UNIT #7: Sequences and Series

Arithmetic Sequences

Learning Goal:

I will learn how to determine the formula for the general term of an arithmetic sequence [i.e. $t_n = a + (n-1)d$]

+3 +3 +3 +3

5, 8, 11, 14, 17, ...

Sequence A

Lesson: Arithmetic Sequences

Sequences

A number sequence is a set of numbers, whose terms are often related by a pattern.

The first term is called t_1, the second t_2, and the n^{th} term t_n.

The symbol t_1 represents the value of the term, the subscript represents the place of the term in the sequence. In this case the subscript 1 indicates that t_1 is the first term in the sequence.

Example 1:
Given the formula for the nth term, write the first four terms of the following sequence:

$t_n = 2n + 1$

$t_1 = 2(1) + 1$

$t_1 = 3$

$t_2 = 2(2) + 1$

$t_2 = 5$

$t_3 = 2(3) + 1$

$t_3 = 7$

$t_4 = 2(4) + 1$

$t_4 = 9$

Some sequences lead to a general formula to determine all terms.

Example 2:
Find the formula for the nth term of the sequence

2, 5, 10, 17, 26
t_1 t_2 t_3 t_4 t_5

$t_n = (n)^2 + 1$ Eg. $t_2 = (2)^2 + 1 = 5$

There are 2 types of sequences:
1. Arithmetic sequences
2. Geometric sequences

Arithmetic Sequences

An arithmetic sequence is a *linear* function. The first term is denoted by an "**a**" and each term after the first is found by adding a constant called the common difference denoted by "**d**".

+3 +3 +3

Eg. For the sequence: 1, 4, 7, 10... **a = 1, d = 3**

The general arithmetic sequence is

a, a+d, a+2d, a+3d. . .**a+(n-1)d**

a -> first term t_1
d -> difference between consecutive terms
nth term -> $t_n = a+(n-1)d$

Finding the formula of the nth term:

Find the formula of the nth term for the sequence 5, 8, 11, 14. . .

+3 +3 +3

a

a = 5
d = 3
$t_n = a + (n-1)d$

$t_n = 5 + (n-1)3$

$t_n = 5 + 3n - 3$

$\boxed{t_n = 3n + 2}$

Test

$t_1 = 3(1) + 2$

$t_1 = 5$ ✓

Finding the number of terms:

How many terms are in the sequence 3, 7, 11, 15. . .83?

a

+4 +4

a = 3

d = 4

t_n = 83 (last term)

$t_n = a + (n-1)d$

$83 = 3 + (n-1)4$

$83 = 3 + 4n - 4$

$\frac{84}{4} = \frac{4n}{4}$

$\boxed{21 = n}$

∴ there are 21 terms in sequence.

Finding t_n given two terms:

In an arithmetic sequence, $t_7 = 9$, $t_{12} = 29$. Find **a** and **d** and write the formula for the nth term.

$t_n = a + (n-1)d$

1st eq: $t_7 = 9$ (n, t_n)

$9 = a + (7-1)d$

$9 = a + 7d - d$

$9 = a + 6d$

2nd eq: $t_{12} = 29$ (n, t_n)

$29 = a + (12-1)d$

$29 = a + 11d$

Find d: Elimination

① $9 = a + 6d$

② $29 = a + 11d$

$-20 = -5d$

$\frac{-20}{-5} = \frac{-5d}{-5}$

$\boxed{4 = d}$

Find a: Sub d=4 into ①

$9 = a + 6(4)$

$9 = a + 24$

$\boxed{-15 = a}$

$t_n = -15 + (n-1)4$ (a, d)

$t_n = -15 + 4n - 4$

$\boxed{t_n = 4n - 19}$ formula

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Success Criteria:

To be successful, I must be able to...

- list the first terms of a sequence when given the formula
- find the indicate terms when given the formula
- find the formula for the nth term $t_n = a+(n-1)d$
- find the number of terms in an arithmetic sequence
- write the formula when given two terms

Practice Work

p. 433 #3, 4 e.o.
p. 442 #6 a - e
#7 a - e, 8