Trig Identities.

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(1) $\tan^2\theta + 1 = \sec^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} + 1 = \frac{1}{\cos^2\theta}$

$\frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

Pythag. $\frac{(1-\cos^2\theta)+\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

$\frac{1}{\cos^2\theta} = \frac{1}{\cos^2\theta}$

(2) $\csc\theta = \sec\theta\cot\theta$

$\frac{1}{\sin\theta} = \frac{1}{\cos\theta}\left(\frac{1}{\tan\theta}\right)$

$\frac{1}{\sin\theta} = \frac{1}{\cos\theta}\left(\frac{\cos\theta}{\sin\theta}\right)$

$\frac{1}{\sin\theta} = \frac{1}{\sin\theta}$.

(3) $\cos\theta = \sin\theta\cot\theta$

$\cos\theta = \sin\theta\left(\frac{\cos\theta}{\sin\theta}\right)$

$\cos\theta = \cos\theta$

(4) $1 + \csc A = \csc A\,(1+\sin A)$

$1 + \frac{1}{\sin A} = \frac{1}{\sin A}(1+\sin A)$

$\frac{\sin A + 1}{\sin A} = \frac{1+\sin A}{\sin A}$

$\frac{1+\sin A}{\sin A} = \frac{1+\sin A}{\sin A}$.

(5) $\cot B \sin B \sec B = 1$

$\left(\frac{\cos B}{\sin B}\right)\sin B\left(\frac{1}{\cos B}\right) = 1$

$\frac{(\cos B)(\sin B)}{(\sin B)(\cos B)} = 1$

$1 = 1$

(6) $\cos C(\sec C - 1) = 1 - \cos C$

$\cos C\left(\frac{1}{\cos C} - 1\right) = 1 - \cos C$

$\frac{\cos C}{\cos C} - \cos C = 1 - \cos C$

$1 - \cos C = 1 - \cos C$.

(7) $1 - \sin^2\theta = \sin\theta\cos\theta\cot\theta$

$\cos^2\theta = \sin\theta\cos\theta\left(\frac{\cos\theta}{\sin\theta}\right)$

$\cos^2\theta = \cos^2\theta$.

(8) $\csc^2\theta = \cot^2\theta + 1$

$\frac{1}{\sin^2\theta} = \frac{\cos^2\theta}{\sin^2\theta} + 1$

$\frac{1}{\sin^2\theta} = \frac{\cos^2\theta + \sin^2\theta}{\sin^2\theta}$

$\frac{1}{\sin^2\theta} = \frac{1}{\sin^2\theta}$.

(9) $\csc^2\theta \cos^2\theta = \csc^2\theta - 1$

$\frac{1}{\sin^2\theta}\cos^2\theta = \frac{1}{\sin^2\theta} - 1$

Puth. $\frac{1}{\sin^2\theta}(1-\sin^2\theta) = \frac{1}{\sin^2\theta} - 1$

Lumman Den. $\frac{1-\sin^2\theta}{\sin^2\theta} = \frac{1-\sin^2\theta}{\sin^2\theta}$

(10) $\tan^2\theta - \sin^2\theta = \sin^2\theta\tan^2\theta$

$\frac{\sin^2\theta}{\cos^2\theta} - \sin^2\theta = \sin^2\theta\frac{\sin^2\theta}{\cos^2\theta}$

Factor ↖ $\frac{\sin^2\theta - \sin^2\theta(\cos^2\theta)}{\cos^2\theta}$

$\frac{\sin^2\theta(1-\cos^2\theta)}{\cos^2\theta} = \frac{\sin^2\theta\sin^2\theta}{\cos^2\theta}$

$\frac{\sin^2\theta\sin^2\theta}{\cos^2\theta} = \frac{\sin^2\theta\sin^2\theta}{\cos^2\theta}$

$\frac{\sin^4\theta}{\cos^2\theta} = \frac{\sin^4\theta}{\cos^2\theta}$

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(1) $\sec x(1+\cos x) = 1+\sec x$

$\frac{1}{\cos x}(1+\cos x) = 1+\frac{1}{\cos x}$

$\frac{1+\cos x}{\cos x} = \frac{\cos x+1}{\cos x}$

(2) $\sin x(1+\csc x) = \sin x + 1$

$\sin x\left(1+\frac{1}{\sin x}\right) = \sin x + 1$

$\sin x + \frac{\sin x}{\sin x} = \sin x + 1$

$\sin x + 1 = \sin x + 1$

(3) $\tan x(1+\cot x) = 1+\tan x$

$\frac{\sin x}{\cos x}\left(1+\frac{\cos x}{\sin x}\right) = 1+\frac{\sin x}{\cos x}$

$\frac{\sin x}{\cos x} + \frac{\sin^2\cos x}{\sin x\cos x} = 1+\frac{\sin x}{\cos x}$

$\frac{\sin x}{\cos x} + 1 = 1+\frac{\sin x}{\cos x}$

(4) $\cos x(\sec x+1) = \cos x + 1$

$\cos x\left(\frac{1}{\cos x}+1\right) = \cos x + 1$

$\frac{\cos x}{\cos x} + \cos x = \cos x + 1$

$1 + \cos x = \cos x + 1$

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(5) $\csc x(\sin x - 1) = 1 - \csc x$

$\frac{1}{\sin x}(\sin x - 1) = 1 - \frac{1}{\sin x}$

$\frac{\sin x}{\sin x} - \frac{1}{\sin x} = 1 - \frac{1}{\sin x}$

$1 - \frac{1}{\sin x} = 1 - \frac{1}{\sin x}$

(6) $\cot x(1 - \tan x) = \cot x - 1$

$\left(\frac{\cos x}{\sin x}\right)\left(1 - \frac{\sin x}{\cos x}\right) = \frac{\cos x}{\sin x} - 1$

$\frac{\cos x}{\sin x} - \frac{\sin x \cos x}{\sin x \cos x} = \frac{\cos x}{\sin x} - 1$

$\frac{\cos x}{\sin x} - 1 = \frac{\cos x}{\sin x} - 1$

(7) $\sin x \tan^2 x \cot^3 x = \cos x$

$\sin x \left(\frac{\sin^2 x}{\cos^2 x}\right)\left(\frac{\cos^3 x}{\sin^3 x}\right) = \cos x$

Cancel: $\frac{\sin^2 x \cos x}{\sin^2 x} = \cos x$

$\cos x = \cos x$

FOIL.

(8) $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$

$\cos^2 x - 2\cos x \sin x + \sin^2 x = 1 - 2\sin x \cos x$

Pythag. $\cos^2 x + \sin^2 x - 2\cos x \sin x = 1 - 2\sin x \cos x$

$1 - 2\cos x \sin x = 1 - 2\sin x \cos x$

(9) $(\sin x + \cos x)(\sin x - \cos x) = 1 - 2\cos^2 x$

$\sin^2 x - \sin x \cos x + \cos x \sin x - \cos^2 x = 1 - 2\cos^2 x$

$\sin^2 x - \cos^2 x = 1 - 2\cos^2 x$

Pythag. $\sin^2 x - (1 - \sin^2 x) = 1 - 2(1 - \sin^2 x)$

$\sin^2 x - 1 + \sin^2 x = 1 - 2 + 2\sin^2 x$

$2\sin^2 x - 1 = 2\sin^2 x - 1$

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(10) $\dfrac{\tan^2 x}{1+\tan^2 x} = \sin^2 x$

$$\frac{\frac{\sin^2 x}{\cos^2 x}}{1+\frac{\sin^2 x}{\cos^2 x}} = \sin^2 x$$

$$\frac{\frac{\sin^2 x}{\cos^2 x}}{\frac{\cos^2 x+\sin^2 x}{\cos^2 x}} = \sin^2 x$$

$$\frac{\sin^2 x}{\cancel{\cos^2 x}} \times \frac{\cancel{\cos^2 x}}{\cos^2 x+\sin^2 x} = \sin^2 x$$

$$\frac{\sin^2 x}{\cos^2 x+\sin^2 x} = \sin^2 x$$

Pyth. → $\cos^2 x+\sin^2 x$

$$\frac{\sin^2 x}{1} = \sin^2 x$$

(11) $\dfrac{1+\cos x}{1-\cos x} = 1 + \dfrac{2\cos x(1+\cos x)}{\sin^2 x}$

$$\frac{1+\cos x}{1-\cos x} = \frac{\sin^2 x + 2\cos x(1+\cos x)}{\sin^2 x}$$

$$\frac{1+\cos x}{1-\cos x} = \frac{(1-\cos^2 x)+2\cos x+2\cos^2 x}{1-\cos^2 x}$$

$$\frac{1+\cos x}{1-\cos x} = \frac{1+\cos^2 x+2\cos x}{1-\cos^2 x}$$

$$\left(\frac{1+\cos x}{1+\cos x}\right)\frac{1+\cos x}{1-\cos x} = \frac{1+2\cos x+\cos^2 x}{1-\cos^2 x}$$

$$\frac{1+2\cos x+\cos^2 x}{(1-\cos^2 x)}$$

(12) $\dfrac{1+\sin^2 x \sec^2 x}{1+\cos^2 x \csc^2 x} = \sin^2 x \sec^2 x$

$\dfrac{1+\sin^2 x\left(\frac{1}{\cos^2 x}\right)}{1+\cos^2 x \frac{1}{\sin^2 x}} = \sin^2 x\left(\frac{1}{\cos^2 x}\right)$

$\dfrac{1+\frac{\sin^2 x}{\cos^2 x}}{1+\frac{\cos^2 x}{\sin^2 x}} = \dfrac{\sin^2 x}{\cos^2 x}$ ✓

$\dfrac{\cos^2 x+\sin^2 x}{\cos^2 x} \div \dfrac{\sin^2 x+\cos^2 x}{\sin^2 x}$

$\dfrac{\cos^2 x+\sin^2 x}{\cos^2 x} \times \dfrac{\sin^2 x}{\sin^2 x+\cos^2 x}$

$\dfrac{1}{\cos^2 x} \times \dfrac{\sin^2 x}{1}$

$\dfrac{\sin^2 x}{\cos^2 x}$ ✓

(13) $2+\dfrac{\sin^4 x+\cos^4 x}{\sin^2 x \cos^2 x} = \sec^2 x \csc^2 x$

$= \dfrac{1}{\cos^2 x \sin^2 x}$ ✓

perfect squares

$\dfrac{2\sin^2 x\cos^2 x+\sin^4 x+\cos^4 x}{\sin^2 x \cos^2 x}$

Pythag $\dfrac{(\sin^2 x+\cos^2 x)(\sin^2 x+\cos^2 x)}{\sin^2 x \cos^2 x}$

$\dfrac{(1)(1)}{\sin^2 x \cos^2 x}$

$\dfrac{1}{\sin^2 x \cos^2 x}$ ✓