize outliers when they exist and learn to investigate their source. Students learn that standard deviation is a measure of spread, that a larger standard deviation means the data are more spread out, and to understand standard deviation as "typical distance from the mean" for a symmetrical distribution. They also understand that interquartile range is a description of variability better-suited to a skewed distribution. Finally, students are introduced to two-way frequency tables and understand how to interpret relative frequencies in the context of the data represented in the tables.

In grades 6 to 8, students were introduced to data sets and different ways to represent data (histograms, dot plots, box plots). Statistics is introduced as a tool to answer questions about a population that have variability in the answer. Students learn about measures of center (median, mean) and measures of variability (interquartile range, mean absolute deviation), using them to draw informal comparative inferences about two populations. Students build on and expand their understandings of statistics in this unit. The key characteristics (measures of shape, center, and spread) are again seen and in addition, students may further describe the shape of a data distribution (symmetric, skewed, flat or bell shaped) and summarize by a statistic measuring center and a statistic measuring spread. Instead of creating representations of data, the emphasis in high school is on interpreting representations and judiciously interpreting measures of center and spread. Students develop more precise understanding of measures of center. They learn that mean and median are equal for symmetrical distributions, explain why mean and median are not equal in examples of skewed distributions, select median as the better measure of center for skewed distributions, and make generalizations about what kinds of distributions have means larger than medians and which have medians larger than means. Students learn that standard deviation is a measure of spread, that a larger standard deviation means the data are more variable or spread out, and the meaning of standard deviation as "typical distance from the mean" for a symmetrical distribution. To aid in developing their understanding, students will calculate a standard deviation by hand for a small data set at least once. Given different visual representations of data (box plots, histograms, dot plots) students draw and justify significant and meaningful conclusions about the given situation. (All of these representations are frequency graphs, however, the Standards do not require students to know or use the term "frequency graph," although they use the term "frequency tables.") Students are introduced to two-way frequency tables and understand how to interpret relative frequencies in the context of the data represented in the tables. In unit S2 (which could take place either before or after this unit), students also build their statistics foundation by learning ways to determine whether two sets of data are correlated, and how strongly. Students identify linear association and interpret slope and intercept in the context of the data. Given different visual representations of data (linear models) students draw and justify significant and meaningful conclusions about the given situation. Students begin to use technology as a means to plot data and generate correlation coefficients. In S3, students revisit two-way frequency tables from a probability standpoint, and use them as a tool for conceptualizing and finding conditional probabilities. In S4, students combine the ideas of distributions and probability. They learn about normal distributions and use them to solve problems, and use the distributions of probability models to find the likelihood of a particular outcome. In doing so, students build on their experience with standard deviations from S1, calculating standard deviations using technology, and interpreting the results. Every high school statistics and probability standard is a modeling standard, hence modeling pervades the four units.

M1.1.0 Pre-unit diagnostic assessment

Assess students' recall of middle grades statistics, specifically their ability to

• recognize a statistical question;
• describe the distribution of data collected to answer a statistical question by its center, spread, and overall shape;
• interpret statistical plots;
• summarize numerical data sets in relation to their context;
• informally assess the degree of visual overlap of two numerical data distributions with similar variabilities.

Instead of creating representations of data, the emphasis in high school is on interpreting representations and judiciously interpreting measures of center and spread. Students describe the shape of a data distribution in more detail (symmetric, skewed, flat, or bell-shaped). Students develop a more precise understanding of measures of center and understand relationships between mean and median for symmetrical and skewed data distributions. They learn that outliers affect the mean of a data set but not the median. They recognize outliers when they exist and learn to investigate their source. Students learn that standard deviation is a measure of spread, that a larger standard deviation means the data are more spread out, and to understand standard deviation as "typical distance from the mean" for a symmetrical distribution. They also understand that interquartile range is a description of variability better-suited to a skewed distribution. Finally, students are introduced to two-way frequency tables and understand how to interpret relative frequencies in the context of the data represented in the tables.

M1.1.1 How can data be represented and summarized meaningfully?

• Revisit various ways to plot data: dot plots, histograms, and box plots.
• Interpret plots of data within the context of the data.
• Use the terms “symmetric” and “skewed” as descriptors of distributions.

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M1.1.8 Summative Assessment

Assess students’ ability to

• calculate mean, median, and mode;
• create box plots given data;
• compare and contrast two frequency distributions;
• articulate reasons to choose mean or use median as a measure of center;

• read and interpret relative frequencies.

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M1.1.2 Analyze data distributions

• Create dot plots, histograms and box plots.
• Use available classroom technology to create histograms and box plots and calculate measures of center and spread.
• Use terms such as “flat,” “skewed,” “bell-shaped,” and “symmetric” to describe data distributions.
• Analyze and compare data sets.

Instead of creating representations of data, the emphasis in high school is on interpreting representations and judiciously interpreting measures of center and spread. Students describe the shape of a data distribution in more detail (symmetric, skewed, flat, or bell-shaped). Students develop a more precise understanding of measures of center and understand relationships between mean and median for symmetrical and skewed data distributions. They learn that outliers affect the mean of a data set but not the median. They recognize outliers when they exist and learn to

investigate their source. Students learn that standard deviation is a measure of spread, that a larger standard deviation means the data are more spread out, and to understand standard deviation as "typical distance from the mean" for a symmetrical distribution. They also understand that interquartile range is a description of variability better-suited to a skewed distribution. Finally, students are introduced to two-way frequency tables and understand how to interpret relative frequencies in the context of the data represented in the tables.

M1.1.3 Measures of center

• Recall how to calculate mean and median.
• Understand mean and median as a "typical value" that can answer a statistical question.
• Know that mean and median are equal for a symmetrical data distribution.
• Explain why mean and median are unequal for a skewed data distribution.
• Select mean as the better measure for symmetrical distributions, and median as the better measure for skewed distributions.
• Make generalization what kinds of distributions have means larger than medians, and what kinds have medians larger than means.
• Recognize outliers when they exist, and know to investigate their source—that data point is way out there, why is that? Is there something weird about it that means we should disregard it?
• Know that outliers affect the mean, but not the median of a data set.

Instead of creating representations of data, the emphasis in high school is on interpreting representations and judiciously interpreting measures of center and spread. Students describe the shape of a data distribution in more detail (symmetric, skewed, flat, or bell-shaped). Students develop a more precise understanding of measures of center and understand relationships between mean and median for symmetrical and skewed data distributions. They learn that outliers affect the mean of a data set but not the median. They recognize outliers when they exist and learn to investigate their source. Students learn that standard deviation is a measure of spread, that a larger standard deviation means the data are more spread out, and to understand standard deviation as "typical distance from the mean" for a symmetrical distribution. They also understand that interquartile range is a description of variability better-suited to a skewed distribution. Finally, students are introduced to two-way frequency tables and understand how to interpret relative frequencies in the context of the data represented in the tables.

M1.1.4 Mid-unit assessment

Assess students' ability to

• describe a set of data given a graph or table;
• identify and calculate spread, center, shape, outliers, quartiles, mean, median, mode;
• construct and interpret a box plot;
• compare, contrast, and draw conclusions when given two data sets.

Instead of creating representations of data, the emphasis in high school is on interpreting representations and judiciously interpreting measures of center and spread. Students describe the shape of a data distribution in more detail (symmetric, skewed, flat, or bell-shaped). Students develop a more precise understanding of measures of center and understand relationships between mean and median for symmetrical and skewed data distributions. They learn that outliers affect the mean of a data set but not the median. They recognize outliers when they exist and learn to investigate their source. Students learn that standard deviation is a measure of spread, that a larger standard deviation means the data are more spread out, and to understand standard deviation as "typical distance from the mean" for a symmetrical distribution. They also understand that interquartile range is a description of variability better-suited to a skewed distribution. Finally, students are introduced to two-way frequency tables and understand how to interpret relative frequencies in the context of the data represented in the tables.

M1.1.5 Standard deviation

• Describe variability by calculating deviations from the mean.

• Compare two data sets with the same means but different variabilities, and contrast them by calculating the deviation of each data point from the mean.
• Interpret sets with greater deviations as having greater variability.
• Calculate a standard deviation by hand for a small data set, and understand standard deviation as an indicator of a typical deviation from the mean of an element of the data set.

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M1.1.6 Bringing it all together

• Represent a data set in different ways and decide which way is most appropriate.
• Select measures of center and spread appropriate to the shape of the distribution.
• Compare and contrast two or more distributions by using appropriate measures to describe center, variability, and shape.

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M1.1.7 Two-way frequency tables

• Interpret a two-way table.
• Understand that the choices made when organizing data can lead to different conclusions.

Instead of creating representations of data, the emphasis in high school is on interpreting representations and judiciously interpreting measures of center and spread. Students describe the shape of a data distribution in more detail (symmetric, skewed, flat, or bell-shaped). Students develop a more precise understanding of measures of center and understand relationships between mean and median for symmetrical and skewed data distributions. They learn that outliers affect the mean of a data set but not the median. They recognize outliers when they exist and learn to investigate their source. Students learn that standard deviation is a measure of spread, that a larger standard deviation means the data are more spread out, and to understand standard deviation as “typical distance from the mean” for a symmetrical distribution. They also understand that interquartile range is a description of variability better-suited to a skewed distribution. Finally, students are introduced to two-way frequency tables and understand how to interpret relative frequencies in the context of the data represented in the tables.

M1.2 Linear Equations, Inequalities and Systems

• Explain each step in solving a simple equation in one variable.
• Create and solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
• Model constraints and relationships between quantities by equations and inequalities, and by systems of equations and inequalities, and interpret solutions.
• Solve systems of linear equations approximately by graphing and exactly by algebraic methods.
• Understand the principles behind the method of elimination.
• Graph the solution set to a linear inequality in two variables as a half-plane.
• Graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Students begin learning about ratios and rates in Grade 6. In Grade 7, they represent proportional relationships by equations of the form $y = kx$, understanding k as the constant of proportionality or unit rate. In Grade 8, they recognize such equations as special kinds of linear equations $y = mx + b$ where m is the constant of proportionality and b is 0. They understand m as the slope of the line obtained from graphing the equation and b as the y-intercept of the line which is the value of y when $x = 0$.
By this point in their mathematical trajectory, then, students should be fairly comfortable with linear equations in two variables. They should be able to create and graph such equations to represent real-world situations that can be modeled by linear equations. Just as importantly, students should be able to describe the fundamental characteristic of linear functions, namely that they have a constant rate of change: the change in the output variable is proportional to the corresponding change in the input variable.

In grade 8, students analyzed and solved pairs of simultaneous linear equations (8.EE.C.8). Students should:
• know that the solutions to a system of two linear equations in two variables correspond to points of intersections of their graphs, because points of intersection satisfy both equations simultaneously (8.EE.C.8.a);

• know how to solve a system of two linear equations in two variables algebraically (8.EE.C.8.b);
• be able to estimate solutions by graphing the equations (8.EE.C.8.b);
• know how to solve real-world and mathematical problems leading to a system of two linear equations in two variables (8.EE.C.8.c).

In this unit, students build on what they know from middle school about linear equations and inequalities and systems of linear equations and expand their understanding to include systems of linear inequalities. They work with more complex modeling problems and become fluent in general methods of solution. They make more sophisticated use of graphical methods of representing and solving equations, inequalities, and systems, and they interpret points or regions in the plane in terms of the context.

Students will apply what they learn in this unit to the study of bivariate statistics. They will revisit the notion of a function and use the techniques learned here to study linear functions. They will come to view linear functions as one of many function families that display predictable characteristics. The technique of substitution, learned to find values that simultaneously satisfy two linear equations, is useful in more general situations, e.g., to solve a system consisting of a linear and a quadratic equation (A-REI.7) and in differential and integral calculus. For students who study calculus, linear functions will be an essential basis for their work with derivatives and differentiation.

M1.2.0 Pre-unit diagnostic assessment

Assess students' ability to
- **solve linear equations in one variable with rational number coefficients;**
- **solve word problems leading to linear inequalities of the form of the form $px + q \geq r$ with rational coefficients;**
- **graph linear equations of the form $y = mx + b$;**
- **solve a real-world problem that leads to a simple case of a system of two linear equations in two variables, where the same variable occurs with coefficient 1 in both equations.**

M1.2.1 Overview of linear equations and inequalities in two variables

Create and graph the solutions of linear equations and inequalities in two variables, and discuss their meaning in a real-world context.

This hook lesson provides an engaging context which allows students to exercise many of the skills they started to learn in middle school and will continue to apply in more sophisticated ways in this unit: setting linear equations in two variables to model a relationship between two quantities, solving for another variable, interpreting and graphing inequalities in two variables. It sets the stage for the units to come.

M1.2.2 Reason about linear equations and inequalities in one variable

- **Explain each step in solving a simple equation in one variable.**
- **Create and solve linear equations in one variable, including equations with coefficients represented by letters.**
- **Create and solve linear inequalities in one variable.**

As preparation for the work with two-variable equations, inequalities and systems in this unit, students must have a strong foundation in working with equations and inequalities in one variable and a clear understanding of what an equation is and what it means for a number to be a solution to the equation (it makes the two sides equal). This section gives students opportunities to practice reasoning, manipulating and solving equations and inequalities.

Tasks

A-REI Same solutions?

A-REI How does the solution change?

A-REI Reasoning with linear inequalities

M1.2.3 Model with systems of linear equations

- **Model constraints and relationships between quantities with systems of linear equations.**
- **Solve systems of linear equations approximately by graphing and exactly by algebraic methods.**

Students have worked with systems of linear equations in middle school and solved simple problems with them. In high school they work with more complex modeling problems and become fluent in general methods of solution. This first section on systems focuses on the modeling aspect. The systems are either solved graphically or the algebraic manipulations required to solve them are relatively simple. Furthermore, the modeling emphasis supports conceptual understanding by emphasizing the quantitative meaning of the variables, the equations, and the solutions to the system. This prepares students to take a thinking approach the solution methods in the next section on systems, rather than a purely formal one.

Tasks

A-REI, A-CED Cash Box

A-REI Quinoa Pasta 2

M1.2.4 Mid-unit assessment

Assess students' ability to

- **solve equations and inequalities in one variable;**
- **explain each step in solving a simple equation;**
- **solve systems of equations graphically and algebraically;**
- **set up and solve systems of equations that model a context;**
- **interpret a solution to a system of equations in terms of the context.**

M1.2.5 Solve general systems of linear equations in two variables

- **Solve systems of linear equations exactly by algebraic methods.**
- **Understand the principles behind the method of elimination.**

Students were introduced to the basic methods of solving systems of equations in middle school. In Section 3 of this unit they used simple systems to solve modeling problems. In this section they become fluent in general methods for solving systems algebraically and reason through the justification for these methods.

Tasks

A-REI Accurately weighing pennies I

A-REI Estimating a Solution via Graphs

A-REI Solving Two Equations in Two Unknowns

A-REI Accurately weighing pennies II

M1.2.6 Model with inequalities in two variables

• Identify constraints from a context, choose relevant variables and model the context with an inequality or system of inequalities.
• Identify coordinates pairs or points in the plane as solutions or non-solutions and interpret them in terms of the context.
• Graph solution sets to a linear inequality or system of inequalities.

In Grades 6–8 students learned about inequalities in one variable, and about equations and systems of equations in two variables. Here they tie these together and study inequalities, or systems of inequalities, in two variables. The emphasis is on modeling and interpreting solutions or non-solutions. Students also represent inequalities by shaded regions and interpret points in the plane; however, graphing should not be overemphasized or reduced to a procedure that does not engage the meaning of the inequality.

Tasks

A-REI Solution Sets

A-REI Fishing Adventures 3

M1.2.7 Summative assessment

Assess students' ability to
• create and solve linear equations and inequalities in one variable;
• solve systems of linear equations exactly by algebraic methods;
• model relationships between quantities and compare different relationships;
• graph the solution set to a linear inequality in two variables as a half-plane, and to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

M1.3 Bivariate Statistics

• Represent data on two quantitative variables on a scatter plot.
• Describe how two quantitative variables on a scatter plot are related.
• Interpret the slope and the intercept of a linear model in the context of the data.
• Use available technology to find lines of best fit.
• Assess the goodness of fit of a line to a small data set by plotting and analyzing residuals.
• Fit a linear function for a scatter plot that suggests a linear association.
• Use available technology to compute correlation coefficients.
• Understand that the correlation coefficient measures the "tightness" of a line fitted to data.
• Understand that correlation does not necessarily imply causality.

In this unit, students build on the statistical work they did in grade 8. Using technology, they represent data on two variables on a scatter plot, find the line of best fit, and interpret the slope and intercepts in context. They assess the fit of a line to data more precisely by plotting and analyzing residuals. Students compare strength of associations between different pairs of variables by interpreting correlation coefficients which they compute using technology. They gain experience in distinguishing between correlation and causality. Modeling is an intrinsic part of the high school statistics and probability standards and of this unit.

From their experiences with linear functions in grade 8, students are familiar with slope and intercept. They gained experience with scatter plots and described patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association. (8.SP.A.1) For scatter plots that suggested a linear association, they informally fit a line and informally assessed its fit. (8.SP.A.2) They wrote equations for these linear models, and interpreted their slopes and intercepts in the context of the data. (8.SP.A.3) In this unit, students build on the statistical work they did in grade 8. They work with bivariate data and find the line of best fit by using a graphing calculator or other software. (These lines of best fit are regression lines (or technologically-generated approximations of them) but the Standards do not require students to learn or use the terms “regression,” “regression line,” “regression equation,” or “least squares.”) They assess the fit of a line to data more precisely by plotting and analyzing residuals. Students compare strength of associations between different pairs of variables by interpreting correlation coefficients, which they compute using technology. And, they gain experience in distinguishing between correlation and causality. Modeling is an intrinsic part of the high school statistics and probability standards, and of this unit. In unit S4, student build on the techniques used this unit. Plotting residuals suggests that not some data sets may be better modeled non-linear functions than linear ones.

M1.3.0 Pre-unit diagnostic assessment

Assess students’ ability to

• write a linear equation and interpret it in a context;
• determine a rate of change and initial value given several data points that exhibit a linear relationship;
• write an equation for a linear relationship;
• use the equation to make predictions;
• interpret the slope and intercept of a linear equation in a context.

In this unit, students build on the statistical work they did in grade 8. Using technology, they represent data on two variables on a scatter plot, find the line of best fit, and interpret the slope and intercepts in context. They assess the fit of a line to data more precisely by plotting and analyzing residuals. Students compare strength of associations between different pairs of variables by interpreting correlation coefficients which they compute using technology. They gain experience in distinguishing between correlation and causality. Modeling is an intrinsic part of the high school statistics and probability standards and of this unit.

M1.3.1 Preview

• Activate prior experience in grade 8 with informally fitting a line to a scatter plot and informally judging its goodness of fit.

• Model the relationship with an equation for a line and use it to make predictions.
• Represent data on two quantitative variables on a scatter plot.
• Describe how two variables on a scatter plot are related.
• Interpret the slope and the intercept of a linear model in the context of the data.

(Note: 8.SP.2 and 8.SP.3 are prerequisites, not target standards in this unit. However, they are standards involved in one of the suggested activities.)

In this unit, students build on the statistical work they did in grade 8. Using technology, they represent data on two variables on a scatter plot, find the line of best fit, and interpret the slope and intercepts in context. They assess the fit of a line to data more precisely by plotting and analyzing residuals. Students compare strength of associations between different pairs of variables by interpreting correlation coefficients which they compute using technology. They gain experience in distinguishing between correlation and causality. Modeling is an intrinsic part of the high school statistics and probability standards and of this unit.

M1.3.2 Lines of best fit and residuals

• Use available technology to find lines of best fit.
• Quantify the goodness of fit by plotting and analyzing residuals.

In this unit, students build on the statistical work they did in grade 8. Using technology, they represent data on two variables on a scatter plot, find the line of best fit, and interpret the slope and intercepts in context. They assess the fit of a line to data more precisely by plotting and analyzing residuals. Students compare strength of associations between different pairs of variables by interpreting correlation coefficients which they compute using technology. They gain experience in distinguishing between correlation and causality. Modeling is an intrinsic part of the high school statistics and probability standards and of this unit.

M1.3.3 Interpreting the correlation coefficient

• Fit a linear function for a scatter plot that suggests a linear association.
• Use available technology to compute correlation coefficients.
• Understand that the correlation coefficient measures the "tightness" of a line fitted to data.
• Understand the significance of correlation coefficients close to 1 or –1.
• Interpret the rate of change and constant term of a line fitted to data in the context of the data.

In this unit, students build on the statistical work they did in grade 8. Using technology, they represent data on two variables on a scatter plot, find the line of best fit, and interpret the slope and intercepts in context. They assess the fit of a line to data more precisely by plotting and analyzing residuals. Students compare strength of associations between different pairs of variables by interpreting correlation coefficients which they compute using technology. They gain experience in distinguishing between correlation and causality. Modeling is an intrinsic part of the high school statistics and probability standards and of this unit.

M1.3.4 Correlation vs causation

Understand that correlation does not necessarily imply causality.

In this unit, students build on the statistical work they did in grade 8. Using technology, they represent data on two variables on a scatter plot, find the line of best fit, and interpret the slope and intercepts in context. They assess the fit of a line to data more precisely by plotting and analyzing residuals. Students compare strength of associations between different pairs of variables by interpreting correlation coefficients which they compute using technology. They gain experience in distinguishing between correlation and causality. Modeling is an intrinsic part of the high school statistics and probability standards and of this unit.

M1.3.5 Bringing it all together

• Describe how two quantitative variables are related.
• Use technology to create a line of best fit.

- **Fit a line to given data and use it to make predictions.**
- **Interpret the coefficients of a line of best fit in the context of the data to which it is fitted.**
- **Compute and interpret a correlation coefficient.**
- **Understand that variables with a high correlation do not necessarily have a causal relationship.**

In this unit, students build on the statistical work they did in grade 8. Using technology, they represent data on two variables on a scatter plot, find the line of best fit, and interpret the slope and intercepts in context. They assess the fit of a line to data more precisely by plotting and analyzing residuals. Students compare strength of associations between different pairs of variables by interpreting correlation coefficients which they compute using technology. They gain experience in distinguishing between correlation and causality. Modeling is an intrinsic part of the high school statistics and probability standards and of this unit.

M1.3.6 Summative Assessment

Assess students' ability to

- **create a scatter plot and a line of best fit using technology;**
- **interpret the coefficients of the line of best fit in the context of the data;**
- **interpret the correlation coefficient;**
- **articulate the difference between correlation and causation.**

In this unit, students build on the statistical work they did in grade 8. Using technology, they represent data on two variables on a scatter plot, find the line of best fit, and interpret the slope and intercepts in context. They assess the fit of a line to data more precisely by plotting and analyzing residuals. Students compare strength of associations between different pairs of variables by interpreting correlation coefficients which they compute using technology. They gain experience in distinguishing between correlation and causality. Modeling is an intrinsic part of the high school statistics and probability standards and of this unit.

M1.4 Functions

• Interpret key features of graphs in terms of the quantities represented. • Sketch graphs showing key features of the graph by hand and using technology. • Understand that a function from one set (the domain) to another set (the range) assigns to each element of the domain exactly one element of the range. • Use function notation. • Interpret statements that use function notation in various contexts. • Work with graphs of piecewise-defined functions, including step functions. • Relate the domain of a function to its graph. • Relate the domain of a function to the quantitative relationship it describes. • Calculate and interpret the average rate of change of a function over a specified interval. • Estimate the average rate of change of a function from its graph. • Solve for x such that f(x) = c, when f is a linear function. • Write an expression for the inverse of a linear function.

In grade 8, students are first introduced to the notion of functions. They understand a function as a rule that assigns to each input exactly one output 8.F.A.1. The main focus is on • linear functions and their representations (equations, graphs, tables, or verbal descriptions); • understanding that the equation y = mx + b defines a linear function, whose graph is a line; • modeling a linear relationship by a function, determining rate of change and initial value and interpreting them in terms of a situation modeled by the function and in terms of its graph or a table of values; • describing qualitatively a functional relationship between two quantities by analyzing a graph, e.g., where it is increasing or decreasing, linear or nonlinear. In this unit, students begin to use formal notation for functions, writing equations such as $f(x) = 2x + 3$ to describe a function. Students develop the understanding that the input/output relationship is a correspondence between two sets, and use the terms domain and range to describe them. They develop fluency with function notation and its use in describing qualitative features of the graph of a function by first interpreting, then writing expressions, equations, and inequalities such as $f(x + 2)$, $f(a) = 2$, $f(x) > 2$, and $f(x) > g(x)$. Students expand their repertoires of functions, working with piecewise-defined functions, including step functions. Building on their experiences with rate of change and slope from grade 8, students examine the behavior of non-linear functions. They describe key aspects of their graphs, and calculate and interpret average rates of change over specified intervals. Students' work with domain and range provides a basis for understanding when a function has an inverse. They examine simple functions that do and do not have inverses and write expressions for inverses of linear functions, but do not draw general conclusions about when a function has an inverse. Functions are a unifying theme in high school mathematics. In statistics, functions play especially prominent supporting roles as lines of best fit for bivariate statistics (units S2 and S4) and the normal distribution (unit S4). In geometry, transformations are viewed as functions sending points in the plane to points in the plane (units G1, G2, and G3), and ratios of sides of right triangles lead to the trigonometric functions on acute angles (unit G4 and A9). In algebra, students study systems of linear equations and inequalities (unit A1), exponential functions and geometric sequences (units A2 and A6), quadratic functions (unit A3), and rational functions, polynomials, and logarithms (unit A7 and A8).

M1.4.0 Pre-unit diagnostic assessment

Assess students' ability to

- **identify functions and non-functions;**
- **identify linear and nonlinear functions;**
- **write an equation for the corresponding linear function when given two points on a line;**
- **find and interpret the rate of change when given a linear function that models a situation;**
- **interpret the graph of a function in terms of the situation it models.**

M1.4.1 Graphing and functions

- **Sketch graphs showing key features.**
- **Interpret key features of graphs in terms of the quantities represented.**

In this unit, students begin their formal study of functions. They are introduced to function notation and gain a more precise understanding of what it means to be a function. They learn how to interpret functions in a given context and how to analyze them using different representations. In this section, they begin by graphing a variety of different functions.

M1.4.2 Introducing function notation

Understand that a function assigns to each element in the domain exactly one element of the range.

In this section, students are introduced to function notation and begin to interpret statements that use it. They begin to build expertise in understanding how equations and inequalities that use function notation correspond to features of graphs of functions. In this section, the focus is on statements about one input and its output (i.e., one point on the graph). In section 7, they return to this correspondence, going in the opposite direction: from features of graphs to equations and inequalities about the functions represented.

Tasks

F-IF Interpreting the graph
F-IF Using Function Notation I
F-IF The Customers
F-IF Points on a graph

M1.4.3 Interpreting function notation in context

Interpret statements that use function notation in terms of the quantities represented.

In this section, students apply and extend their understanding of function notation in various contexts by interpreting statements that use function notation in terms of the quantities represented.

Tasks

F-IF Cell phones
F-IF Yam in the Oven

M1.4.4 Mid-unit assessment

Assess students' ability to

• use function notation to represent points on a graph and to describe features of a graph;

• **understand function notation and how to interpret statements in function notation in terms of a context.**

M1.4.5 Domain, range, piecewise-defined functions

• **Use the notions of domain and range.**
• **Interpret a graph of a piecewise-defined function.**
• **Graph step functions.**

Tasks
F-IF The restaurant
F-IF Finding the domain
F-IF The Parking Lot
F-IF Bank Account Balance

M1.4.6 Interpreting graphs in context

• **Interpret key features of graphs in terms of the quantities represented.**
• **Sketch graphs showing key features.**
• **Relate the domain of a function to its graph.**
• **Relate the domain to the quantitative relationship it describes.**

In this section, students read and interpret graphs of functions. These include graphs of functions that arise from data, including functions that arise from using trend lines to join data points. Students also have an opportunity to apply their understanding of domain and range. The tasks in this section could be approached as a jigsaw activity. In small groups, students become experts on one particular task, then work in new groups which include at least one expert for each task.
Tasks
F-IF Oakland Coliseum
F-IF Warming and Cooling
F-IF Influenza epidemic
F-IF How is the Weather?
F-IF Telling a Story With Graphs

M1.4.7 Average rate of change

• Calculate the average rate of change of a function over a specified interval;
•Interpret the average rate of change of a function over a specified interval.

In this section, students are introduced to the notion of average rate of change over an interval. They work with expressions for average rates of change, and compute and estimate average rates of change. In grade 8, students learned that the rate of change of a linear function is the slope of its graph and that the slope can be computed from the coordinates of any two distinct points on the line. For nonlinear functions, the story is more complicated because their rates of change vary depending on the interval chosen. As for linear functions, average rates of change over an interval can be computed from the coordinates of two points on their graphs, namely those corresponding to the endpoints of the interval.
Tasks
F-IF Temperature Change
F-IF The High School Gym
F-IF Mathemafish Population

M1.4.8 Inverse functions

• Solve for x such that $f(x) = c$, when f is a linear function.
• Write an expression for an inverse.

WHAT: This post describes two days of instruction. First, students approach the idea of an inverse operation in the context of ciphers. They then write rules by listing out operations for a given function and then writing a new expression which “undoes” those operations F-BF.B.4a. The next day, students create tables and graphs for functions and their inverses. They look for commonalities as they consider pairs of functions and their inverses. Note: This lesson was created for an Algebra 2 class, so depending on students’ prior experience with various function types, it may need to be modified to make it appropriate for a given classroom. WHY: This lesson highlights fundamental characteristics about functions and their inverses: that it can generally be thought of as a process of "undoing," and also that the inputs and outputs trade places. This lesson does not address the idea that a function only has an inverse function if it is one to one, but that idea is addressed in later activities in this section.

Tasks

F-IF Your Father

F-BF Temperatures in degrees Fahrenheit and Celsius

F-BF US Households

M1.4.9 Summative assessment

Assess students' ability to

- **identify the domain and range of a given function;**
- **create a step function that represents pairs of given values;**
- **interpret statements that use function notation in a context;**
- **identify key features of graphs;**
- **identify referents of expressions that use function notation in a given graph;**
- **identify and relate the domain and range of a function in a given context;**
- **calculate the average rate of change of a function over a specified interval;**
- **interpret the average rate of change of a function in terms of the quantities represented.**

M1.5 Exponential Functions 1

• Distinguish between the growth laws of linear and exponential functions and recognize when a situation can be modeled by a linear function versus an exponential function.
• Graph exponential functions and understand how changing by a constant factor over equal intervals affects the graph.
• Model situations of growth and decay with exponential functions expressed in various different forms given a graph, a description of the situation, or two input-output pairs (including reading these from a table)
• Understand that over time a quantity increasing exponentially will eventually exceed a quantity increasing linearly.
• Understand the form of different expressions for exponential functions in terms of change by a constant factor over equal intervals.

In Grade 9, students should be familiar with linear functions from Grade 8 and from the F1 unit. They have been formally introduced to functions and function notation and have explored the behaviors and traits of both linear and non-linear functions. Additionally, students have spent significant time graphing, interpreting graphs and have explored how to compare the graphs of two linear functions to each other.
In this unit, students are introduced to exponential functions. Students learn the fundamental growth law for exponential functions and compare it with the law for linear functions. They recognize exponential functions when presented with data, graphs and real-world contexts. They construct exponential functions and use them to model situations and solve problems. They distinguish between situations that should be modeled with a linear function vs. an exponential function. They know various forms ways of expressing an exponential function (e.g. ,). They know that an increasing exponential function eventually is greater than an increasing linear function.

In A3, students will be introduced to quadratic functions. Students will focus on the basic nature of quadratic functions, contexts in the real world that can be modeled by quadratic functions, and the different forms of the expression for a quadratic function and what those forms tell you about the behavior of the function and the shape of its graph. Students will also extend their understanding of exponential functions and how they relate to quadratic functions; understanding that an exponential growth function will eventually exceed both a linear and a quadratic function. In the A7 unit, students will focus on a more in-depth understanding of exponential and log functions.

M1.5.0 Pre-unit diagnostic assessment

Diagnose students' ability to
• use the exponent laws to find equivalent expressions;
• solve problems involving percent increase/decrease;
• solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form;
• construct a linear function;
• describe a non-linear function;
• compare functions

M1.5.1 The Key to Exponential Growth

Learn the difference between growth by a constant multiplicative factor and growth by a constant additive factor.

In this section, students are introduced the underlying growth law for an exponential function, namely that the output changes by a constant multiplicative factor for a constant additive change in the input variable. They gain a quantitative sense of the difference this makes through an application to population growth.
Tasks
F-LE, A-CED Paper Folding

M1.5.2 Introduction to exponential functions

• Distinguish between the growth laws of linear and exponential functions.
• Construct simple exponential models.
• Create tables and graphs of exponential functions and understand their behavior in terms of the fundamental growth law.
• Understand the form of the expression $f(x) = ab^x$ for an exponential function in terms of the fundamental growth law.

In this section students construct and interpret exponential functions expressed in the form f(x)=ab^x to model various context. They work with contexts where the initial value a and the growth factor b are either given or are directly inferable from the context, or where they must interpret those values in terms of the context. They make tables and graphs of exponential functions and begin to acquire a quantitative sense of exponential growth both numerically and graphically.

Tasks

F-LE U.S. Population 1790-1860
F-IF Identifying Exponential Functions
F-IF Exponential Parameters
F-LE Basketball Rebounds
F-LE Equal Differences over Equal Intervals 1
F-LE Equal Factors over Equal Intervals

M1.5.3 Model with Exponential Functions

• Model situations of growth and decay with exponential functions expressed in various different forms given a graph, a description of the situation, or two input-output pairs (including reading these from a table).
• Recognize that exponential functions have a constant percent growth or decay rate per unit interval.
• Interpret the parameters of an exponential function in a context.

Now that they are familiar with the basic form $f(x) = ab^x$ of an exponential function, students start to work with exponential functions expressed in other ways. They learn the relationship between the growth (or decay) factor and the growth (or decay) rate; if r is the growth rate then $1 + r$ is the growth factor. They model more complex situations where they must derive the growth factor in various ways given data about the context.

Tasks

F-LE Predicting the Past
F-LE Moore's Law and Computers
F-LE DDT-cay
F-LE All Your Base Are Belong to Us

M1.5.4 Compare exponential and linear functions

Understand that over time a quantity increasing exponentially will eventually exceed a quantity increasing linearly.

In this section, students will compare linear and exponential functions. Students are familiar with linear functions and linear growth so students will use this base understanding to develop the notion that increasing exponential functions will eventually exceed increasing linear functions.

Tasks

F-LE Linear or exponential?
F-LE, A-REI Population and Food Supply
F-LE Exponential growth versus linear growth I
F-LE Exponential growth versus linear growth II

M1.5.5 Bringing it Together

- **Interpret graphs and expressions for exponential functions.**
- **Model depreciation with linear and exponential functions.**
- **Compare linear and exponential models.**

In this section, students bring together much of what they have learned in this unit. They use their ability to write linear and exponential functions given two input-output pairs, they compare different models, both linear and exponential, and draw conclusions from them.

M1.5.6 Summative assessment

Assess students' ability to

- **construct and compare linear and exponential functions given data;**
- **recognize a situation in which a quantity grows by a constant percent rate per unit interval relative to another;**
- **given an exponential model in the form $f(t) = ab^t$, interpret the constants a and b in terms of the context;**
- **explain in words the similarities and differences between linear and exponential models;**
- **recognize situations that can be modeled with linear functions and with exponential functions, and solve problems.**

M1.6 Constructions and Rigid Motions

• Know and be able to use precise definitions of geometric terms.
• Make formal geometric constructions by hand and using geometry software.
• Given a geometric figure and a rotation, reflection, or translation draw the transformed figure.
• Develop definitions of rotation, reflection, and translation.
• Represent transformations in the plane; describe transformations as functions whose inputs and outputs are points in the plane.
• Describe the rotations and reflections that carry a given quadrilateral or regular polygon onto itself.
• Prove that the measures of the interior angles of a triangle have sum 180°.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

Students have worked with geometric shapes since kindergarten. In grade 4, they classified two-dimensional shapes by properties of their sides and angles, and in grade 5, they classified these shapes in a hierarchy based on properties. In grade 8, they were introduced to the concepts of rotation, reflection, and translation via physical models, transparencies, or geometry software. They defined congruence of two-dimensional figures in terms of these rigid motions, understanding that two figures were congruent if one could be obtained from the other via a sequence of rigid motions. Given a two-dimensional figure, students described the effect of a rotation, reflection, or translation on the figure in terms of coordinates. Students also worked with determining distances in the coordinate plane using the Pythagorean Theorem.

M1.6.0 Pre-unit diagnostic assessment

Assess students' ability to

• draw geometric shapes that satisfy given conditions;
• given two congruent figures, describe a sequence that exhibits the congruence between them;
• describe the effects of rigid motions on two-dimensional figures using coordinates.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

M1.6.1 Definitions and Geometry

Recall and reconnect with the meanings of geometry terms that will be used in this unit.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

M1.6.2 Introduction to Constructions

• Use a compass and a straightedge to construct various geometric figures.
• Use geometric software to construct various geometric figures.
• Begin to understand the formal definitions of geometric figures through constructions.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

M1.6.3 Representing transformations

• Draw the result of a rotation, reflection, or translation on given geometric figures by hand.
• Draw the result of a rotation, reflection, or translation on given geometric figures using geometric software.
• Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

M1.6.4 Mid-Unit Assessment

Assess students' ability to

• make formal geometric constructions (copying a segment);
• construct a square;
• given a geometric figure and a rotation, reflection or translation draw the transformed figure;
• understand and explain the formal definition of rotation.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

M1.6.5 Mappings of the plane

• Represent transformations in the plane using various tools (transparencies, geometry software, etc.).
• Describe transformations as functions that take points in the plane as inputs and given other points as outputs.
• Compare transformations that preserve distance and angle and those that do not (translation vs. horizontal stretch).

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

M1.6.6 Symmetries

• Understand the idea of reflection symmetry.
• Understand the idea of rotation symmetry.
• Be able to describe the rotations and reflections that carry a given rectangle, parallelogram, trapezoid, or regular polygon onto itself.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

M1.6.7 Bringing it all together

Create a given design through construction and rigid motions.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

M1.6.8 Summative Assessment

Assess students' ability to

• give definitions of geometric terms;
• make formal geometric constructions (parallel and perpendicular lines);
• construct a square and explain why the construction yields a square;
• given a geometric figure and a rotation, reflection or translation draw the transformed figure;
• understand and explain the formal definition of rotation;
• describe transformations as functions;
• given a square or trapezoid, describe the rotations and reflections that map it onto itself.

Students construct (with both compass and straight-edge and technology, rather than the physical models and transparencies used in eighth grade) perpendicular lines, parallel lines, and regular polygons, and develop formal definitions of these objects. Students then develop more precise definitions for translations, rotations, and reflections, and use these to describe symmetries - single rigid transformations that carry objects to themselves. Careful attention is given to properties of figures that are preserved (for example, as the result of a translation, a line segment is both parallel and congruent to the pre-image), as they will be important to following work with transformational proofs. Additionally, coordinates are used to represent rigid motions as functions that map points in the plane to points in the plane.

M1.7 Congruence

• Specify sequences of rigid motions that will carry a figure onto another.
• Understand that there can be more than one sequence of rigid motions that carries a figure onto another figure.
• Use the definition of congruence in terms of rigid motions to decide if two figures are congruent.
• Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent (CPCTC).
• Be able to explain how the criteria for triangle congruence follow from the definition of congruence in terms of rigid motions.
• Prove theorems about lines and angles.
• Prove theorems about parallelograms.
• Prove base angles of isosceles triangles are congruent.

Students build on their understanding of rigid motions to formalize the definition of congruence that they developed in grade 8. They specify a series of rigid motions that carries one figure onto another and use the definition to determine whether two objects are congruent. Students show two triangles are congruent if and only if corresponding pairs of sides and angles are congruent. They explain how the criteria for triangle congruence (ASA, SAS, SSS) follow from the definition of congruence in terms of rigid motions. Armed with these criteria, students are able to prove theorems about triangles, lines, angles, and parallelograms.

In the preceding unit, students built on their middle grades experiences with geometry. They made formal geometric constructions and developed definitions for rigid motions (translations, rotations, reflections) and several key geometric terms. Students examined the effects of single rigid motions on figures, considering rigid motions as mappings of the coordinate plane to itself. Given simple figures, they described symmetries of those figures—the rotations and reflections that mapped the figures onto themselves. In this unit, students build on their understanding of rigid motions to strengthen the understanding of congruence that they developed in grade 8. Students use this definition to show that two triangles are congruent if and only if corresponding pairs of sides and angles are congruent. They explain how the criteria for triangle congruence (ASA, SAS, SSS) follow from the definition of congruence in terms of rigid motions. They use these criteria to prove theorems about triangles, lines, angles, and parallelograms. In the next unit, students extend their knowledge of transformations to include dilations. They explore properties of dilations, use this knowledge to understand similarity in triangles and other shapes in terms of transformations, and use this understanding to solve problems.

M1.7.0 Pre-unit diagnostic assessment

Assess students' ability to

• make formal geometric constructions (parallel and perpendicular lines;
• construct a square and explain why the construction yields a square.

Students build on their understanding of rigid motions to formalize the definition of congruence that they developed in grade 8. They specify a series of rigid motions that carries one figure onto another and use the definition to determine whether two objects are congruent. Students show two triangles are congruent if and only if corresponding pairs of sides and angles are congruent. They explain how the criteria for triangle congruence (ASA, SAS, SSS) follow from the definition of congruence in terms of rigid motions. Armed with these criteria, students are able to prove theorems about triangles, lines, angles, and parallelograms.

M1.7.1 Sequences of rigid motions

• Specify sequences of rigid motions that will carry a figure onto another.
• Find different ways to transform one figure into another.
• Given a sequence of rigid motions, try to find a shorter sequence of rigid motions with the same outcome.

Students build on their understanding of rigid motions to formalize the definition of congruence that they developed in grade 8. They specify a series of rigid motions that carries one figure onto another and use the definition to determine whether two objects are congruent. Students show two triangles are congruent if and only if corresponding pairs of sides and angles are congruent. They explain how the criteria for triangle congruence (ASA, SAS, SSS) follow from the definition of congruence in terms of rigid motions. Armed with these criteria, students are able to prove theorems about triangles, lines, angles, and parallelograms.

M1.7.2 Defining Congruence

• Use the definition of congruence in terms of rigid motions.
• Use the definition of congruence to decide if two figures are congruent or not.

Students build on their understanding of rigid motions to formalize the definition of congruence that they developed in grade 8. They specify a series of rigid motions that carries one figure onto another and use the definition to determine whether two objects are congruent. Students show two triangles are congruent if and only if corresponding pairs of sides and angles are congruent. They explain how the criteria for triangle congruence (ASA, SAS, SSS) follow from the definition of congruence in terms of rigid motions. Armed with these criteria, students are able to prove theorems about triangles, lines, angles, and parallelograms.

M1.7.3 Triangle Congruence Criteria

• Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent (CPCTC).
• Explain how the criteria for triangle congruence follow from the definition of congruence.
• Prove that base angles of isosceles triangles are congruent.

Students build on their understanding of rigid motions to formalize the definition of congruence that they developed in grade 8. They specify a series of rigid motions that carries one figure onto another and use the definition to determine whether two objects are congruent. Students show two triangles are congruent if and only if corresponding pairs of sides and angles are congruent. They explain how the criteria for triangle congruence (ASA, SAS, SSS) follow from the definition of congruence in terms of rigid motions. Armed with these criteria, students are able to prove theorems about triangles, lines, angles, and parallelograms.

M1.7.4 More Congruence Theorems

• Prove theorems about lines and angles.
• Prove theorems about parallelograms.

Students build on their understanding of rigid motions to formalize the definition of congruence that they developed in grade 8. They specify a series of rigid motions that carries one figure onto another and use the definition to determine whether two objects are congruent. Students show two triangles are congruent if and only if corresponding pairs of sides and angles are congruent. They explain how the criteria for triangle congruence (ASA, SAS, SSS) follow from the definition of congruence in terms of rigid motions. Armed with these criteria, students are able to prove theorems about triangles, lines, angles, and parallelograms.

M1.7.5 Summative Assessment

Assess students' ability to

• show and explain sequences of rigid motions and analyze different sequences of rigid motions that carry one shape to another;
• explain the criteria for triangle congruence and whether given triangles are congruent or not;
• understand and explain that opposite sides of a parallelogram are congruent;
• explain that vertical angles are congruent;
• explain that when a transversal crosses parallel lines, alternate interior and corresponding angles are congruent.

Students build on their understanding of rigid motions to formalize the definition of congruence that they developed in grade 8. They specify a series of rigid motions that carries one figure onto another and use the definition to determine whether two objects are congruent. Students show two triangles are congruent if and only if corresponding pairs of sides and angles are congruent. They explain how the criteria for triangle congruence (ASA, SAS, SSS) follow from the definition of congruence in terms of rigid motions. Armed with these criteria, students are able to prove theorems about triangles, lines, angles, and parallelograms.