Algebra 1

In Algebra 1, students build on the descriptive statistics, expressions and equations, and functions work first encountered in the middle grades while using more formal reasoning and precise language as they think deeper about the mathematics. Students add to the statistical work from the middle grades by working with standard deviation, describing statistical distributions more precisely, and measuring goodness-of-fit with residuals and the correlation coefficient. Students further their work with linear equations and inequalities as they transition from representations tied to tangible objects to working with abstract expressions. Students develop their abilities to see structure in expressions to show that expressions involving several operations are equivalent (for example, grasping that "substitution" works at various levels of complexity), and they solve linear and quadratic equations by writing a series of equivalent statements, justifying each step. Students formalize their concept of function and encounter exponential and quadratic functions as well as other examples of non-linear functions. A function that arises from a real context requires students to attend to an appropriate domain and to the meaning of various features of the function in the context. As they explore various functions, students should also leverage the power of making connections between graphical, tabular, symbolic, and contextual representations.

A1.1 One Variable Statistics

• Create dot plots, histograms, and box plots (S-ID.A.1).
• Use available classroom technology to create histograms and box plots and calculate measures of center and spread (S-ID.A.1).
• Use terms such as "flat," "skewed," "bell-shaped," and "symmetric" to describe data distributions (S-ID.A.2).
• Analyze and compare data sets (S-ID.A.3).
• Understand relationships between mean and median for symmetrical and skewed data distributions (S-ID.A.2).
• Recognize outliers when they exist, and know to investigate their source (S-ID.A.3).
• Know that outliers affect the mean, but not the median of a data set (S-ID.A.3).
• Describe variability by calculating deviations from the mean (S-ID.A.2).
• Compare two data sets with the same means but different variabilities, and contrast them by calculating the deviation of each data point from the mean (S-ID.A.2).
• Understand that IQR is a description of variability better suited to a skewed distribution (S-ID.A.3).
• Work with two-way tables (S-ID.B.5).

The story before this unit:
In grades 6 to 8, students were introduced to data sets and different ways to represent data (histograms, dot plots, box plots). Statistics is introduced as a tool to answer questions about a population that have variability in the answer. Students learn about measures of center (median, mean) and measures of variability (interquartile range, mean absolute deviation), using them to draw informal comparative inferences about two populations.

The part of the story happening in this unit:
Students build on and expand their understandings of statistics in this unit. The key characteristics (measures of shape, center, and spread) are again seen and in addition, students may further describe the shape of a data distribution (symmetric, skewed, flat or bell shaped) and summarize by a statistic measuring center and a statistic measuring spread. Instead of creating representations of data, the emphasis in high school is on interpreting representations and judiciously interpreting measures of center and spread.

Students develop more precise understanding of measures of center. They learn that mean and median are equal for symmetrical distributions, explain why mean and median are not equal in examples of skewed distributions, select median as the better measure of center for skewed distributions, and make generalizations about what kinds of distributions have means larger than medians and which have medians larger than means.

Students learn that standard deviation is a measure of spread, that a larger standard deviation means the data are more variable or spread out, and the meaning of standard deviation as “typical distance from the mean” for a symmetrical distribution. To aid in developing their understanding, students will calculate a standard deviation by hand for a small data set at least once. Given different visual representations of data (box plots, histograms, dot plots) students draw and justify significant and meaningful conclusions about the given situation. (All of these representations are frequency graphs, however, the Standards do not require students to know or use the term “frequency graph,” although they use the term “frequency tables.”) Students are introduced to two-way frequency tables and understand how to interpret relative frequencies in the context of the data represented in the tables. In unit S2 (which could take place either before or after this unit), students also build their statistics foundation by learning ways to determine whether two sets of data are correlated, and how strongly. Students identify linear association and interpret slope and intercept in the context of the data.

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Students are introduced to two-way frequency tables and understand how to interpret relative frequencies in the context of the data represented in the tables.

The story after this unit:
In unit S2 (which could take place either before or after this unit), students also build their statistics foundation by learning ways to determine whether two sets of data are correlated, and how strongly. Students identify linear association and interpret slope and intercept in the context of the data. Given different visual representations of data (linear models) students draw and justify significant and meaningful conclusions about the given situation. Students begin to use technology as a means to plot data and generate correlation coefficients.

In S3, students revisit two-way frequency tables from a probability standpoint, and use them as a tool for conceptualizing and finding conditional probabilities.

In S4, students combine the ideas of distributions and probability. They learn about normal distributions and use them to solve problems, and use the distributions of probability models to find the likelihood of a particular outcome. In doing so, students build on their experience with standard deviations from S1, calculating standard deviations using technology, and interpreting the results.

Every high school statistics and probability standard is a modeling standard, hence modeling pervades the four units.

A1.1.0 Pre-unit diagnostic assessment

Diagnose students' recall of middle grades statistics, specifically their ability to:

• recognize a statistical question (6.SP.A.1);
• describe the distribution of data collected to answer a statistical question by its center, spread, and overall shape (6.SP.A.2);
• interpret statistical plots (6.SP.B.4);
• summarize numerical data sets in relation to their context (6.SP.B.5);
• informally assess the degree of visual overlap of two numerical data distributions with similar variabilities (7.SP.B.3).

A1.1.1 How can data be represented and summarized meaningfully?

• Revisit various ways to plot data: dot plots, histograms, and box plots (S-ID.A.1).
• Interpret plots of data within the context of the data (S-ID.A.3).
• Use the terms "symmetric" and "skewed" as descriptors of distributions (S-ID.A.2).

Students revisit the methods for representing and summarizing data that they learned in grades 6 to 8. They interpret dot plots, histograms, and box plots, attending to the referents of symbols used. Not just what are the quartile values, but what do they indicate about the data? Not just whether the graph is symmetric or skewed, but what does that tell us about the data?

A1.1.2 Analyze data distributions

• Create dot plots, histograms and box plots (S-ID.A.1).
• Use available classroom technology to create histograms and box plots and calculate measures of center and spread (S-ID.A.1).
• Use terms such as "flat," "skewed," "bell-shaped," and "symmetric" to describe data distributions (S-ID.A.2).
• Analyze and compare data sets (S-ID.A.3).

Much of this section reviews ideas developed in grades 6 and 7 (but not 8), in order to allow the teacher to address any gaps in understanding revealed in the pre-unit assessment. If students are not already familiar with terms used to describe distributions (e.g., flat, skewed, bell-shaped, symmetric), then these terms should be introduced. With any task used, students should always be asked to interpret the statistical terms and measures in the context of the data set being described.

The teacher may also opt to use the tasks in this section to demonstrate how to use available technology to create histograms and box plots, and to calculate measures of center and spread, helping students to choose appropriate tools strategically when they analyze data in the future.

Tasks
S-ID.1,2,3 Speed Trap

A1.1.3 Measures of center

• Recall how to calculate mean and median.
• Understand mean and median as a "typical value" that can answer a statistical question.
• Know that mean and median are equal for a symmetrical data distribution (S-ID.A.2).
• Explain why mean and median are unequal for a skewed data distribution (S-ID.A.2).
• Select mean as the better measure for symmetrical distributions, and median as the better measure for skewed distributions (S-ID.A.2).
• Make generalization what kinds of distributions have means larger than medians, and what kinds have medians larger than means (S-ID.A.2).
• Recognize outliers when they exist, and know to investigate their source—

that data point is way out there, why is that? Is there something weird about it that means we should disregard it (S-ID.A.3)?
• Know that outliers affect the mean, but not the median of a data set (S-ID.A.3).

Students deepen their understanding of mean and median as measures of center, gaining a better understanding which to use in summarizing a given data distribution. They work with data sets where mean and median are equal and where they are different, and explain, using the context of the data, why this occurs. Teachers may continue to use the class's available technology to calculate summary statistics and make plots.

Tasks
S-ID Haircut Costs
Identifying Outliers
S-ID.3 Describing Data Sets with Outliers

A1.1.4 Mid-unit assessment

Assess students' ability to
• describe a set of data given a graph or table (S-ID.A.2);
• identify and calculate spread, center, shape, outliers, quartiles, mean, median, mode (S-ID.A.2);
• construct and interpret a box plot (S-ID.A.1);
• compare, contrast, and draw conclusions when given two data sets (S-ID.A.3).

A1.1.5 Standard deviation

• Describe variability by calculating deviations from the mean (S-ID.A.2).
• Compare two data sets with the same means but different variabilities, and contrast them by calculating the deviation of each data point from the mean (S-ID.A.2).
• Interpret sets with greater deviations as having greater variability (S-ID.A.2).
• Calculate a standard deviation by hand for a small data set, and understand standard deviation as an indicator of a typical deviation from the mean of an element of the data set (S-ID.A.2).

In the previous section, students interpreted the meaning of the various measures of center. Measures of center are important because they are single numbers that show what value is typical for a data set. However, data involves variation. How much data varies is an important question. In this section, students examine variability, the other major feature of measurements taken to answer a statistical question.

In grades 6 to 8, students learned that interquartile range (IQR) and mean absolute deviation (MAD) are ways to describe spread. In this section, they learn to calculate MAD's more sophisticated cousin, standard deviation. They start by looking at how much each data point deviates from the mean, and use these calculations to describe different data sets as more or less variable. Then, they go through the procedure for calculating standard deviation. (Although the Standards do not insist that students do such calculations or learn this procedure, doing the calculation a few times can help to illustrate what the standard deviation measures.) They come to understand standard deviation as "typical distance from the mean," and that higher values for standard deviation imply that a distribution is more spread out, whereas lower values imply that data are more closely clustered about the mean.

Tasks

S-ID Understanding the Standard Deviation

S-ID Measuring Variability in a Data Set

A1.1.6 Bringing it all together

• Represent a data set in different ways and decide which way is most appropriate (S-ID.A.1).
• Select measures of center and spread appropriate to the shape of the distribution (S-ID.A.3).
• Compare and contrast two or more distributions by using appropriate measures to describe center, variability, and shape (S-ID.A.2).

In this section, students are presented with summary statistics (mean, median, standard deviation, Q1, Q3, and minimum and maximum values) for five sets of data. Students must decide how best to represent the data, interpret and understand differences in center and spread, and determine if there are any outliers in order to accurately compare the given data sets.

A1.1.7 Two-way frequency tables

• Interpret a two-way table (S-ID.B.5).
• Understand that the choices made when organizing data can lead to different conclusions (S-ID.B.5).

Sometimes, it is illuminating to categorize univariate data, especially when the categories might influence the values of the variable. For example, in health studies, participants are often categorized as smokers and non-smokers, or men and women.

Tasks
S-IC, S-ID Musical Preferences

A1.1.8 Summative Assessment

Assess students' ability to
• calculate mean, median, and mode (S-ID.A.2);
• create box plots given data (S-ID.A.1);
• compare and contrast two frequency distributions (S-ID.A.3);
• articulate reasons to choose mean or use median as a measure of center (S-ID.A.2);
• read and interpret relative frequencies (S-ID.B.5).

A1.2 Linear Equations, Inequalities and Systems

• Explain each step in solving a simple equation in one variable (A-REI.A.1).
• Create and solve linear equations and inequalities in one variable, including equations with coefficients represented by letters (A-CED.A.1★, A-REI.B.3).
• Model constraints and relationships between quantities by equations and inequalities, and by systems of equations and inequalities, and interpret solutions (A-CED.A.2★, A-CED.A.3★).
• Solve systems of linear equations approximately by graphing and exactly by algebraic methods (A-REI.C.6).
• Understand the principles behind the method of elimination (A-REI.C.5).
• Graph the solution set to a linear inequality in two variables as a half-plane (A-REI.D.12).
• Graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes (A-REI.D.12).

The story before this unit:

Students begin learning about ratios and rates in Grade 6. In Grade 7, they represent proportional relationships by equations of the form $y = kx$, understanding k as the constant of proportionality or unit rate. In Grade 8, they recognize such equations as special kinds of linear equations $y = mx + b$ where m is the constant of proportionality and b is 0. They understand m as the slope of the line obtained from graphing the equation and b as the y-intercept of the line which is the value of y when $x = 0$.

By this point in their mathematical trajectory, then, students should be fairly comfortable with linear equations in two variables. They should be able to create and graph such equations to represent real-world situations that can be modeled by linear equations. Just as importantly, students should be able to describe the fundamental characteristic of linear functions, namely that they have a constant rate of change: the change in the output variable is proportional to the corresponding change in the input variable.

In grade 8, students analyzed and solved pairs of simultaneous linear equations (8.EE.C.8). Students should:
• know that the solutions to a system of two linear equations in two variables

correspond to points of intersections of their graphs, because points of intersection satisfy both equations simultaneously (8.EE.C.8.a);
• know how to solve a system of two linear equations in two variables algebraically (8.EE.C.8.b);
• be able to estimate solutions by graphing the equations (8.EE.C.8.b);
• know how to solve real-world and mathematical problems leading to a system of two linear equations in two variables (8.EE.C.8.c).

The part of the story happening in this unit:

In this unit, students build on what they know from middle school about linear equations and inequalities and systems of linear equations and expand their understanding to include systems of linear inequalities. They work with more complex modeling problems and become fluent in general methods of solution. They make more sophisticated use of graphical methods of representing and solving equations, inequalities, and systems, and they interpret points or regions in the plane in terms of the context.

The story after this unit:

Students will apply what they learn in this unit to the study of bivariate statistics. They will revisit the notion of a function and use the techniques learned here to study linear functions. They will come to view linear functions as one of many function families that display predictable characteristics. The technique of substitution, learned to find values that simultaneously satisfy two linear equations, is useful in more general situations, e.g., to solve a system consisting of a linear and a quadratic equation (A-REI.7) and in differential and integral calculus. For students who study calculus, linear functions will be an essential basis for their work with derivatives and differentiation.

A1.2.0 Pre-unit diagnostic assessment

Assess students' ability to
• solve linear equations in one variable with rational number coefficients (8.EE.C.7.b);
• solve word problems leading to linear inequalities of the form of the form $px + q \geq r$ with rational coefficients (7.EE.B.4.b);
• graph linear equations of the form $y = mx + b$ (8.F.A.3);
• solve a real-world problem that leads to a simple case of a system of two linear equations in two variables, where the same variable occurs with coefficient 1 in both equations (8.EE.C.8).

A1.2.1 Overview of linear equations and inequalities in two variables

Create and graph the solutions of linear equations and inequalities in two variables, and discuss their meaning in a real-world context (A-CED.A.2★, A-CED.A.3★, A-REI.B.3, A-REI.D.12).

This hook lesson provides an engaging context which allows students to exercise many of the skills they started to learn in middle school and will continue to apply in more sophisticated ways in this unit: setting linear equations in two variables to model a relationship between two quantities, solving for another variable, interpreting and graphing inequalities in two variables. It sets the stage for the units to come.

A1.2.2 Reason about linear equations and inequalities in one variable

• Explain each step in solving a simple equation in one variable (A-REI.A.1).
• Create and solve linear equations in one variable, including equations with coefficients represented by letters (A-REI.B.3).
• Create and solve linear inequalities in one variable (A-REI.B.3).

As preparation for the work with two-variable equations, inequalities and systems in this unit, students must have a strong foundation in working with equations and inequalities in one variable and a clear understanding of what an equation is and what it means for a number to be a solution to the equation (it makes the two sides equal). This section gives students opportunities to practice reasoning, manipulating and solving equations and inequalities.

Tasks

A-REI How does the solution change?
A-REI Same solutions?
A-REI Reasoning with linear inequalities

A1.2.3 Model with systems of linear equations

• Model constraints and relationships between quantities with systems of linear equations (A-CED.A.2★, A-CED.A.3★).

• Solve systems of linear equations approximately by graphing and exactly by algebraic methods (A-REI.C.6).

Students have worked with systems of linear equations in middle school and solved simple problems with them. In high school they work with more complex modeling problems and become fluent in general methods of solution. This first section on systems focuses on the modeling aspect. The systems are either solved graphically or the algebraic manipulations required to solve them are relatively simple. Furthermore, the modeling emphasis supports conceptual understanding by emphasizing the quantitative meaning of the variables, the equations, and the solutions to the system. This prepares students to take a thinking approach the solution methods in the next section on systems, rather than a purely formal one.

Tasks
A-REI, A-CED Cash Box
A-REI Quinoa Pasta 2

A1.2.4 Mid-unit assessment

Assess students' ability to
• solve equations and inequalities in one variable (A-REI.B.3);
• explain each step in solving a simple equation (A-REI.A.1);
• solve systems of equations graphically and algebraically (A-REI.C.6);
• set up and solve systems of equations that model a context (A-CED.A.3★);
• interpret a solution to a system of equations in terms of the context (A-CED.A.3★).

A1.2.5 Solve general systems of linear equations in two variables

• Solve systems of linear equations exactly by algebraic methods (A-REI.C.6).
• Understand the principles behind the method of elimination (A-REI.C.5).

Students were introduced to the basic methods of solving systems of equations in middle school. In Section 3 of this unit they used simple systems to solve modeling problems. In this section they become fluent in general methods for solving systems algebraically and reason through the justification for these methods.

Tasks

A-REI Accurately weighing pennies I
A-REI Estimating a Solution via Graphs
A-REI Solving Two Equations in Two Unknowns
A-REI Accurately weighing pennies II

A1.2.6 Model with inequalities in two variables

• Identify constraints from a context, choose relevant variables and model the context with an inequality or system of inequalities (A-CED.A.3⋆).
• Identify coordinates pairs or points in the plane as solutions or non-solutions and interpret them in terms of the context (A-CED.A.3⋆).
• Graph solution sets to a linear inequality or system of inequalities (A-REI.D.12).

In Grades 6–8 students learned about inequalities in one variable, and about equations and systems of equations in two variables. Here they tie these together and study inequalities, or systems of inequalities, in two variables. The emphasis is on modeling and interpreting solutions or non-solutions. Students also represent inequalities by shaded regions and interpret points in the plane; however, graphing should not be overemphasized or reduced to a procedure that does not engage the meaning of the inequality.

Tasks

A-REI Solution Sets
A-REI Fishing Adventures 3

A1.2.7 Summative assessment

Assess students' ability to
• create and solve linear equations and inequalities in one variable (A-REI.B.3);
• solve systems of linear equations exactly by algebraic methods (A-REI.C.6);
• model relationships between quantities and compare different relationships (A-CED.A.2⋆, A-REI.C.6);
• graph the solution set to a linear inequality in two variables as a half-plane, and to a system of linear inequalities in two variables as the intersection of the corresponding half-planes (A-REI.D.12).

A1.3 Bivariate Statistics

• Represent data on two quantitative variables on a scatter plot (S-ID.B.6).
• Describe how two quantitative variables on a scatter plot are related (S-ID.B.6).
• Interpret the slope and the intercept of a linear model in the context of the data (S-ID.B.7).
• Use available technology to find lines of best fit (S-ID.B.6a).
• Assess the goodness of fit of a line to a small data set by plotting and analyzing residuals (S-ID.B.6b).
• Fit a linear function for a scatter plot that suggests a linear association (S-ID.B.6c).
• Use available technology to compute correlation coefficients (S-ID.B.8).
• Understand that the correlation coefficient measures the "tightness" of a line fitted to data (S-ID.B.8).
• Understand that correlation does not necessarily imply causality (S-ID.B.9).

The story before this unit:

From their experiences with linear functions in grade 8, students are familiar with slope and intercept. They gained experience with scatter plots and described patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association (8.SP.A.1). For scatter plots that suggested a linear association, they informally fit a line and informally assessed its fit (8.SP.A.2). They wrote equations for these linear models, and interpreted their slopes and intercepts in the context of the data (8.SP.A.3).

The part of the story happening in this unit:

In this unit, students build on the statistical work they did in grade 8. They work with bivariate data and find the line of best fit by using a graphing calculator or other software. (These lines of best fit are regression lines (or technologically-generated approximations of them) but the Standards do not require students to learn or use the terms "regression," "regression line," "regression equation," or "least squares.") They assess the fit of a line to data more precisely by plotting and analyzing residuals. Students compare strength of associations between different pairs of variables by interpreting correlation coefficients, which they compute using technology. And, they gain experience in distinguishing between correlation and causality. Modeling is an intrinsic part of the high school statistics and probability standards, and of this unit.

The story after this unit:

In a future statistics unit on Inferences, student build on the techniques used this unit. Plotting residuals suggests that not some data sets may be better modeled non-linear functions than linear ones.

A1.3.0 Pre-unit diagnostic assessment

Diagnose students' ability to

• write a linear equation and interpret it in a context (8.F.B.4)
• determine a rate of change and initial value given several data points that exhibit a linear relationship;
• write an equation for a linear relationship;
• use the equation to make predictions;
• interpret the slope and intercept of a linear equation in a context.

A1.3.1 Preview

• Activate prior experience in grade 8 with informally fitting a line to a scatter plot and informally judging its goodness of fit (8.SP.A.2).
• Model the relationship with an equation for a line and use it to make predictions (8.SP.A.3).
• Represent data on two quantitative variables on a scatter plot (S-ID.B.6).
• Describe how two variables on a scatter plot are related (S-ID.B.6).
• Interpret the slope and the intercept of a linear model in the context of the data (S-ID.C.7).

(Note: 8.SP.A.2 and 8.SP.A.3 are prerequisites, not target standards in this unit. However, they are standards involved in one of the suggested activities.)

Students are presented with bivariate data that suggest a strong linear correlation. They plot the data points by hand or with technology, informally create a line of best fit, write an equation for the line, and use the equation to make predictions and interpret them in the context. Other tasks in this section focus on the last step: using the equation of a line fitted to data to make predictions and interpret them in the context of the data. Familiarity with informal fitting a line to data and assessing that fit is a basis for work in this unit:

assessing fit more precisely by analyzing residuals.

Tasks

S-ID.7 Texting and Grades II

S-ID.6a,7 Olympic Men's 100-meter dash

A1.3.2 Lines of best fit and residuals

• Use available technology to find lines of best fit (S-ID.B.6a).
• Quantify the goodness of fit by plotting and analyzing residuals (S-ID.B.6b).

Until now, students have estimated lines of best fit by eyeballing. This section shows how to use technology to plot lines of best fit and to quantify goodness (or badness) of their fit by analyzing residuals.

Tasks

S-ID, F-IF Laptop Battery Charge 2

A1.3.3 Interpreting the correlation coefficient

• Fit a linear function for a scatter plot that suggests a linear association (S-ID.B.6c).
• Use available technology to compute correlation coefficients (S-ID.C.8).
• Understand that the correlation coefficient measures the “tightness” of a line fitted to data (S-ID.C.8).
• Understand the significance of correlation coefficients close to 1 or –1 (S-ID.C.8).
• Interpret the rate of change and constant term of a line fitted to data in the context of the data (S-ID.C.7).

Students now know that a calculator has the power to find the line of best fit, and that best fit has something to do with residuals. But how can we gauge just how good a fit that “best” line is? Conversely, how close to linear is a linear-ish set of data? When they use their technology of choice to compute the line of best fit, students will learn to read the correlation coefficient and how to interpret its value.

Tasks

S-ID Used Subaru Foresters I

A1.3.4 Correlation vs causation

Understand that correlation does not necessarily imply causality (S-ID.C.9).

Students are now experts at figuring out how closely two sets of data are correlated, but they may be susceptible to the logical fallacy that correlation implies causality. Some well-chosen tasks can warn them against drawing such a false conclusion.

Tasks
S-IC, S-ID High blood pressure

A1.3.5 Bringing it all together

• Describe how two quantitative variables are related (S-ID.B.6).
• Use technology to create a line of best fit (S-ID.B.6c).
• Fit a line to given data and use it to make predictions (S-ID.B.6a).
• Interpret the coefficients of a line of best fit in the context of the data to which it is fitted (S-ID.B.7).
• Compute and interpret a correlation coefficient (S-ID.B.8).
• Understand that variables with a high correlation do not necessarily have a causal relationship (S-ID.B.9).

In this section, students use all that they have learned in this unit. They are given data or they collect it, and use technology to create the line of best fit and calculate the correlation coefficient. The final step is to analyze the situation and draw conclusions based on their findings.

Tasks
S-ID Coffee and Crime

A1.3.6 Summative Assessment

Assess students' ability to
• create a scatter plot and a line of best fit using technology (S-ID.B.6);
• interpret the coefficients of the line of best fit in the context of the data

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(S-ID.C.7);
• interpret the correlation coefficient (S-ID.C.8);
• articulate the difference between correlation and causation (S-ID.C.9).

A1.4 Functions

• Interpret key features of graphs in terms of the quantities represented (F-IF.B.4 ⋆).
• Sketch graphs showing key features of the graph by hand and using technology (F-IF.C.7⋆).
• Understand that a function from one set (the domain) to another set (the range) assigns to each element of the domain exactly one element of the range (F-IF.A.1).
• Use function notation (F-IF.A.2).
• Interpret statements that use function notation in various contexts (F-IF.A.2).
• Work with graphs of piecewise-defined functions, including step functions (F-IF.C.7b⋆).
• Relate the domain of a function to its graph (F-IF.B.5⋆).
• Relate the domain of a function to the quantitative relationship it describes (F-IF.B.5⋆).
• Calculate and interpret the average rate of change of a function over a specified interval (F-IF.B.6⋆).
• Estimate the average rate of change of a function from its graph (F-IF.B.6⋆).
• Solve for x such that f(x) = c, when f is a linear function (F-BF.B.4a).
• Write an expression for the inverse of a linear function (F-BF.B.4a).

The story before this unit:

In grade 8, students are first introduced to the notion of functions. They understand a function as a rule that assigns to each input exactly one output 8.F.A.1. The main focus is on

- linear functions and their representations (equations, graphs, tables, or verbal descriptions);
- understanding that the equation $y = mx + b$ defines a linear function, whose graph is a line;
- modeling a linear relationship by a function, determining rate of change and initial value and interpreting them in terms of a situation modeled by the function and in terms of its graph or a table of values;
- describing qualitatively a functional relationship between two quantities by analyzing a graph, e.g., where it is increasing or decreasing, linear or nonlinear.

The part of the story happening in this unit:

In this unit, students begin to use formal notation for functions, writing equations such as $f(x) = 2x + 3$ to describe a function. Students develop the understanding that the input/output relationship is a correspondence between two sets, and use the terms domain and range to describe them. They develop fluency with function notation and its use in describing qualitative features of the graph of a function by first interpreting, then writing expressions, equations, and inequalities such as $f(x + 2)$, $f(a) = 2$, $f(x) > 2$, and $f(x) > g(x)$.

Students expand their repertoires of functions, working with piecewise-defined functions, including step functions.

Building on their experiences with rate of change and slope from grade 8, students examine the behavior of non-linear functions. They describe key aspects of their graphs, and calculate and interpret average rates of change over specified intervals.

Students' work with domain and range provides a basis for understanding when a function has an inverse. They examine simple functions that do and do not have inverses and write expressions for inverses of linear functions, but do not draw general conclusions about when a function has an inverse.

The story after this unit:

Functions are a unifying theme in high school mathematics. In statistics, functions play especially prominent supporting roles as lines of best fit for bivariate statistics and the normal distribution. In geometry, transformations are viewed as functions sending points in the plane to points in the plane, and ratios of sides of right triangles lead to the trigonometric functions on acute angles. In algebra, students study systems of linear equations and inequalities, exponential functions and geometric sequences, quadratic functions, and rational functions, polynomials, and logarithms.

A1.4.0 Pre-unit diagnostic assessment

Assess students' ability to
• identify functions and non-functions (8.F.A.1);
• identify linear and nonlinear functions (8.F.A.3);
• write an equation for the corresponding linear function when given two points on a line (8.F.B.4);

• find and interpret the rate of change when given a linear function that models a situation (8.F.B.4);
• interpret the graph of a function in terms of the situation it models (8.F.B.4).

A1.4.1 Graphing and functions

• Sketch graphs showing key features (F-IF.B.4★, F-IF.C.7★).
• Interpret key features of graphs in terms of the quantities represented (F-IF.B.4★, F-IF.C.7★).

In this unit, students begin their formal study of functions. They are introduced to function notation and gain a more precise understanding of what it means to be a function. They learn how to interpret functions in a given context and how to analyze them using different representations. In this section, they begin by graphing a variety of different functions.

A1.4.2 Introducing function notation

Understand that a function assigns to each element in the domain exactly one element of the range (F-IF.A.1).

In this section, students are introduced to function notation and begin to interpret statements that use it. They begin to build expertise in understanding how equations and inequalities that use function notation correspond to features of graphs of functions. In this section, the focus is on statements about one input and its output (i.e., one point on the graph). In section 7, they return to this correspondence, going in the opposite direction: from features of graphs to equations and inequalities about the functions represented.

Tasks
F-IF Interpreting the graph
F-IF Using Function Notation I
F-IF The Customers
F-IF Points on a graph

A1.4.3 Interpreting function notation in context

Interpret statements that use function notation in terms of the quantities represented (F-IF.A.2).

In this section, students apply and extend their understanding of function notation in various contexts by interpreting statements that use function notation in terms of the quantities represented.

Tasks
F-IF Cell phones
F-IF Yam in the Oven

A1.4.4 Mid-unit assessment

Assess students' ability to
• use function notation to represent points on a graph and to describe features of a graph (F-IF.A.1);
• understand function notation and how to interpret statements in function notation in terms of a context (F-IF.A.2).

A1.4.5 Domain, range, piecewise-defined functions

• Use the notions of domain and range (F-IF.A.1, F-IF.B.5★).
• Interpret a graph of a piecewise-defined function (F-IF.C.7b★).
• Graph step functions (F-IF.C.7b★).

In this section, students are introduced to the notions of domain and range. They work with graphs of two types of functions that may not be familiar: continuous piecewise-defined functions and step functions. Because the ranges of step functions are discrete, students may find the notion of range especially salient in the case of step functions.

Tasks
F-IF The restaurant
F-IF Finding the domain
F-IF The Parking Lot
F-IF Bank Account Balance

A1.4.6 Interpreting graphs in context

• Interpret key features of graphs in terms of the quantities represented (F-IF.B.4★).
• Sketch graphs showing key features (F-IF.B.4★).
• Relate the domain of a function to its graph (F-IF.B.5★).
• Relate the domain to the quantitative relationship it describes (F-IF.B.5★).

In this section, students read and interpret graphs of functions. These include graphs of functions that arise from data, including functions that arise from using trend lines to join data points. Students also have an opportunity to apply their understanding of domain and range.

The tasks in this section could be approached as a jigsaw activity. In small groups, students become experts on one particular task, then work in new groups which include at least one expert for each task.

Tasks
F-IF Oakland Coliseum
F-IF Warming and Cooling
F-IF Influenza epidemic
F-IF How is the Weather?
F-IF Telling a Story With Graphs

A1.4.7 Average rate of change

• Calculate the average rate of change of a function over a specified interval (F-IF.B.6★)
• Interpret the average rate of change of a function over a specified interval (F-IF.B.6★).

In this section, students are introduced to the notion of average rate of change over an interval. They work with expressions for average rates of change, and compute and estimate average rates of change. In grade 8, students learned that the rate of change of a linear function is the slope of its graph and that the slope can be computed from the coordinates of any two distinct points on the line. For nonlinear functions, the story is more complicated because their rates of change vary depending on the interval chosen. As for linear functions, average rates of change over an interval can be computed from the coordinates of two points on their graphs, namely those corresponding to the endpoints of the interval.

Tasks
F-IF Temperature Change
F-IF The High School Gym
F-IF Mathemafish Population

A1.4.8 Inverse functions

• Solve for x such that $f(x) = c$, when f is a linear function (F-BF.B.4a).
• Write an expression for an inverse (F-BF.4a).

In this section, students learn about inverse functions. The tasks illustrate some real-world contexts in which inverse functions occur.

Tasks
F-IF Your Father
F-BF Temperatures in degrees Fahrenheit and Celsius
F-BF US Households

A1.4.9 Summative assessment

Assess students' ability to
• identify the domain and range of a given function (F-IF.A.1);
• create a step function that represents pairs of given values (F-IF.C.7b*);
• interpret statements that use function notation in a context (f-IF.A.2);
• identify key features of graphs (F-IF.B.4*);
• identify referents of expressions that use function notation in a given graph (F-IF.B.4*);
• identify and relate the domain and range of a function in a given context (F-IF.B.5);
• calculate the average rate of change of a function over a specified interval (F-IF.B.6*);
• interpret the average rate of change of a function in terms of the quantities represented (F-IF.B.6*).

A1.5 Exponential Functions 1

• Distinguish between the growth laws of linear and exponential functions and recognize when a situation can be modeled by a linear function versus an exponential function (F-LE.A.1★).
• Graph exponential functions and understand how changing by a constant factor over equal intervals affects the graph (F-IF.C.7e★).
• Model situations of growth and decay with exponential functions expressed in various different forms given a graph, a description of the situation, or two input-output pairs (including reading these from a table) (F-LE.A.2★).
• Understand that over time a quantity increasing exponentially will eventually exceed a quantity increasing linearly (F-LE.A.3★).
• Understand the form of different expressions for exponential functions in terms of change by a constant factor over equal intervals (F-LE.B.5★).

The story before this unit:

In Grade 9, students should be familiar with linear functions from Grade 8 and from the F1 unit. They have been formally introduced to functions and function notation and have explored the behaviors and traits of both linear and non-linear functions. Additionally, students have spent significant time graphing, interpreting graphs and have explored how to compare the graphs of two linear functions to each other.

The part of the story happening in this unit:

In this unit, students are introduced to exponential functions. Students learn the fundamental growth law for exponential functions and compare it with the law for linear functions. They recognize exponential functions when presented with data, graphs and real-world contexts. They construct exponential functions and use them to model situations and solve problems. They distinguish between situations that should be modeled with a linear function vs. an exponential function. They know various forms ways of expressing an exponential function. They know that an increasing exponential function eventually is greater than an increasing linear function.

The story after this unit:

In the next unit, students will be introduced to quadratic functions. Students will focus on the basic nature of quadratic functions, contexts in the real world that can be modeled by quadratic functions, and the different forms of the expression for a quadratic function and what those forms tell you about the behavior of the function and the shape of its graph. Students will also extend their understanding of exponential functions and how they relate to quadratic functions; understanding that an exponential growth function will eventually exceed both a linear and a quadratic function.

In the Logarithms unit, students will focus on a more in-depth understanding of exponential and log functions.

A1.5.0 Pre-unit diagnostic assessment

Diagnose students' ability to
• use the exponent laws to find equivalent expressions (8.EE.A.1);
• solve problems involving percent increase/decrease (7.RP.A.3);
• solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (7.EE.B.3);
• construct a linear function (8.F.B.4);
• describe a non-linear function (8.F.B.5);
• compare functions (8.F.A.2).

A1.5.1 The key to exponential growth

Learn the difference between growth by a constant multiplicative factor and growth by a constant additive factor (F-LE.A.1★).

In this section, students are introduced the underlying growth law for an exponential function, namely that the output changes by a constant multiplicative factor for a constant additive change in the input variable. They gain a quantitative sense of the difference this makes through an application to population growth.

Tasks
F-LE, A-CED Paper Folding

A1.5.2 Introduction to exponential functions

• Distinguish between the growth laws of linear and exponential functions (F-LE.A.1⋆).
• Construct simple exponential models (F-LE.A.2⋆).
• Create tables and graphs of exponential functions and understand their behavior in terms of the fundamental growth law(F-IF.C.7e⋆).
• Understand the form of the expression $f(x) = ab^x$ for an exponential function in terms of the fundamental growth law (F-LE.B.5⋆).

In this section students construct and interpret exponential functions expressed in the form f(x)=ab^x to model various context. They work with contexts where the initial value a and the growth factor b are either given or are directly inferable from the context, or where they must interpret those values in terms of the context. They make tables and graphs of exponential functions and begin to acquire a quantitative sense of exponential growth both numerically and graphically.

Tasks
F-IF Identifying Exponential Functions
F-IF Exponential Parameters
F-LE U.S. Population 1790-1860
F-LE Basketball Rebounds
F-LE Equal Differences over Equal Intervals 1
F-LE Equal Factors over Equal Intervals

A1.5.3 Model with exponential functions

• Model situations of growth and decay with exponential functions expressed in various different forms given a graph, a description of the situation, or two input-output pairs (including reading these from a table) (F-LE.A.2⋆).
• Recognize that exponential functions have a constant percent growth or decay rate per unit interval (F-LE.A.1a⋆).
• Interpret the parameters of an exponential function in a context (F-LE.B.5 ⋆).

Now that they are familiar with the basic form $f(x) = ab^x$ of an exponential function, students start to work with exponential functions expressed in other ways. They learn the relationship between the growth (or decay) factor and the growth (or decay) rate; if r is the growth rate then $1 + r$ is the growth factor. They model more complex situations where they must derive the growth factor in various ways given data about the context.

Tasks
F-LE Predicting the Past
F-LE Moore's Law and Computers
F-LE DDT-cay
F-LE All Your Base Are Belong to Us

A1.5.4 Compare exponential and linear functions

Understand that over time a quantity increasing exponentially will eventually exceed a quantity increasing linearly (F-LE.A.3★).

In this section, students will compare linear and exponential functions. Students are familiar with linear functions and linear growth so students will use this base understanding to develop the notion that increasing exponential functions will eventually exceed increasing linear functions.

Tasks
F-LE Linear or exponential?
F-LE, A-REI Population and Food Supply
F-LE Exponential growth versus linear growth I
F-LE Exponential growth versus linear growth II

A1.5.5 Bringing it together

• Interpret graphs and expressions for exponential functions (F-IF.C.7e★, F-LE.B.5★).
• Model depreciation with linear and exponential functions(F-LE.A.2★).
• Compare linear and exponential models (F-LE.A.1★).

In this section, students bring together much of what they have learned in this unit. They use their ability to write linear and exponential functions given two input-output pairs, they compare different models, both linear and exponential, and draw conclusions from them.

A1.5.6 Summative assessment

Assess students' ability to
• construct and compare linear and exponential functions given data (F-LE.A.2★, F-IF.C.7e★);
• recognize a situation in which a quantity grows by a constant percent rate per unit interval relative to another (F-LE.A.1c★);
• given an exponential model in the form $f(t) = ab^t$, interpret the constants a and b in terms of the context (F-LE.B.5);
• explain in words the similarities and differences between linear and exponential models (F-LE.a.1a★);
• recognize situations that can be modeled with linear functions and with exponential functions, and solve problems (F-LE.A.1★).

A1.6 Quadratic Functions

• Construct quadratic functions and quadratic sequences (A-CED.A.2★, F-IF.A.3, F-BF.A.1a★).
• Represent quadratic functions using recursive formulas, expressions, tables, and graphs (A-SSE.A.1★, A-CED.A.2★, F-IF.A.7a★, F-BF.A.1a★).
• Express quadratic functions in equivalent forms for different purposes; understand the relation between vertex form and the shape of the graph (A-SSE.A.1, A-SSE.B.3★, F-IF.A.7a★, F-IF.C.8, F-BF.B.3).
• Find the average rate of change of a quadratic function over a unit interval and compare rates for successive intervals (F-IF.B.6★).
• Describe properties that distinguish linear, exponential, and quadratic functions (F-LE.A.3★).
• Model with quadratic functions (A-CED.A.2★, F-IF.B.4★, F-IF.C.7a★, F-BF.A.1a★).

The story before this unit:

In the unit Linear Equations, Inequalities, and Systems students developed fluency with linear functions and in unit A2 they learned about simple exponential functions. Students can use graphing technology to plot a function, find intercepts and intersection points, and find a linear regression. They know exponent rules and the application of the distributive property to combining like terms or factoring out a common factor.

The part of the story happening in this unit:

In this unit students build and interpret quadratic functions. They work with contexts that can be modeled by quadratic functions and compare them with contexts that can be modeled linear and exponential functions. They express quadratic functions using recursive equations (e.g. $f(n+1) = f(n) + 2n + 1$), equations in two variables (e.g. $y = x^2 + 2x + 3$), or function notation (e.g. $f(x) = x^2 + 2x + 3$). They understand the purpose of different forms for the quadratic expression on the right hand side in the last two cases. They graph quadratic functions expressed in different forms, and construct functions expressed in factored or vertex form for a given learn how to put a function in vertex form, and see what information can be most easily obtained from it.

The story after this unit:

In the unit Quadratic Equations students solve quadratic equations in one variable approximately and exactly using various methods. They continue to develop facility in algebraic manipulation of quadratic expressions and equations. In unit A5, they explore complex numbers and revisit quadratic equations to solve for complex roots.

A1.6.0 Pre-unit diagnostic assessment

Diagnose students' ability to

• distinguish linear from exponential functions and identify if a function is neither (F-LE.A.1★);
• evaluate quadratic expressions and solve simple quadratic equations by inspection (6.EE.A.1, 6.EE.A.2c);
• solve simple quadratic equations by inspection (6.EE.B.5, 8.EE.A.2, A-REI.A.1).

A1.6.1 It's neither linear nor exponential

• Model a context with a quadratic function and interpret values of the function in context (A-CED.A.2★, F-BF.A.1a★).
• Graph a quadratic function and interpret the graph (F-IF.C.7a★).
• Find the average rate of change over a unit interval and compare rates for successive intervals (F-IF.B.6★).

This hook lesson touches on topics that arise throughout the unit: modeling with quadratic functions, interpreting their graphs in terms of a context, the way quadratic functions grow, and solving quadratic equations. The specific quadratic function used is of the simplest type and students do not have to carry out extensive manipulations. Rather, the context provides a motivation for learning those manipulations.

A1.6.2 Visual patterns and quadratic sequences

• Look for structure in number sequences arising from visual patterns (A-SSE.A.1★, F-IF.A.3).
• Model the patterns with quadratic functions given by recursive descriptions, expressions, or equations (A-SSE.A.1★, A-CED.A.2★, F-BF.A.1a★).

In the previous section students saw a quadratic function where the change over successive intervals of a fixed length grows linearly. It is not until calculus that students will be able to describe precisely the growth law for quadratic functions (namely that their derivatives are linear). However, they can see a discrete version of this law by restricting to quadratic functions whose domains are the whole numbers, that is, quadratic sequences. This allows students to focus on some basic features of quadratic functions without getting bogged down in analysis of real world data and contexts. In this and subsequent sections these sequences often arise from sequences of visual patterns. Each visual pattern lends itself to a geometric interpretation for how it grows, in such a way that the number of additional squares (or other objects) in each successive pattern grows linearly. It is then natural to model the number with a quadratic sequence given what students learned in the previous section. Students conjecture the number of squares at a given stage, create a table, and express the number of squares algebraically. Since the prompts are visual patterns, there are multiple points of entry and students have a context with which to check their conjectures.

A note on equations, expressions, and functions: students might initially write an expression, for example n^2, to represent the number of squares in the pattern. This is a natural thing to do. At some later point, in order to help them see that a sequence is a function, it might be helpful to introduce another variable S for the number of squares in the pattern and write an equation $S = n^2$, or to use function notation $f(n) = n^2$. All are acceptable, but it is important to use terminology correctly and not refer to expressions as equations or vice versa.

A1.6.3 Compare equivalent expressions for quadratic eunctions

• Understand that different ways of seeing a pattern give rise to different but equivalent expressions for the function arising from the pattern (A-SSE.B.3⋆).
• Reason through the equivalence of expressions (A-SSE.A.2).

This section explores equivalence of quadratic expressions. Visual patterns that are more complicated than in the previous section lead to different ways of expressing a quadratic function and prompt a discussion about the meaning of equivalence (equivalent expressions define the same function) and what purpose each of the equivalent forms might be useful for. Various equivalences can be explored in this section. Students draw on their previous knowledge of the distributive property to multiply binomials (the principle of multiplying “each by each"), including the special case of expanding the square in an expression given in vertex form. They also begin to think about how these processes might be reversed (factoring and completing the square).

Tasks

A-SSE Seeing Dots
A-SSE Equivalent Expressions

A1.6.4 Compare quadratic functions with linear and exponential functions

• Distinguish between tables representing linear, exponential, and quadratic functions (F-LE.A.3*).
• Distinguish between graphs of linear, exponential, and quadratic functions (F-LE.A.3*).
• Use precise language to describe properties that distinguish linear, exponential, and quadratic functions (F-LE.A.3*).

Now that students have been introduced to quadratic functions they compare them with the two other families of functions they have studied, linear and exponential functions. They compare tables and graphs and describe the differences. They also compare the growth of increasing quadratic functions with the growth of increasing linear functions or increasing exponential functions.

Tasks

F-LE Identifying Functions

A1.6.5 Model simple contexts with quadratic functions

• Construct a simple quadratic model (F-BF.A.1a*).
• Use the model to solve problems and make predictions (F-IF.B.4*, F-IF.C.7a *).

Now that students have constructed quadratic functions and compared them with linear and exponential functions, they start to explore contexts that can be modeled by quadratic functions. Different contexts naturally lead to different forms, and students use the skills developed in Section 3 to convert between forms. They also consider what shape graph is suggested by the context and use that information to narrow down the possibilities for expressing the function. This section could revisit the context from Section 1 in a deeper way, or bring in completely new contexts as listed here.

Tasks
F-BF Skeleton Tower

A1.6.6 Mid-unit assessment

Assess students' ability to

• write, use and interpret quadratic function that models a given context (A-CED.A.2★, F-IF.B.4★, F-BF.A.1a★);
• express quadratic functions in equivalent forms and choose an appropriate form for a given purpose (A-SSE.A.1★, A-SSE.A.2, A-SSE.A.3);
• differentiate between graphs and tables of values for linear, exponential, and quadratic functions (F-LE.A.3★);
• label features of the graph of a quadratic function with key vocabulary words (axis of symmetry, vertex, maximum, minimum, x-intercept, y-intercept) (F-IF.C.7a★).

A1.6.7 Express quadratic functions in vertex form

• Understand how the structure of vertex form is related to the maximum or minimum value of the function and to the vertex of its graph (A-SSE.A.1★, F-BF.B.3).
• Use vertex form to write a possible quadratic function given the maximum or minimum of the function or the vertex of its graph (A-SSE.B.3★, F-IF.C.7a★).

The standard form of a quadratic is not always the most useful form for a given situation. When modeling the path of a projectile, for example, it may be useful to express a function in vertex form in order to find the maximum height. In this section students interpret and construct functions expressed in vertex form. It is possible, but not necessary, that students begin work with converting a function from standard form to vertex form by completing the square. They are not expected to gain fluency in this operation until the next unit.

Tasks

F-BF Building a quadratic function from $f(x) = x^2$

A1.6.8 Work with the three basic forms

• Understand how the structure of factored form is related to the zeros of a function and the x-intercepts of its graph (A-SSE.A.1★, F-IF.C.7a★).
• Select the best form for expressing a quadratic function to illuminate specific features of graphs (A-SSE.B.3★, F-IF.C.8).

In this section, students write quadratic functions in different forms to illuminate different features of the function. Through the use of graphing technology or by hand, they explore ways to sketch quadratic functions given key features of the graph. Factored form, which has not been explicitly discussed before now, comes into play in these activities and students explore its equivalence to other forms through graphing. They explore expressions in vertex, factored, or standard form and interpret those expressions in terms of a model. By the end of this section students should be fluent with converting from factored or vertex form to standard form. They are also in the process of learning how to construct factored and vertex form from standard form, and become fluent with this in the next unit.

Tasks

A-SSE Graphs of Quadratic Functions
A-SSE Profit of a company
F-IF Which Function?

A1.6.9 Model richer contexts with quadratic functions

• Fit functions to verbal descriptions and graphs using key features (F-IF.C.7a★, F-BF.B.3).
• Solve modeling problems (F-IF.B.4★, F-LE.A.1, F-LE.A.2).

Students are now ready to start applying their knowledge of quadratic functions. They draw on their ability to construct expressions for functions to meet a given purpose. They identify intercepts and vertices in order to write functions that match a given situation. The work in this section prepares students for the work on solving quadratic equations in one variable but does not require students to find exact solutions to them.

Tasks

F-BF Medieval Archer

F-LE Choosing an appropriate growth model

A1.6.10 Choosing the most convenient form (optional)

Select find a function that matches a given graph (F-IF.B.4★, F-LE.A.3).

This section can be used at the end of the unit or intermittently throughout the unit to reinforce different aspects of quadratic functions (and other functions) using technology. The Daily Desmos challenges here mostly contain quadratics, but a few have either linear or exponential components making them a perfect opportunity to think through other types of functions they learned more about in previous units or grade levels and compare them with their new knowledge of quadratics.

A1.6.11 Summative assessment

Assess students' ability to

- **generate functions given graphs and graphs given functions (A-CED.A.2★, F-IF.C.7a★, F-IF.C.8, F-BF.A.1a★, F-BF.B.3);**
- **interpret expressions for quadratic functions in terms of a context it represents (A-SSE.A.1★);**
- **express quadratic functions in different forms for different purposes (F-IF.C.8);**
- **solve modeling problems using quadratic functions (F-IF.B.4★, F-BF.A.1a★).**

A1.7 Quadratic Equations

• Connect solving quadratic equations to finding zeros of quadratic functions (A-SSE.B.3a★, F-IF.C.8a★).
• Explore forms of quadratic equations that can be solved by seeing structure (A-REI.B.4b).
• Understand and be able to use the method of factoring to solve factorable quadratic equations (A-REI.A.1, A-REI.B.4b).
• Understand and be able to use the method of completing the square to solve quadratic equations, and derive the quadratic formula (A-SSE.B.3b★, A-REI.A.1, A-REI.B.4).
• Construct and solve quadratic equations by the most strategic method to solve problems in various contexts (A-CED.A.1★, A-REI.B.4b).
• Express a quadratic function in the appropriate form for a given purpose, including vertex form (A-SSE.B.3b★, F-IF.C.8a★).
• Solve problems using systems consisting of a linear and a quadratic equation in two variables (A-REI.C.7).
• Derive the equation of a parabola given the focus and a directrix parallel to one of the axes (G-GPE.A.2).

The story before this unit:

Students have just completed a study of quadratic functions where they explored tables, graphs, and expressions for quadratic functions in situations that can be modeled by them. The various forms of a quadratic expression were necessitated and explored.

The part of the story happening in this unit:

In this unit, students see quadratic equations arise naturally out of modeling problems involving quadratic functions. Any time you want to know the input to a quadratic function that produces a specified output, you have to solve a quadratic equation. Students understand solving quadratic equations as a process of reasoning and use the properties of operations to form equivalent quadratic expressions and equations. They convert between standard, vertex, and factored form by factoring, completing the square, and distributing. This unit does not treat factoring as a systematic method, but rather as an opportunistic method to be used when a factorization is readily available. The method of completing the square is a systematic method that works in all cases, and leads to the quadratic formula. Students should be able to solve quadratic equations by many different methods, making strategic choices of the best method for the situation at hand.

Students then adapt the method of completing the square to putting a quadratic function in vertex form. The unit ends with a section on deriving the equation for a parabola from its geometric definition.

The story after this unit:

Students will start to encounter situations where the roots are not real numbers, which will prepare them for a full investigation of complex numbers in the next unit.

A1.7.0 Pre-unit diagnostic assessment

Diagnose students' ability to

• recognize a solution to a quadratic equation (A-REI.A.1);
• perform operations on rational numbers (7.NS.A);
• work with irrational numbers, primarily by approximating on the number line (N-RN.B.3).

A1.7.1 Motivate solving quadratic equations

• Generate an intellectual need to solve a quadratic equation.
• Understand the graphical method for solving equations approximately.

In this section students develop an intellectual need for a general method for solving quadratic equations, through an encounter with a problem that is not obviously modeled by a quadratic equation and where the solution isn't obvious through reasoning or inspection. They solve the problem approximately using graphing technology. Some students may have experience with solving quadratic equations and offer solutions. Solving the equation exactly can be delayed until Section 3, where students can solve it by completing the square.

A1.7.2 What do we already know about solving quadratic equations?

• Understand the Zero Product Property and use it to justify steps to solve a factorable quadratic equation (A-REI.A.1).
• Explore forms of quadratic equations that can be solved by seeing structure (A-SSE.A.2, A-REI.B.4b).
• Connect solving quadratic equations to finding zeros of quadratic functions (A-SSE.B.3A★, F-IF.C.8a★).

In the Quadratic Functions unit students saw that the factored form of a quadratic function makes it easy to see the zeros of the function. Here they see that fact in a different light, learning that finding the zeros of a quadratic function is the same as solving the quadratic equation obtained by setting the function equal to zero. Students use factoring as an opportunistic method to solve quadratic equations, taking advantage of situations where a factorization is readily available from seeing the structure of a quadratic expression (e.g. a difference of squares) (MP7). Then they see how the vertex form of a quadratic function can also be useful in solving quadratic equations by taking square roots, as preparation for the general method of completing the square in the next section.

Note: The fact that a number has at most two square roots is connected to the factored form. For example, the fact that $x^2 = 4$ has only the two solutions 2 and -2 follows from the fact that the equation $x^2 - 4 = 0$ can be converted to factored form $(x + 2)(x - 2) = 0$.

Tasks
A-REI Zero Product Property 1
A-REI, A-APR Solving a Simple Cubic Equation
Quadratic Sequence 1
Quadratic Sequence 2

A1.7.3 Solve quadratic equations by completing the square

Understand and be able to use the method of completing the square to solve quadratic equations (A-REI.A.1, A-REI.B.4).

This section continues to develop the method of completing the square with a carefully sequenced set of activities. Initially students look at equations in which an expression of the form $x^2 + 2ax$ is clearly visible; then they consider equations in general form. By the end of the section students arrive at completing the square as a general method for solving quadratic equations, both executing the procedure and understanding how it works.

Tasks

Quadratic Sequence 3

A-REI Visualizing Completing the Square

A1.7.4 Put quadratic functions in vertex form

Put quadratic functions in vertex form (A-SSE.B.3b★, F-IF.C.8a★).

In this section, students adapt the method of completing the square from solving quadratic equations to putting into vertex form the expressions that define quadratic functions. This connects with the work in the Quadratic Functions unit Section 7, where students saw the geometric meaning of vertex form but did not necessarily learn how to put a quadratic function in that form.

Tasks

F-BF Identifying Quadratic Functions (Standard Form)

A-SSE Rewriting a Quadratic Expression

A1.7.5 Deriving the quadratic formula

Use completing the square to derive the quadratic formula (A-SSE.B.3b★, A-REI.B.4a).

Now that students have practice with completing the square, they have all of the tools they need to look at solving quadratic equations more abstractly and derive the quadratic formula. For some students, the algebraic manipulation in this section will seem overwhelming, but for others this section can bring home the connectedness between all they have learned about solving quadratic equations thus far. The main purpose of this section is to derive and build a conceptual understanding of the quadratic formula before using it to solve equations in the next section.

Tasks

A-REI, A-SSE, F-BF Building a General Quadratic Function

A1.7.6 Practice with all methods, picking the best one to use, and drawing connections between them

• Construct and solve quadratic equations by the most strategic method in various contexts (A-CED.A.1⋆, A-REI.B.4b).
• Express a quadratic function in the appropriate form for a given purpose (F-IF.C.8a⋆).

Now that students have a full toolbox for solving quadratic equations and expressing quadratic functions in different forms, their task becomes selecting which one to use in varying situations. In this section students see a variety of mathematical and real world problems, with the most strategic solution method varying from one problem to the next.

Tasks

A-REI Braking Distance
F-IF, A-REI Springboard Dive
F-IF Throwing Baseballs
A-REI Two Squares are Equal

A1.7.7 Connect algebra to geometry

• Solve problems using systems of a linear and a quadratic equation in two variables (A-REI.C.7).
• Derive equation of a parabola given the focus and a directrix parallel to one of the axes (G-GPE.A.2).

In this section students connect equations in two variables to the geometry of the curves they define. They study systems of linear and quadratic equations. In Grade 8 they used similar triangles to explain why a line has constant slope, and thus derived the equation $y = mx + b$ for a non-vertical line. In this unit they extend this work to parabolas, explaining why the geometric definition of a parabola leads to a quadratic equation in two variables.

Tasks

A-REI A Linear and Quadratic System
G-GPE Defining Parabolas Geometrically

A1.7.8 Culminating activity

Apply the tools learned in this unit to a real-world situation.

In this section students have opportunities to strengthen and deepen their understanding by applying their new learning to a novel context.

A1.7.9 Summative assessment

Assess students' ability to

• reason with algebraic properties to solve quadratic equations (A-REI.A.1);
• select the best method to solve a quadratic equation (factor, complete the square, quadratic formula) (A-REI.B.4);
• relate solutions of quadratic equations to graphs and interpret solutions in terms of a context (A-SSE.B.3a★, F-IF.C.8a★).