Algebra 2

In Algebra 2, students extend the algebra and function work done in Algebra 1. Students continue to develop their picture of the complex number system by investigating how non-real solutions arise and how non-real numbers behave. Exponential functions are considered over a domain of real numbers, necessitating work with fractional exponents. The logarithm is defined as the inverse of exponentiation and from this definition students consider the properties of logarithms in addition to using them to solve for unknown exponents. Students extend their previous work with quadratics and polynomials to achieve a more general understanding of polynomials. Work done in geometry with sin, cos, and tan as operations is extended in a study of the unit circle and sin(x), cos(x), and tan(x) as functions. Students build on their work with probability from grade 7 to admit the notions of independence and conditional probability. Finally, students do further work in statistics where they revisit and extend their understanding of variability in data and of ways to describe variability in data. The normal distribution is studied, and students explore the reasoning that allows them to draw conclusions based on data from statistical studies.

A2.1 Extending the Number System

• Work with infinite decimal expansions of numbers on the number line.
• Reason about operations with rational and irrational numbers (N-RN.B.3).
• Extend properties of integer exponents to rational exponents and write expressions with rational exponents as radicals (N-NR.A.1, N-RN.A.2).
• Solve equations and real-world problems involving radicals and fractional exponents (A-REI.A.2).
• Note extraneous solutions and explain where they come from (A-REI.A.2).
• Discover a new type of number that is outside previously known number systems (N-CN.A.1).
• Perform operations with complex numbers (N-CN.A.2).
• Solve quadratic equations with complex solutions (N-CN.C.7).

In Grade 8, students discovered there are numbers that are not rational. They approximated them using rational numbers to locate them approximately on the number line. They also worked with integer exponents and became familiar with the basic exponent rules. Finally, in their work with quadratics, students encountered situations where they came to a negative under the square root sign, which was interpreted until now as meaning the equation had no solutions.

The theme of this unit is extending number systems and operations on numbers, first from rational to real numbers and then from real to complex numbers. Along the way students extend the operation of raising a number to a power to situations where the exponent is not an integer. They define the meaning of a numerical expression with rational exponents in terms of radicals, by reasoning about how to extend the exponent rules. They use this new understanding to solve equations involving rational exponents and radicals. Finally they extend the real number system and the operations on real numbers to include complex numbers and use them to solve quadratic equations with no real solutions.

In future units on exponential functions, students will extend their knowledge of rational exponents and apply them to even more complicated situations. Students pursuing careers in STEM fields might use imaginary and complex numbers in many of their future college courses.

A2.1.0 Pre-unit diagnostic assessment

Assess students' ability to
• recall, from eighth grade, the definition of rational numbers and well-known examples of irrational numbers (8.NS.A.2);
• mentally compute and work with exponents, including 0, negative exponents, and negative bases (8.EE.A.1);
• solve a quadratic equation by various methods (A-REI.B.4).

A2.1.1 From rational numbers to the real number line

• Work with infinite decimal expansions of numbers on the number line.
• Reason about operations with rational and irrational numbers (N-RN.B.3).

In this section, students take a deeper look at the number line. In elementary school they learned how to place fractions on the number line and in middle school they added negative numbers and learned about the existence of a few isolated irrational numbers, such as π and $\sqrt{2}$. Now they see infinite decimal expansions as a way of locating any number, rational or irrational, on the number line. They also expand their repertoire of irrational numbers by considering sums and products of rational with irrational numbers.

Tasks
8.NS Identifying Rational Numbers
N-RN, 8-NS Calculating the square root of 2
N-RN Rational or Irrational?

A2.1.2 Extend the properties of exponents to rational exponents

• Extend properties of integer exponents to rational exponents and write expressions with rational exponents as radicals (N-RN.A.1, N-RN.A.2).
• Solve real-world problems in which rational exponents arise (N-RN.A.1).

Students have encountered square roots and cube roots in Grades 6–8. Now that they have the real numbers at their disposal they can contemplate more complicated numerical expressions involving radicals and fractional exponents. In this section they learn the rules for manipulating such expressions. They first review familiar exponent rules and remind themselves how they rewrite exponential expressions, particularly the rule $(x^a)^b = x^{ab}$. They investigate the consequences of extending this rule to rational exponents and see how it implies that $x^{(a/b)} = \sqrt[b]{x^a}$. The section continues with a modeling task using Kepler's Law and then wraps up with a short reasoning task where students can work with rational exponents in decimal form.

Tasks

N-RN Extending the Definitions of Exponents, Variation 2
N-RN Evaluating Exponential Expressions
N-RN Kepler's Third Law of Motion
N-RN Checking a calculation of a decimal exponent

A2.1.3 Solve radical equations

• Solve radical equations and equations with fractional exponents (A-REI.A.2).
• Note extraneous solutions and describe where they came from(A-REI.A.2).

In this section, students will move from reasoning about expressions with rational exponents to solving equations involving such expressions. They reason through to the solution step by step. They explore extraneous solutions and explain why they occur.

Tasks

A-REI Who wins the Race?
A-REI Radical Equations

A2.1.4 Mid-unit formative assessment

Assess students' ability to
• identify extraneous solutions when solving radical equations (A-REI.A.2);
• rewrite expressions with rational exponents and radicals (N-RN.A.2);
• demonstrate understanding about the outcomes of operations on rational and irrational numbers (N-RN.B.3).

A2.1.5 Beyond the number line: complex numbers

Discover a new type of number that is outside previously known number systems (N-CN.A.1).

In this section, a new type of number is necessitated and then defined collaboratively. By hinging their understanding on their previous knowledge of numbers and then posing a question that doesn't fit into that system, students hit a point of disequilibrium where they need to define imaginary numbers and then seek to understand how they behave, particularly taking note of patterns that emerge with them.

Tasks

N-CN Complex number patterns

A2.1.6 Operations on complex numbers

• Explore how the new number, i, behaves under certain operations (N-CN.A.2).
• Perform operations with complex numbers and draw conclusions about patterns that emerge (N-CN.A.2).

With a new type of number in their world, students can begin exploring the properties of these numbers and how to add, subtract, multiply and divide them using the properties of operations to guide the work. They explore patterns that emerge when i is raised to positive integer powers, and build on this work by investigating similar patterns with other complex numbers.

Tasks

N-CN, A-SSE Computations with Complex Numbers
N-CN Powers of a complex number

A2.1.7 Solve quadratic equations with complex roots

Apply knowledge of complex numbers to solve quadratic equations with complex solutions (N-CN.C.7).

In this final section of the unit, students connect their knowledge of complex numbers to their previous work with quadratics. They learn that quadratic equations always have complex solutions even when they have no real solutions, unifying the class of quadratic equations when the complex number system is considered.

Tasks

N-CN, A-REI Completing the square

A2.1.8 Summative assessment

Assess students' ability to
• solve radical equations and demonstrate awareness of the possibility of extraneous roots resulting from an algebraic solution (A-REI.A.2);
• describe and show examples of how rational and irrational numbers behave under certain operations (N-RN.B.3);
• rewrite expressions involving rational exponents and radicals (N-RN.A.2);
• demonstrate understanding of the meaning of *i* (N-CN.A.1);
• perform operations with *i* and with complex numbers in the form *a* + *bi* (N-CN.A.2);
• solve quadratic equations with real coefficients that have complex solutions (N-CN.C.7).

A2.2 Exponential Functions 2

• Create and analyze a simple exponential function arising from a real-world or mathematical context (F-LE.A.2★).
• Evaluate and interpret exponential functions at non-integer inputs (N-RN.A.1, F-LE.A.2★).
• Understand functions of the form $f(t) = P(1 + r/n)^{nt}$ and solve problems with different compounding intervals (A-SSE.A.1★, F-LE.A.2★).
• Understand informally how the base e is used in functions to model a quantity that compounds continuously (F-BF.A.1a★).
• Write exponential expressions in different forms (F-LE.A.2★, F-BF.A.1★).
• Explain what the parameters of an exponential function mean in different contexts (F-LE.B.5★).
• Use the properties of exponents to write expressions in equivalent forms (A-SSE.B.3a★).
• Build exponential functions to model real world contexts (F-LE.A.1★, F-LE.A.2★, F-BF.A.1★).
• Analyze situations that involve geometric sequences and series (A-SSE.A.1★).
• Derive the formula for the sum of a finite geometric series (A-SSE.B.4★).

In previous units students have worked with geometric sequences and understand they change by a constant ratio over a constant interval. They are able to write both recursive and closed equations for them. Students understand the difference between a linear and exponential function, can recognize situations and tables described by each, and know that an exponential function will always overtake a linear function. They know that an exponential function grows increasingly rapidly in one direction, and approaches a value asymptotically in the other direction. They have solved exponential equations of the form $ab^x = c$ by graphing. They can construct an exponential function given a graph, description of a relationship, or two input output pairs with integer inputs (including in a table). Given an expression defining an exponential function, they can interpret its parameters in a context. They can also fit a simple exponential function to a scatterplot. Every exponential function until now has only involved integer inputs.

In this unit students broaden their view of exponential functions to include the entire real number line as a possible domain. They learn about functions with base e. An approach using compound interest that shows e arising as the natural base for a quantity being compounded continuously can serve as a way to develop understanding appropriate to this level. First, students must understand functions of the form $f(x) = P(1 + r/n)^{nt}$ which show a given compounding frequency, n.

Students examine some different forms of exponential functions and learn to interpret the parameters in terms of a context. They learn the concept of doubling time and see functions expressed in a form that shows the doubling time; they work algebraically with functions expressed in a form like $f(x) = A(1 + r/n)^{nt}$ that shows the compounding period; and they work with functions written with the base e, $g(x) = Ae^{rt}$, in many continuous growth contexts. Students build functions in those forms in order model real-world contexts. Contexts may include Moore's law for computer processor speeds, population growth, and temperature change. Students should also consider whether an exponential or a linear model is appropriate in various contexts.

Students analyze situations that involve summing an exponential sequence (which is generally called a geometric series) These arise naturally in some saving and banking problems as well as in some interesting geometric contexts. Students will build on previous work with geometric sequences to derive a formula for the sum of a geometric series.

A later unit introduces logarithms as the solutions to exponential equations. Ultimately, students are proficient at graphing, analyzing, solving, and modeling exponential situations. Students are comfortable with exponentials from a functions viewpoint. This lays the foundations for success in calculus.

A2.2.0 Pre-unit diagnostic assessment

Diagnose students' ability to
- **create basic exponential functions (F-LE.A.2★);**
- **create an exponential model for a simple context (F-BF.A.1★);**
- **apply exponent rules (N-RN.A.1).**

A2.2.1 Understanding exponential growth and decay

Create and analyze a simple exponential function arising from a real-world or mathematical context (F-lE.A.2★).

Students have been introduced to exponential functions in Exponential Functions

1. For some students this introduction could have occurred a year or two previously. Therefore this unit starts with an activity that re-introduces exponential growth and decay in an engaging real-world or mathematical context. The context can be modeled by an exponential function with domain contained in the integers, thus providing a review of previous experience as needed for students.

A2.2.2 Exponential functions on the real numbers

Evaluate and interpret exponential functions at non-integer inputs(N-RN.A.1, F-LE.A.2★).

In Exponential Functions 1 students considered exponential functions at integer inputs only. Now that they understand how to determine the value of b^x for any rational number x, they can approximate b^x to any degree of accuracy for any real number x. In this section they broaden their view of exponential functions to include the entire real number line as a possible domain. They evaluate functions and interpret their values at real inputs in terms of a context, in preparation for the more sophisticated work in the following sections.

Tasks
F-LE Allergy medication
F-LE Boom Town

A2.2.3 Changing compounding intervals, continuous compounding, and the base e

- **Understand functions of the form $f(t) = P(1 + r/n)^{nt}$ which use a given compounding frequency, n (A-SSE.A.1★).**
- **Solve problems given different compounding intervals (F-LE.A.2★).**
- **Understand (informally) how the base e arises in the context of compounding intervals as n becomes arbitrarily large (F-BF.A.1a★).**

The base e is very commonly used in scientific and other modeling applications. The fundamental reason that e is a useful base for an exponential function is beyond the scope of this course. However, an approach using compound interest that shows e arising as the natural base for a quantity being compounded continuously can serve as a way to develop understanding at this level. First, students must understand functions of the form $f(t) = P(1 + r/n)^{nt}$ which show a given compounding interval, n. This section examines that approach.

Tasks

A-SSE The Bank Account

F-BF Compounding with a 5% Interest Rate

A2.2.4 Interpreting exponential functions

- **Solve problems involving exponential functions in many different contexts (F-LE.A.2★).**
- **Write exponential expressions in different forms (F-BF.A.1★).**
- **Explain what the parameters of an exponential function mean in different contexts (F-LE.B.5★).**
- **Use the properties of exponents to write expressions in equivalent forms (A-SSE.B.3c★).**

In Exponential Functions 1, students worked with exponential functions in the form $f(t) = ab^t$ or $f(t) = a(1 + r)^t$ and interpreted the parameters a, b, and r in terms of a context. In this unit they see more complicated forms. In the previous section, students developed an understanding of different compounding intervals and continuous compounding using base e. The purpose of this section is to examine some of these different forms and learn to interpret the parameters in terms of a context. Students learn the concept of doubling time and see functions expressed in a form that shows the doubling time; they work algebraically with functions expressed in a form like $f(x) = A(1 + r/n)^{nt}$ that shows the compounding period; and they work with functions written with the base e, $g(x) = Ae^{rt}$, in many continuous growth contexts. In this section they do not build functions in any of these forms.

Tasks

F-LE Bacteria Populations

F-BF Lake Algae

F-LE Rising Gas Prices – Compounding and Inflation

A-SSE Forms of exponential expressions

A2.2.5 Modeling with exponential functions

Build exponential functions to model real world contexts (F-LE.A.1★, F-LE.A.2★, F-BF.A.1★).

Having seen the purpose of various different expressions for exponential functions, students now start to build functions in those forms in order model real-world contexts. Contexts may include Moore's law for computer processor speeds, population growth, and temperature change. Students should also consider whether an exponential or a linear model is appropriate in various contexts.

Tasks
F-LE In the Billions and Exponential Modeling
F-LE Boiling Water

A2.2.6 Geometric sequences and series

- **Analyze situations that involve geometric sequences and series (A-SSE.A.1★).**
- **Derive the formula for the sum of a finite geometric series (A-SSE.B.4★).**

Analyze situations that involve summing a exponential sequence (which is generally called a geometric series) These arise naturally in some saving and banking problems as well as in some interesting geometric contexts.

Tasks
A-SSE A Lifetime of Savings
A-SSE Course of Antibiotics
A-SSE Triangle Series
A-SSE Cantor Set

A2.2.7 Summative assessment

Assess students' ability to
- **evaluate and interpret exponential functions at non-integer inputs (N-RN.A.1, F-LE.A.2★);**
- **construct exponential functions given a description or data from a table (F-LE.A.2★);**
- **manipulate exponential expressions (A-SSE.B.3★);**

• use the formula for the sum of a finite geometric series to solve a problem (A-SSE.B.4★);
• create and analyze simple exponential functions arising from real-word data (F-LE.B.5★).

A2.3 Logarithms

• Understand the definition of a logarithm as the solution to an exponential equation (F-LE.A.4*).
• Practice evaluating logarithmic expressions and converting between the exponential form of an equation and the logarithmic form (F-LE.A.4*).
• Solve exponential equations using logarithms (F-LE.A.4*).
• Understand the natural logarithm as a special case (F-LE.A.4*).
• Graph exponential and logarithmic functions, both by hand and using technology (F-IF.C.7e*, F-BF.5(+)).
• (Optional) Understand and explain the properties of logarithms.
• (Optional) Use properties of logarithms to solve problems.
• (Optional) Solve problems using the properties of logarithms.

Before this unit, students can construct exponential functions given a graph, table, or description, and interpret exponential functions expressed in different forms in terms of a context. They can graph exponential functions and note key features such as end behavior, asymptotes, and intercepts. They recognize real-world contexts that can be modeled by exponential functions, such as savings accounts and population growth, and they construct functions to model them. For example, they might describe the amount in a savings account with a $1000 initial balance that earns 10% a year compounded monthly using the function $A(t) = 1000(1.0083)^{12t}$. They can solve problems involving exponential equations like $20,000 = 1000(1.0083)^{12t}$ graphically, but not algebraically.

In this unit, students understand the logarithm defined operationally as the inverse of exponentiation, and have opportunities to practice interpreting logarithm notation and evaluating logarithms. They use logarithms to solve for an unknown exponent in situations modeled by exponential functions. These functions include ones expressed with base e, necessitating the introduction of the natural logarithm. Students graph logarithmic functions along with the exponential functions that are their inverses, developing an understanding of logarithmic functions as the inverses of exponential functions.

The Common Core State Standards do not explicitly require students to know and use the properties of logarithms. Student intending to pursue STEM careers in college should go deeper and learn these topics. Two optional sections at the end of the unit develop and apply the properties of logarithms.

After this unit, logarithms become a natural part of the toolkit in working with situations modeled by exponential functions. Applications abound in calculus, engineering, and the sciences. There are many sets of data that reveal their structure when plotted on logarithmic scales.

A2.3.0 Pre-unit diagnostic assessment

Assess students' ability to
• use reasoning and exponent properties to solve for an unknown exponents (A-REI.A.1);
• graph a relatively simple exponential function, showing correct end behavior and intercepts (F-IF.C.7e⋆);
• solve an exponential function at a given value graphically (A-REI.D.11⋆);
• summarize an exponential relation by writing an equation, given a table or several points (F-LE.A.2⋆).

A2.3.1 Motivate the need to undo exponentiation

Generate a need to find an unknown exponent which is not easy to guess and check.

The goal of this section is to help students see why a logarithm might be useful in their mathematical toolkit. Students are presented with a situation where they develop an exponential model and need to find an unknown exponent that yields a specified value. Teachers can revisit and assess solving such a problem by graphing with technology or by guess and check. They can then point out that students can solve all other kinds of equations that they know about by rewriting them in a helpful, equivalent form (for example, $x^3 = 1000$ can be rewritten as $x = 1000^{1/3}$) and suggest that there should be a way to do that to find an unknown exponent (as in, for example, $3^x = 1000$). Teachers can either introduce the logarithm at this point, or leave students in suspense until the next section.

A2.3.2 Understand the definition of a logarithm

• Understand the definition of a logarithm as the solution to an exponential equation (F-LE.A.4⋆).
• Practice evaluating log expressions and converting between the exponential form of an equation and the logarithmic form (F-LE.A.4⋆).

The logarithm can be defined operationally. Just as $\sqrt[3]{2}$ is defined to be the positive real number that when multiplied by itself three times is equal to 2, the solution to $2^x = k$ is defined to be $x = \log_2(k)$. In this section students develop this definition through an exploratory activity. They analyze a number of true statements about logarithms without having been told the meaning of the notation, make conjectures about the pattern they fit using their knowledge of exponents, and express their meaning in terms of an equivalent exponential equation. They come up with a definition of the logarithm by precisely describing what they see and generalizing it. They then practice interpreting and converting logarithmic expressions.

A2.3.3 Use logarithms to solve problems

- **Solve exponential equations using logarithms (F-LE.A.4★).**
- **Understand the natural logarithm as a special case (F-LE.A.4★).**

Once students understand what a logarithm is, they need ample opportunity to apply that understanding to solve problems in various contexts. All of the problems in this section should involve exponential functions with base 2, 10, or e and should be solvable by reasoning directly from the definition of the logarithm, using the equivalence between $b^x = y$ and $x = \log_b y$. In particular many of them involve modeling continuous growth using an exponential function with base e, so this is the section where the natural logarithm is introduced. Students do not need to know the property $\log(a^b) = b\log(a)$ or solve equations using this property ("taking logs of both sides").

Note on calculating logarithms: some scientific calculators have buttons for base 10 logarithms and natural logarithms, but not base 2 logarithms. There are many online calculators that can calculate the latter, such as Desmos or the Google calculator, which is activated by typing \log_2(x)$ into the search engine.

Tasks

F-LE Algae Blooms
F-LE Newton's Law of Cooling
F-LE Moore's Law and Computers
F-LE Snail Invasion

A2.3.4 The logarithm function

• Graph exponential and logarithmic functions, both by hand and using technology (F-IF.7e★, F-BF.B.5(+)).
• Verify that $f(x) = 10x$ and $g(x) = log_{10}(x)$ are inverses of one another (F-BF.B.4b(+)).

The previous sections focus on the definition of the logarithm as a notation for an unknown exponent and on solving equations involving exponentials and logarithms. In this section students start to view the logarithm as a function. This is analogous to the progression from learning about the square root symbol to studying the square root function. Students create graphs of logarithm functions by viewing them as the inverse of exponential functions, with inputs and outputs reversed. They apply their previous understanding of the nature of exponential functions to draw conclusions about the behavior of the graphs of logarithmic functions. For example, students who understand debt as an exponential function of time view the same situation as time being a logarithmic function of debt.

Tasks
F-BF Exponentials and Logarithms II
F-LE Exponential Kiss

A2.3.5 Going deeper: properties of logarithms (optional)

• Understand and explain the properties of logarithms.
• Use properties of logarithms to solve problems.

The sections previous to this one are sufficient to meet the standards about logarithms in CCSSM including the (+) standards. Students pursuing STEM careers will need to go further. This section and the next one show the bridging material needed for STEM readiness.

The properties of logarithms arise naturally from the properties of exponents and the fact that logarithmic functions are inverse to exponential functions. In this section students are given opportunities to notice patterns when combining logarithmic expressions with operations (for example, $\log(A) + \log(B)$ is equal to $\log(AB)$), conjecture that these patterns could be useful shortcuts, and then justify why the patterns always work.

A2.3.6 Going deeper: using the properties of logarithms (optional)

Solve problems using the properties of logarithms.

Students solve problems involving exponential functions with bases other than 2, e, or 10, or involving more than one exponential function with different bases, so that it is natural to use the property $\log(a^b) = b\log(a)$. The problems here are more complex than the ones in Section 3 and are suitable for students who are preparing for STEM majors in college.

Tasks
F-LE Rumors
F-LE Comparing Exponentials

A2.3.7 Summative assessment

Assess students' ability to
• solve equations with unknown exponents using logarithms (F-BF.B.5(+), F-LE.A.4★);
• graph a simple logarithmic function by hand showing intercepts and end behavior (F-IF.C.7e★);
• demonstrate understanding of the inverse nature of exponential and logarithmic functions (F-BF.B.4(+));
• solve a real-world problemw ith an unknown exponent using logarithms (F-LE.A.4★).

A2.4 Polynomials and Rational Functions

• Add, subtract, and multiply polynomials and express them in standard form using the properties of operations (A-APR.A.1).
• Prove and make use of polynomial identities (A-APR.C.4).
• Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior (A-APR.B.3★, F-IF.C.7c★).
• Use the remainder theorem to find factors of polynomials (A-APR.B.2, A-APR.D6).
• Use various strategies including graphing and factoring to solve problems in contexts that can be modeled by polynomials in one variable.
• Build a rational function that describes a relationship between two quantities (F-BF.A.1).
• Graph rational functions, interpret features of the graph in terms of a context, and use the graphs to solve problems (A-SSE.A.1a, A-REI.D.11★, F-IF.B.4, F-IF.C.7d(+)).
• Express rational functions in different forms to see different aspects of the situation they model (A-APR.D.6).
• Solve simple rational equations and understand why extraneous roots can arise (A-REI.A.2).

Coming into this unit, students understand that one can do arithmetic on quadratic expressions, and have generalized that understanding to polynomial expressions. They have solved quadratic equations with complex solutions using the methods of factoring, completing the square, and the quadratic formula. They have graphed quadratic functions and understand the relationship between zeros and factors of quadratics. They are familiar with modeling situations from which quadratic relations arise.

Students have modeled contexts with simple rational functions. They have graphed these functions and interpreted features of the graphs in context, including vertical asymptotes and end behavior. They can relate the domain of a function to its graph.

In this unit, students extend their previous work with quadratics and polynomials (in unit A4) to achieve a more general understanding of polynomials. They work with polynomials as a system where you can add, subtract, and multiply, analogous to the integers. They graph polynomial functions and they understand and use the relationship between factors, zeros, and intercepts on the graph. They model with polynomial functions.

Students study the graphs of simple rational functions. They consider contexts which can be modeled with rational functions and interpret vertical and horizontal asymptotes in terms of the context. They rewrite simple rational expressions in different forms to see different aspects of the context, and they find approximate solutions using graphical methods to rational equations that arise from the context. They also solve simple rational equations algebraically and study how and why extraneous solutions may arise.

After this unit, students going into STEM fields will see more examples, including polynomial approximations to other functions. Power series expansions (like Taylor series) show up in calculus. The characteristic polynomial of a matrix is a useful tool in university level linear algebra. Students going into higher mathematics will encounter polynomials as objects that help form abstract algebraic structures such as rings and fields.

Students may encounter simple rational relationships in real life. If they take a calculus course with a rigorous algebra component, they should do more work with more complicated rational expressions and equations beforehand.

A2.4.0 Pre-unit diagnostic assessment

Assess students' ability to
• graph a quadratic by factoring to find zeros, and correctly interpreting end behavior (A-APR.B.3, F-IR.C.7c★);
• solve a quadratic equation with a method suitable to the given form of the equation (A-REI.B.4b);
• write a simple rational equation that models a situation A-CED.A.1★, and use the equation to solve problems (A-REI.A.2).

A2.4.1 What is a polynomial?

• Add, subtract, and multiply polynomials and express them in standard form using the properties of operations (A-APR.A.1).
• Prove and make use of polynomial identities (A-APR.C.4).

In this section students become familiar with the arithmetic of polynomials. They add, subtract, and multiply them, and they use the properties of operations, particularly the distributive law, to express them as a sum of powers with coefficients. They recognize that every polynomial can be put in this form. They use polynomials to express and verify numerical patterns.

The emphasis in this section should not be on formal definitions or formal proofs of closure properties. Rather the emphasis is on preparing students for the manipulations they will be using the coming sections, when they start to study polynomial functions.

Tasks
A-APR Powers of 11
A-APR Trina's Triangles
A-APR Non-Negative Polynomials

A2.4.2 Graphing polynomials

• Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior (A-APR.B.3, F-IF.C.7c ⋆).
• Use the remainder theorem to find factors of polynomials (A-APR.B.2, A-APR.D.6).

The last activity in the previous section prepared students to start viewing polynomials in one variable as defining functions. In this section they study the graphs of polynomial functions. They see that the long run behavior of a polynomial is determined by its highest degree term. The use the relationship between factors and zeros to sketch the graph of a polynomial or to choose an appropriate viewing window for a graph produced by technology. The learn The Remainder Theorem and use it to find factors of polynomials.

Tasks
F-IF Graphs of Power Functions
F-IF Running Time
A-APR Graphing from Factors I
A-APR Graphing from Factors II
A-APR Graphing from Factors III
A-APR Zeroes and factorization of a quadratic polynomial I

A2.4.3 Modeling with polynomials

Use various strategies including graphing and factoring to solve problems in contexts that can be modeled by polynomials in one variable.

The main focus of modeling in this course is situations that can be modeled by linear, exponential, and quadratic functions. However, some contexts naturally give rise to polynomial models. In this section students make use of all that they have learned about polynomial functions to solve problems in such contexts.

Tasks

A-CED, A-REI Introduction to Polynomials - College Fund

A2.4.4 Rational Functions

- **Build a rational function that describes a relationship between two quantities (F-BF.A.1★).**
- **Graph rational functions (A-SSE.A.1a★, F-IF.C.7d★).**
- **Interpret the graph of a rational function in terms of a context (F-IF.B.4★).**

In this section students study simple rational functions. The emphasis is on rational functions that arise naturally out of a real-world context, and on interpreting features of their graphs in terms of that context. Students experiment with graphs using technology to learn the relationship between features of the graph and the structure of the expression defining the function.

Tasks

F-BF,IF The Canoe Trip, Variation 1
F-IF Average Cost
F-IF Graphing Rational Functions

A2.4.5 Modeling with rational functions

- **Graph rational functions and use the graphs to solve problems (A-REI.D.11 ★).**
- **Express rational functions in different forms to see different aspects of the situation they model (A-APR.D.6).**
- **Solve simple rational equations and understand why extraneous roots can**

arise (A-REI.A.2).

In the previous section students encountered simple rational functions. Here, they extend that work. They consider contexts which can be modeled with rational functions. They rewrite simple rational expressions in different forms to see different aspects of the context, and they find approximate solutions using graphical methods to rational equations that arise from the context. They also solve simple rational equations algebraically and study how and why extraneous solutions may arise.

Tasks
A-REI Ideal Gas Law
A.APR Combined Fuel Efficiency
A-REI An Extraneous Solution

A2.4.6 End Assessment

Assess students' ability to
• graph a polynomial by factoring to find zeros, and correctly interpreting end behavior (A-APR.B.3, F-IF.C.7c★);
• apply the remainder theorem to solve a mathematical problem (A-APR.B.2);
• model with a polynomial (A-CED.A.1★), and use the model to solve a problem (A-REI.D.11★);
• rewrite a rational expression in a different form to solve a problem (A-REI.D.11★);
• write a rational equation that models a situation (A-CED.A.1★), and use the equation to solve problems (A-REI.A.2);
• demonstrate understanding that when solving a rational equation, extraneous roots may emerge as a result of the solution process (A-REI.A.2).

A2.5 Trigonometric Functions

• Understand some real-world situations that demonstrate periodic behavior.
• Define coordinates on the unit circle as the sine and cosine of an angle (F-TF.A.2).
• Understand radian measure and convert between radians and degree (F-TF.A.1).
• Graph basic trigonometric functions using radians as the x-axis scale (F-IF.C.7e ⋆).
• Understand the relationship between parameters in a trigonometric function and the graph (F-IF.C.7e⋆).
• Model with trigonometric functions, including fitting them to data (F-TF.B.5⋆).
• Prove the Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$ **(F-TF.C.8).**
• Use the unit circle to prove trigonometric identities and relate them to symmetries of the graphs of sine and cosine (F-TF.A.4(+)).
• Use the Pythagorean Identity to calculate trigonometric ratios (F-TF.C.8).

Coming into this unit, students will already be familiar with right triangle trigonometry. They will have used sine, cosine and tangent to solve the angles and sides of a right triangle. Students will also have already seen that similar triangles preserve angles, as well as side ratios. For instance, they will have seen that a 3-4-5 right triangle has the same angles as a 6-8-10 triangle. They may have seen the “special right triangles” in the context of similar triangles. Students will also be familiar with Pythagorean Theorem, and will have used it to solve sides of right triangles. They will be familiar with the graphs of more than one type of function, including linear, quadratic, and exponential. Depending on timing, they may also have seen graphs of rational and polynomial functions.

In this unit, students make a big transition to thinking of trigonometric ratios as functions rather than a relationship between angles and side ratios in a right triangle. There are two important steps in this transition. First, students re-envision trigonometric ratios on the unit circle, and then use the idea of an angle as a movement around a circle to extend the definition to angles of any measure. Second, they use the unit circle to understand a new way of measuring angles, radian measure, which uses distance around the circumference of the circle to measure an angle, rather than an arbitrary division of the circle into 360 degrees. From now on students will be thinking of sine and cosine both as functions with numerical inputs, as well as ratios related to angles.

Students learn the basic shape of the graph of a trigonometric function, and then examine graphs of functions with parameters controlling the period, amplitude, and phase shift. They study the effect of varying these parameters and fit trigonometric functions to data. Addressing some (+) standards, students explore further the consequences of the unit circle definition of sine and cosine. They make a connection to the Pythagorean theorem. Then, they see how the symmetries of the circle give rise to symmetries of the graphs of sine and cosine, and represent these symmetries as identities.

In later math courses, students may see more complex uses of trigonometry. For instance, they may go on to learn that the derivative of the sinθ is the cosθ, and use radians to prove some of the properties of trigonometric derivatives.

A2.5.0 Pre-unit diagnostic assessment

Assess students' ability to
•using basic right triangle trigonometry to solve sides and angles of a right triangle (G-SRT.C.8★);
•use Pythagorean Theorem to solve for the sides of a right triangle (G-SRT.C.8★);
•view the vertical distance between the x-axis and a given point as the value of the point's y-coordinate, and the horizontal distance between the y-axis and a given point as the value of the point's x-coordinate.

A2.5.1 Introduction to periodic behavior

See a basic real-world model of periodic behavior and make sense of what data or graph it might generate.

Students have had a lot of exposure to graphing linear, quadratic, and exponential functions. They have yet to see a function that behaves periodically and understand how it might connect to a specific context. The ferris wheel provides a familiar scenario for students to see how the height of the cart will go up and down continuously, and to connect this information to a possible graph of the height. Students can make a rough sketch after watching the demo, or can use the more specific tools available in the Desmos activity to attempt to get a more accurate graph.

A2.5.2 Extending trigonometric functions to the real numbers

• See coordinates on the unit circle as the sine and cosine of an angle (F-TF.A.2).
• Understand radian measure and convert between radians and degrees.
• Graph basic trigonometric functions using radians as the x-axis scale (F-IF.C.7e⋆).

Students have understood trigonometric ratios in terms of right triangles, using them to solve for various sides and angles. In this section they make the big transition to thinking of trigonometric ratios as functions. There are two important steps in this transition. First, students re-envision trigonometric ratios on the unit circle, and then use the idea of an angle as a movement around a circle to extend the definition to angles of any measure. Second, they use the unit circle to understand a new way of measuring angles, radian measure, which uses distance around the circumference of the circle to measure an angle, rather than an arbitrary division of the circle into 360 degrees. From now on students will be thinking of sine and cosine as functions with numerical inputs, in addition to than ratios related to angles.

Tasks
F-TF.1 What exactly is a radian?
F-TF Bicycle Wheel
F-TF Trig Functions and the Unit Circle

A2.5.3 Modeling periodic behavior

• Understand the relationship between parameters in a trigonometric function and the shape of the graph (F-IF.C.7e⋆).
• Model with trigonometric functions, including fitting them to data (F-TF.B.5⋆).

Students have learned the basic shape of the graph of a trigonometric function, and now begin examining graphs of functions with parameters controlling the period, amplitude, and phase shift. They study the effect of varying these parameters and fit trigonometric functions to data.

Tasks

F-TF, F-BF Exploring Sinusoidal Functions

A2.5.4 Identities and special values for trigonometric functions

• **Prove the Pythagorean Identity $\sin^2\theta + \cos^2\theta = 1$ (F-TF.C.8).**
• **Use the unit circle to prove trigonometric identities and relate them to symmetries of the graphs of sine and cosine (F-TF.A.4(+)).**
• **Use the Pythagorean Identity to calculate trigonometric ratios (F-TF.C.8).**

In this section, which includes some (+) standards, students explore further the consequences of the unit circle definition of sine and cosine. They make a connection between the Pythagorean theorem. Then, they see how the symmetries of the circle give rise to symmetries of the graphs of sine and cosine, and represent these symmetries as identities.

Tasks
F-TF Trigonometric Ratios and the Pythagorean Theorem
F-TF Properties of Trigonometric Functions
F-TF, G-CO, Trigonometric Identities and Rigid Motions
F-TF Special Triangles 1

A2.5.5 Summative Assessment

Assess students' ability to
• **understand radians and angles on a unit circle (F-TF.A.1, F-TF.A.3(+));**
• **graph basic and transformed trigonometric functions (F-IF.C.7e*);**
• **compare trigonometric functions algebraically and graphically (F-IF.C.9);**
• **interpret and graph data within a given context (F-TF.B.5*).**

Tasks
F-TF Foxes and Rabbits 2

A2.6 Probability

• Describe events as subsets of a sample space (the set of outcomes) using characteristics of the outcomes or as unions, intersections, or complements of other subsets ("or," "and," "not") (S-CP.A.1).
• Use the Addition Rule to compute probabilities of compound events in a uniform probability model, and interpret the result in terms of the model (S-CP.B.7).
• In a uniform probability model, understand the probability of A given B as the fraction of B's outcomes that also belong to A (S-CP.B.6).
• Understand the conditional probability of event A given event B as P(A and B)/P(B) (S-CP.A.3).
• Understand that A and B are independent if P(A and B) = P(A) • P(B) (S-CP.A.2).
• Interpret independence of A and B as saying that the probability of A given B is equal to the probability of A, and the probability of B given A is equal to the probability of B, i.e. P(A|B) = P(A) and P(B|A) = P(B) (S-CP.A.5).
• Recognize independence in everyday situations and explain it in everyday language (S-CP.A.5).
• Determine whether events are independent (S-CP.A.2, S-CP.A.4).
• Use data presented in two-way frequency tables to approximate conditional probabilities (S-CP.A.4).

In Grade 7, students encountered theoretical probability in the form of probability models and sample spaces, and experimental probability as in the form of long-run relative frequency. They found probabilities of compound events using lists, tables, tree diagrams, and simulation.

In this unit, the study of probability is extended to the notions of independence and conditional probability. Students learn about conditional probability and independence. Rather than summing probabilities of simple events or using simulations, they calculate probabilities of compound events in terms of probabilities of other compound events or by using frequency tables. The latter builds on work with two-way frequency tables from the One Variable Statistics unit.

Later, in the Making Inferences unit, students draw on their work with probability as well as their work with data in the One Variable Statistics and Bivariate Statistics unit to solve problems.

A2.6.0 Pre-unit diagnostic assessment

Assess students' ability to
• write about probability as a numerical measure of likelihood (7.SP.C.5);
• describe events as subsets of a sample space (S-CP.A.1);
• calculate probability of a compound event in a uniform probability model (S-CP.A.2);
• calculate probability of a compound event when relative frequencies are given (S-CP.A.2).

A2.6.1 Chance

Recall ideas they've already learned about probability and get excited about learning more.

This section is intended to activate students' prior knowledge of probability. The activities in this section are intended to create an intellectual need for the more precise terminology to be developed in the unit. Depending on time constraints, one, two, or three of these activities could be used to achieve the goals of this section.

Tasks
7.SP Red, Green, or Blue?

A2.6.2 The Addition Rule

• Describe events as subsets of a sample space (the set of outcomes) using characteristics of the outcomes or as unions, intersections, or complements of other events ("or", "and," "not") (S-CP.A.1).
• Develop the Addition Rule to compute probabilities of compound events in a uniform probability model (S-CP.B.7).
• Apply the Addition Rule in a uniform probability model and interpret the answer in terms of the model (S-CP.B.7).

In grade 7, students described sample spaces by creating lists, tables, and diagrams. They determined P(A or B) by counting occurrences of simple events in A ∪ B. In this section, students learn to calculate P(A or B) in terms of P(A), P(B), and P(A and B). They begin by considering compound events as subsets of sample spaces, noting that in the sample space: the event "not A" is the complement of A; that the event “A and B” is the intersection of sets A and B; and that the event “A or B” is the union of A and B. The section culminates in a task where the Addition Rule is used to compute a probability.

Tasks

Describing Events
The Addition Rule
S-CP.7 Coffee at Mom's Diner

A2.6.3 Understanding independence and conditional probability

• Understand that two events A and B are independent if P(A and B) = P(A) • P(B) (S-CP.A.2).
• Understand the conditional probability of A given B as P(A and B)/P(B) (S-CP.A.3).
• Determine whether pairs of events are independent (S-CP.A.2).
• Find the conditional probability of A given B as the fraction of B’s outcomes that also belong in A (S-CP.B.6).
• Recognize independence in everyday situations and explain it in everyday language (S-CP.A.5).

In this section, uniform probability models are the main context. Students examine situations involving pairs of independent and non-independent events, allowing them to learn to distinguish between such pairs. They calculate probabilities of compound events in uniform probability models, finding the conditional probability of A given B as the fraction of B’s outcomes that are also in A. They note relationships such as P(A|B) = P(A and B)/P(B), and, when A and B are independent, P(A) = P(A|B).

Tasks

S-CP Lucky Envelopes
S-CP Cards and Independence
S-CP Breakfast Before School

A2.6.4 Using conditional probability to interpret data

Revisit two-way frequency tables and use them to approximate conditional probabilities (S-CP.A.4).

In the One Variable Statistics unit, students encountered two-way frequency tables. In this section, students revisit these tables, using them to approximate conditional probabilities by using the tables as a sample space.

Tasks

S-CP The Titanic 1

A2.6.5 Using conditional probability and independence to interpret data

Use data presented in two-way frequency tables to approximate conditional probabilities and to decide if events are independent (S-CP.A.4).

Students continue their work with two-way tables, using them to approximate conditional probabilities by using the tables as a sample space, and extending their work to consider independence.

Tasks

S-CP The Titanic 2

S-CP Rain and Lightning.

A2.6.6 Tying ideas together

• Interpret two-way frequency tables of data when two categories are associated with each object being classified, using the tables as a sample space in determining conditional probabilities and independence.
• Construct two-way frequency tables, use them to determine probabilities, and interpret these probabilities in the context of the data.

Students apply their knowledge of conditional probability and independence in tasks which are less structured than earlier tasks in the unit.

Tasks

S-CP The Titanic 3

A2.6.7 Summative Assessment

Assess students' ability to

• use the rules of probability to solve problems (multiplication and addition rules) (S-CP.A.1, S-CP.A.2, S-CP.B.7);
• use conditional probability to solve problems (S-CP.A.3);
• determine independence of events using the product rule and conditional probability (S-CP.A.4, S-CP.B.6);
• describe solutions to probabilistic situations using everyday language (S-CP.A.5).

A2.7 Making Inferences

• Understand that statistical methods are used to draw conclusions from data.
• Understand that the validity of data-based conclusions depends on the quality of the data and how the data were collected.
• Critique and evaluate data-based claims that appear in popular media.
• Distinguish between observational studies, surveys and experiments.
• Explain why random selection is important in the design of observational studies and surveys.
• Explain why random assignment is important in the design of statistical experiments.
• Calculate and interpret the standard deviation as a measure of variability.
• Use the normal distribution as a model for data distributions that are approximately symmetric and bell-shaped.
• Use the least squares regression line to model linear relationships in bivariate numerical data.
• Understand sampling variability in the context of estimating a population or a population mean.
• Use data from a random sample to estimate a population proportion.
• Use data from a random sample to estimate a population mean.
• Calculate and interpret margin of error in context.
• Understand the relationship between sample size and margin of error.
• Given data from a statistical experiment, create a randomization distribution.
• Use a randomization distribution to determine if there is a significant difference between two experimental conditions.

This unit builds on the foundation of One Variable Statistics and Bivariate Statistics, as well as students' work with statistics in grade 7. In particular, the concepts of sampling variability and distributions introduced in earlier units are critical to understanding the process of drawing conclusions from data, which is central to this unit.

In this unit, students revisit and extend their understanding of variability in data and of ways to describe variability in data. Students calculate and interpret the standard deviation, first introduced at a conceptual level in One Variable Statistics, as a way to quantify variability.

Students use distributions to describe variability. Normal distributions are introduced as a way to model data distributions that are bell-shaped and approximately symmetric. Students calculate and interpret areas under a normal curve in the context of modeling a data distribution.

Students explore the reasoning that allows them to draw conclusions based on data from statistical studies. They learn the distinction between an observational study and a statistical experiment. They then use data from a random sample from a population to estimate the values of population characteristics such as a population mean or a population proportion. Students develop the understanding that such estimates are subject to sampling variability. The notion of margin of error is introduced as a way of quantifying the uncertainty associated with an estimate of a population characteristic.

Students also explore the important idea of “statistical significance” as they use data from statistical experiments to determine if there is a significant difference between experimental conditions.

The standards addressed in this unit (especially S-IC.B.4 and S-IC.B.5) are conceptually complex and will require several weeks to fully develop.

Together with One Variable Statistics, Bivariate Statistics, and Probability, this unit provides a capstone experience in statistics for grades 6–12 and a solid foundation for an AP Statistics course or a college level introductory statistics course.

A2.7.0 Pre-unit diagnostic assessment

Assess students’ ability to
• construct and interpret a graphical display;
• calculate and interpret a sample proportion;
• calculate and interpret a sample mean;
• fit a line to bivariate data.

A2.7.1 Drawing reasonable conclusions

• Understand that statistical methods are used to draw conclusions from data.
• Understand that the validity of data-based conclusions depends on the quality of the data and how the data were collected.
• Critique and evaluate data-based claims that appear in popular media.

Pick up any newspaper or magazine and you are likely to see statements that claim to be based on data. But were the data collected in a reasonable way? Are the conclusions drawn from the data reasonable and if so, to whom do they apply?

The goal of this section is to get students to think about conclusions based on data and to begin to think about aspects of study design that will be studied in more detail in the later sections of this unit. In this section, you can present students with statements for a recent newspaper or magazine, or you can use some of the ones that follow here. Encourage students to think about what data were collected and how the data were collected. Ask students if they have any reason to question the statement that was made. What additional information would they want to know before making a decision about the reasonableness of the claim?

You can ask students to bring in a headline or data-based claim that they find in a newspaper or online and use them as the basis for discussion on the second day of this section.

Sample data-based statements for discussion:

- *Women's World* (September 27, 2010): Eating cheese before going to bed will help you sleep better. Eating garlic prevents colds.
- *Associated Press* (September 1, 2002): Vitamins found to prevent blocked arteries.
- *Women's World* (November 1, 2010): Strengthen your marriage with prayer.
- *Food Network Magazine* (January 2012): People who push a shopping cart at a grocery store are less likely to purchase junk food that those who use a hand-held basket.
- *The Los Angeles Times* (September 25, 2009): Spanking lowers a child's IQ.

The following article might also be of interest: "Health Freaks on Trial: Duct Tape, Bull Semen and the Call of Television," *Significance*, April 2014, Volume 11, Issue 2.

A2.7.2 Collecting data and types of statistical studies

• Distinguish between observational studies, surveys and experiments.
• Explain why random selection is important in the design of observational studies and surveys.
• Explain why random assignment is important in the design of statistical experiments.

Data are usually collected in order to answer some question. Depending on the nature of the question, data collection usually involves either observing characteristics of a sample from some population or carrying out a statistical experiment. This section introduces three types of statistical studies: observational studies, surveys, and experiments.

Observational studies and surveys are designed to answer questions about a population, and usually involve generalizing from a sample to a larger population of interest. This means that it is important that the sample be selected in a way that is likely to produce data representative of the population. Remind students of their prior work with random sampling in grade 7, and if necessary, review methods for selecting a random sample and discuss why random selection is important in the design of observational studies and surveys.

Statistical experiments are usually designed to answer questions of the form "What happens if...?" or "What is the effect of ...?" Provide examples of questions that could be answered by using data from a statistical experiment (such as "What is the effect of listening to music while studying on exam performance?" or "What happens to the moisture content of tortilla chips if the frying time is increased from 15 seconds to 20 seconds?")

Provide examples of statistical experiments that compare two or more experimental conditions. In a statistical experiment, it is important to start with comparable experimental groups. Discuss how random assignment of subjects to experimental groups is one way to ensure this.

Emphasize the difference between random selection and random assignment, and focus on the purpose of random selection and of random assignment in study design.

Provide examples of each type of study and allow students to practice distinguishing between the different types of studies. You might also ask students to bring in study descriptions from the newspaper or other media sources and have them explain to the class what type of study is being described and whether it involves random selection or random assignment.

Tasks

S-IC Why Randomize?
S-IC Words and Music II
S-IC Strict Parents

A2.7.3 Describing data distributions

• Calculate and interpret the standard deviation as a measure of variability.
• Use the normal distribution as a model for data distributions that are approximately symmetric and bell-shaped.
• Use the least squares regression line to model linear relationships in bivariate numerical data.

This section reviews material from grades 6–8 and previous units and forms a bridge between the previous section on collecting data and later sections on drawing conclusions from data.

The focus of this section is on three main ideas: (1) quantifying variability in a data set, (2) the normal distribution as a model for data distributions, and (3) the least squares regression line as a summary of a bivariate linear relationship. (Note that students are not required to know the term "least squares regression line," referring to it instead as "line of best fit.")

In the unit One Variable Statistics, students were introduced to the idea of standard deviation as a measure of variability. The focus in this unit should be on calculating the standard deviation (using technology) and on interpreting the standard deviation in context.

Students will probably not have seen the normal distribution before this unit, so it will need to be introduced here. Focus on properties of the normal distribution. Students will need to be able to find areas under a normal curve and percentiles for a normal distribution. The use of technology to assist in these calculations is encouraged. Students should be asked to interpret areas under a normal curve and percentiles for a normal distribution in a variety of different contexts.

In the unit Bivariate Statistics, students used technology to find the line of best fit (least squares regression line). This is revisited here with a focus on interpretation and modeling.

A2.7.4 Mid-unit assessment

Assess students' ability to
• given a description of a statistical study, identify the study type (observational study, survey, or experiment);

• determine what type of statistical study would produce data that could be used to answer a given question;
• distinguish between data distributions for which it would be reasonable to use the normal distribution as a model and those for which it would not be reasonable;
• find an area under a normal curve and interpret it in the context of modeling a data distribution.

A2.7.5 Drawing conclusions based on data from a random sample

• Understand sampling variability in the context of estimating a population or a population mean.
• Use data from a random sample to estimate a population proportion.
• Use data from a random sample to estimate a population mean.
• Calculate and interpret margin of error in context.
• Understand the relationship between sample size and margin of error.

A good description of how this section might play out in the classroom can be found on pp. 8–10 of Progressions for the Common Core State Standards in Mathematics: High School Statistics and Probability.

The focus of this section is on standards S-IC.B.3 and S-IC.B.4. This section will require a substantial time commitment. Students build on what they have learned about distributions and sampling variability as they use data from a random sample to learn about the value of a population proportion or a population mean.

It is probably easiest to begin by focusing on estimating a population proportion. Simulation can be used to approximate the sampling distribution of a sample proportion. This can be done using physical simulations (for example from a bag of beads that contains 40% blue beads) or using one of the many technology applets (for example, rossmanchance.com/applets/Reeses3/ReesesPieces.html which builds up a sampling distribution of the proportion of orange candies in a random sample after the user specifies the population proportion and the sample size). It is recommended that at least one physical simulation be carried out before turning to technology so that students understand the process that the technology is implementing.

Simulated sampling distributions become the basis for the important discussion of how the sampling distribution provides information about the anticipated accuracy of estimates based on random samples.

Margin of error associated with an estimate of a population proportion can be motivated based on the simulated sampling distribution in one of two ways:

1. In the context of the sampling distribution of a sample proportion, general properties can be described. These include:

- The sampling distribution is approximately normal if the sample size is large enough and the population proportion is not too close to 0 or 1.
- The sampling distribution is centered at the value of the population proportion, meaning that sample proportions tend to cluster around the actual value of the population proportion.
- The standard deviation of the sampling distribution (the standard deviation of the sample proportion) is approximately equal to $\sqrt{p(1-p)/n}$.

Based on these properties, the margin of error for estimating a population proportion is approximately $2\sqrt{p(1-p)/n}$.

2. A less formal approach introduces $1/\sqrt{n}$ as a conservative estimate of the margin of error (this is the maximum value of $2\sqrt{p(1-p)/n}$, which occurs when $p = ½$. The simulated sampling distributions can then be used to convince students that this value is reasonable (and at times a bit large—this is the “conservative” part) as a way to describe error.

Once margin of error has been developed, the formula and/or simulation results can be used to explore the relationship between sample size and margin of error.

Students should practice interpreting margin of error in context. This is a good place to bring in statements from the media where a margin of error is reported and to have students explain what is meant by the margin of error.

The sampling distribution also provides information that allows testing of claims about a population proportion. By comparing an observed sample proportion to what would be expected under a specified model (for example, a model that specifies that the population proportion is 0.6), a decision can be reached about whether the observed data are consistent with the model or whether it provides evidence that the model is not a believable description of the population.

The final part of this section should consider using sample data to estimate a population mean. Simulating to obtain a sampling distribution is not as easy here as in the case of proportions, but there are a number of good applets that can be used to carry out such simulations, e.g., rossmanchance.com/applets/SampleMeans/SampleMeans.html).

Tasks

S-IC Sarah, the chimpanzee
S-IC Block Scheduling

A2.7.6 Drawing conclusions based on data from a statistical experiment

• Given data from a statistical experiment, create a randomization distribution.
• Use a randomization distribution to determine if there is a significant difference between two experimental conditions.

A good description of how this section might play out in the classroom can be found on pp. 10–12 of Progressions for the Common Core State Standards in Mathematics: High School Statistics and Probability.

This section focuses on using data from statistical experiments to determine if there is a significant difference between two experimental conditions. By investigating what group differences might be expected due to chance alone when subjects are randomly partitioned into two groups, students are able to determine if chance alone is a plausible explanation for an observed difference. It is important that students understand that when a single group is partitioned into two groups, the two groups will tend to differ just by chance. This idea is fundamental in distinguishing “significant differences” from differences that might be due only to chance.

In this section, students should use data from an experiment to create a randomization distribution as a way of exploring group differences that are consistent with chance. This distribution is then used to determine if an observed difference is consistent with chance or whether the difference is large enough to indicate a significance difference due to the effects of the experimental conditions. Both physical and technology assisted simulations can be used to develop randomization distributions.

A2.7.7 Summative assessment

Assess students' ability to
• distinguish between an observational study and an experiment;
• use an appropriate normal distribution to model a data distribution;
• estimate a population proportion and interpret a margin of error in context;
• given a simulated sampling distribution, estimate a margin of error;
• given data from a statistical experiment, create a randomization distribution and use it to determine if there is a significant difference between experimental conditions.

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