MCR 3U1 **Trigonometric Identities**

Pythagorean Identity:

$\sin^2\theta + \cos^2\theta = 1$

$\sin^2\theta = 1 - \cos^2\theta$

$\cos^2\theta = 1 - \sin^2\theta$

Quotient Identity:

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

Reciprocal Identities:

$\csc\theta = \dfrac{1}{\sin\theta}$

$\sec\theta = \dfrac{1}{\cos\theta}$

$\cot\theta = \dfrac{1}{\tan\theta}$

Prove each trigonometric identity.

1. $\tan^2\theta + 1 = \sec^2\theta$
2. $\csc\theta = \sec\theta \cot\theta$
3. $\cos\theta = \sin\theta \cot\theta$
4. $1 + \csc A = \csc A(1 + \sin A)$
5. $\cot B \sin B \sec B = 1$
6. $\cos C(\sec C - 1) = 1 - \cos C$
7. $1 - \sin^2\theta = \sin\theta \cos\theta \cot\theta$
8. $\csc^2\theta = \cot^2\theta + 1$
9. $\csc^2\theta \cos^2\theta = \csc^2\theta - 1$
10. $\tan^2\theta - \sin^2\theta = \sin^2\theta \tan^2\theta$

MCR3U Trigonometric Identities Worksheet

Using the following basic identities:

$$\frac{\sin x}{\cos x} = \tan x$$

$$\sin^2 x + \cos^2 x = 1$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\cos^2 x = 1 - \sin^2 x$$

$$\csc x = \frac{1}{\sin x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$$

Prove:

1. $\sec x(1 + \cos x) = 1 + \sec x$

2. $\sin x(1 + \csc x) = \sin x + 1$

3. $\tan x(1 + \cot x) = 1 + \tan x$

4. $\cos x(\sec x + 1) = \cos x + 1$

5. $\csc x(\sin x - 1) = 1 - \csc x$

6. $\cot x(1 - \tan x) = \cot x - 1$

7. $\sin x \tan^2 x \cot^3 x = \cos x$

8. $(\cos x - \sin x)^2 = 1 - 2\sin x \cos x$

9. $(\sin x + \cos x)(\sin x - \cos x) = 1 - 2\cos^2 x$

10. $\dfrac{\tan^2 x}{1 + \tan^2 x} = \sin^2 x$

11. $\dfrac{1 + \cos x}{1 - \cos x} = 1 + \dfrac{2\cos x(1 + \cos x)}{\sin^2 x}$

12. $\dfrac{1 + \sin^2 x \sec^2 x}{1 + \cos^2 x \csc^2 x} = \sin^2 x \sec^2 x$

13. $2 + \dfrac{\sin^4 x + \cos^4 x}{\sin^2 x \cos^2 x} = \sec^2 x \csc^2 x$