

Arithmetic Series

Date: Notes

An arithmetic series is the SUM of the terms of an arithmetic sequence.

ie. an arithmetic sequence is 2, 5, 8, 11...

ie. an arithmetic series is 2 + 5 + 8 + 11

For these series, the sum of the first four terms is:

$$\begin{aligned} S_4 &= 2 + 5 + 8 + 11 \\ &= 26 \end{aligned}$$

* Formula 1: *

$$S_n = \frac{n}{2} (a + t_n) \quad \text{MEMORIZE !!}$$

Where S_n is the sum of n terms, a is the first term, n is the number of terms, t_n is the final term.

Example 1:

Find the sum of the series $(-4) + (-7) + (-10) \dots + (-118)$ which has 39 terms. Find S_{39} .

$$\begin{aligned} a &= -4 & S_n &= \frac{n}{2} (a + t_n) \\ n &= 39 & S_{39} &= \frac{39}{2} (-4 + (-118)) \\ t_n &= -118 & S_{39} &= \frac{39}{2} (-122) \\ & & S_{39} &= \boxed{-2379} \end{aligned}$$

Example 2:

Find the sum of the first eight terms of the arithmetic series given the first term is -3 and the $t_8 = 39$. Find S_8 .

$$\begin{aligned} a &= -3 & S_8 &= \frac{8}{2} (-3 + 39) \\ n &= 8 & S_8 &= 4 (36) \\ t_n &= 39 & S_8 &= \boxed{144} \end{aligned}$$

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* Formula 2: *

We can substitute t_n in formula 1 with the arithmetic sequence formula $t_n = a + (n-1)d$ to give:

$$S_n = \frac{n}{2} (a + t_n)$$

$$S_n = \frac{n}{2} [(a + (a + (n-1)d))] \rightarrow \text{simplify to...}$$

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

MEMORIZE!!

* MOST IMPORTANT *

Example 3:

For the series $7 + 13 + 19 + 25 \dots$ determine the sum of the first 100 terms. Find S_{100} .

$$a = 7$$

$$d = 6$$

$$n = 100$$

$$S_{100} = \frac{100}{2} [2(7) + (100-1)6]$$

$$= 50 (14 + 594)$$

$$= 50 (608)$$

$$= \boxed{30400}$$

Example 4:

Find the sum of $5 + 8 + 11 + \dots + 107$

① First, find the # terms in series.

$$t_n = a + (n-1)d$$

$$107 = 5 + (n-1)3$$

$$107 = 5 + (3n-3)$$

$$107 = 3n + 2$$

$$\frac{105}{3} = \frac{3n}{3}$$

$$\boxed{35 = n}$$

② Use formula for sum of series. $n=35$

$$S_{35} = \frac{35}{2} [2(5) + (35-1)3]$$

$$= 17.5 (10 + 102)$$

$$= \boxed{1960}$$

Example 5:

Ryan has a new job that pays \$24 000 the first year. He will receive an increase of \$800 at the end of each year for four years.

a) What will Ryan's income be the fifth year? Find t_5 !

$$a = \$24000$$

$$d = 800$$

$$n = 5$$

$$t_n = a + (n-1)d$$

$$t_5 = 24000 + 4(800)$$

$$t_5 = \boxed{\$27200}$$

\therefore his income will be \$27 200 in the fifth year.

b) What will his total income be for this first five years? Find S_5

$$S_5 = \frac{5}{2} [2(24000) + 4(800)]$$

$$= \frac{5}{2} (48000 + 3200)$$

$$= \frac{5}{2} (51200)$$

$$S_5 = \boxed{\$128000}$$

\therefore his total income will be \$128 000 for his first 5 years.