APPENDIX
Modal analysis background
The dynamics of the mechanical systems that we are modeling are ruled by the following differential equation (written here in a linearized version):

\[ K \mathbf{u} + D \dot{\mathbf{u}} + M \ddot{\mathbf{u}} = \mathbf{f} \quad (1) \]

Where \( K, D \) and \( M \) are the stiffness, damping and inertia matrices of the system. The vectors \( \mathbf{u} \) and \( \mathbf{f} \) represent the displacement field and sum of external forces applied on the system. This coupled system of ordinary differential equations has no general analytical solution. The main purpose of modal analysis is to diagonalize equation (1) to get a set of isolated differential equations. Rayleigh damping: \( D = \alpha K + \beta M \) is assumed for modal analysis. One has to solve for eigenvectors \( W_i \) and eigenvalues \( \lambda_i \) of the generalized eigenproblem \( Kx + \lambda Mx = 0 \). The matrix \( W \) that embeds all the eigenvectors diagonalizes both \( K \) and \( M \):

\[ W^T K W = \Lambda \quad W^T M W = ID \quad (2) \]

where \( \Lambda = \text{diag}(\lambda_1, \ldots, \lambda_n) \) gathers all eigenvalues in one vector. Equation (1) can now be transformed into:

\[ \Lambda(z + \alpha \dot{z}) + (\beta \dot{z} + \ddot{z}) = \mathbf{g} \quad (3) \]

where \( z = W^{-1} \mathbf{u} \) is the vector of modal coordinate and \( \mathbf{g} = W^T \mathbf{f} \) is the force vector transformed in the modal basis. Equation (3) represents a set of independent equations (one for each row \( i \)) for which we have an analytical solution of the form:

\[ z_i(t) = c_{1i} e^{i \omega_i t} + c_{2i} e^{-i \omega_i t} \quad (4) \]

where \( c_{1i} \) and \( c_{2i} \) are complex constants, and the \( \omega_i \) are the angular frequencies of mode \( i \) \( (t \) is a scalar representing time). The response of a force \( g \) (expressed in modal coordinates) applied on the system for a duration \( \Delta t \), is modeled through the constants \( c_{1i} \) and \( c_{2i} \) of each mode \( i \) with:

\[ c_{1i} = \frac{2 \Delta t g_i}{\omega_i^+ - \omega_i^-} \quad c_{2i} = \frac{2 \Delta t g_i}{\omega_i^- - \omega_i^+} \quad (5) \]

Selection of the modes
Selecting a subset of the modes of deformation leads to great computational savings, while at the same time to only a small loss of accuracy. We choose to select the lower frequency modes, because they are the most "easily" excited (they have lower \( \lambda_i \) and generate large displacements). Moreover, higher frequency modes are damped faster. Our criterion consists in taking the mode \( l \) with the smallest frequency \( \lambda_l \) (\( \lambda \) representing the ratio between the stiffness of the mode and its inertia), and to keep only modes \( i \) that have: \( \lambda_i < n. \lambda_l \). We measured that if \( n = 1000 \) or more, the outcomes of our fracture simulation are the same. In practice, choosing \( n = 1000 \) leads to approximately retain the 100 first modes of deformation.