$y = a\sin k(x-c)+d$

amp. | horiz. stretch/comp | phase shift | vertical shift

Notes

MCR3U1 Trigonometric Functions – Applications

Sine – Cosine functions are used to model physical situations that fluctuate and display cyclical patterns. Examples: Tides, electricity, sound waves, ferris wheel , temperature, blood pressure. The graphs are a function of time (rather than angle). The horizontal axis displays time. Degrees are found in the equations as the k value.

Example 1 The average monthly temperature in a region in Australia is modelled by the function $T(t) = 23\cos 30°t + 9$ where T is temperature in °C, t is time (month of the year), t = 0 is January.

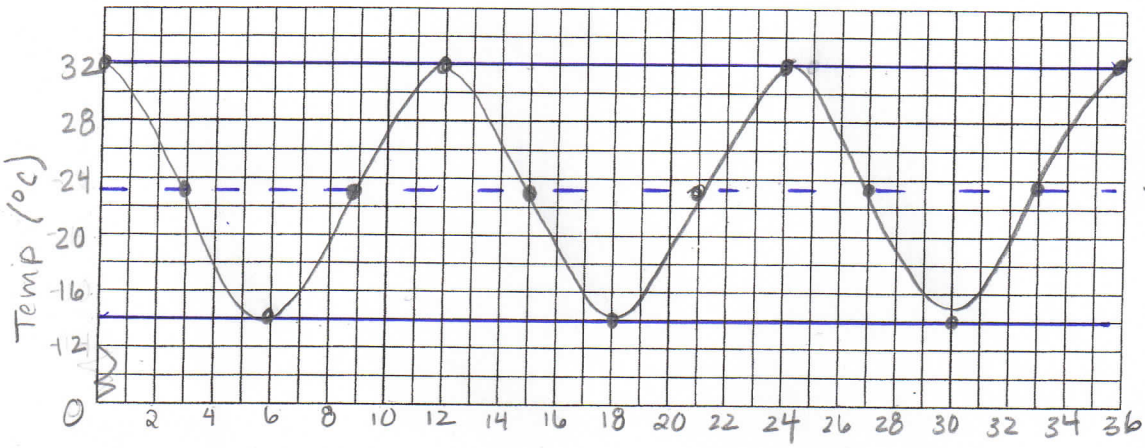
a) What is the period? Period $= \frac{360°}{k} = \frac{360°}{30°} = 12$ months

b) What does the period mean? 12 month cycle, 1 per year

c) What is the amplitude? $a = 23$

d) What is the maximum temperature? max = $23 + 9 = 32°C$ → amplitude + vertical shift

e) What is the minimum temperature? min = $23 - 9 = 14°C$

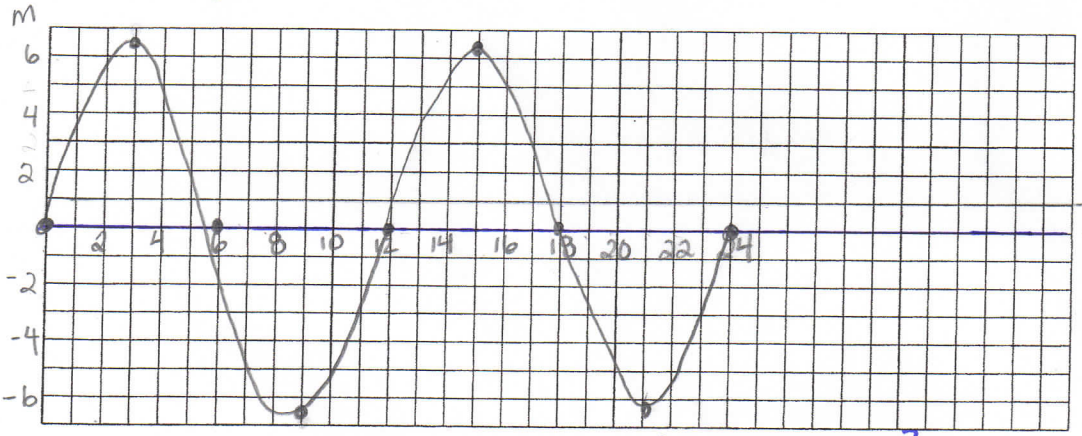


midline $= \frac{14+32}{2} = 23$, $y = 23$

2. Tides: In the Bay of Fundy, tides run 6.5 m above sea level and drop 6.5 m below sea level. The tide completes one cycle every 12 h. Assume the height (h) with respect to mean sea level can be modelled by a sine function.

a) Draw a graph for a 24 h period.

b) Find the equation.



max = 6.5

min = -6.5

$a = \frac{6.5-(-6.5)}{2} = 6.5$

period $= \frac{360}{k}$

$12 = \frac{360}{k}$

$k = \frac{360}{12} = 30°$

How do you know sine or cosine?

Test $t = 0$

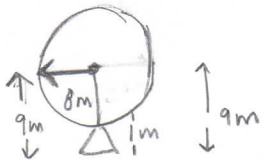
$y = 6.5\sin 30(0) = 0$ ✓

$y = 6.5\cos 30(0) = 6.5$ ✗

$y = 6.5\sin 30t$

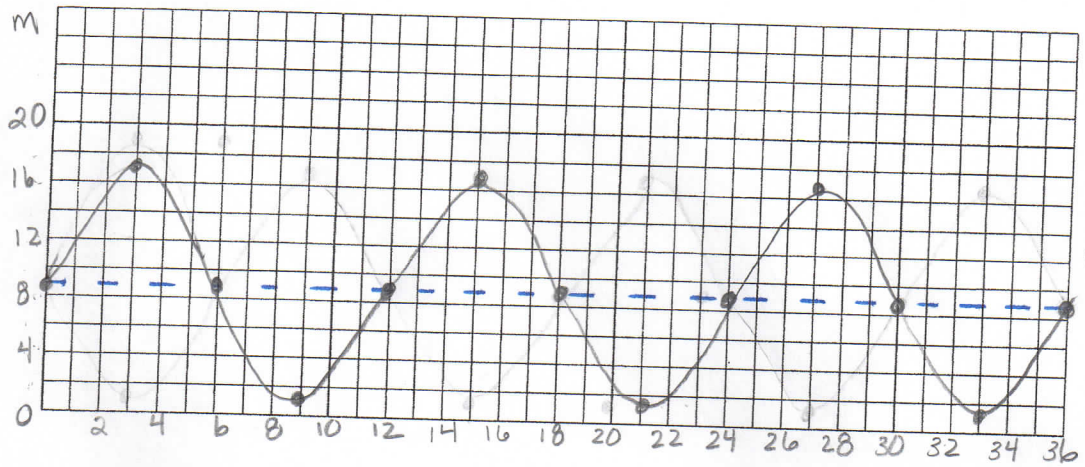
$y = a \sin k(x-c) + d$

Notes



3. A ferris wheel has a radius of 8 m and rotates once every 12 s. The bottom of the ferris wheel is 1 m above the ground.

a) Draw the graph starting at the position level to the centre.
b) Find the equation.



max = 1+8+8 = 17

min = 1

$a = \frac{17-1}{2} = 8$

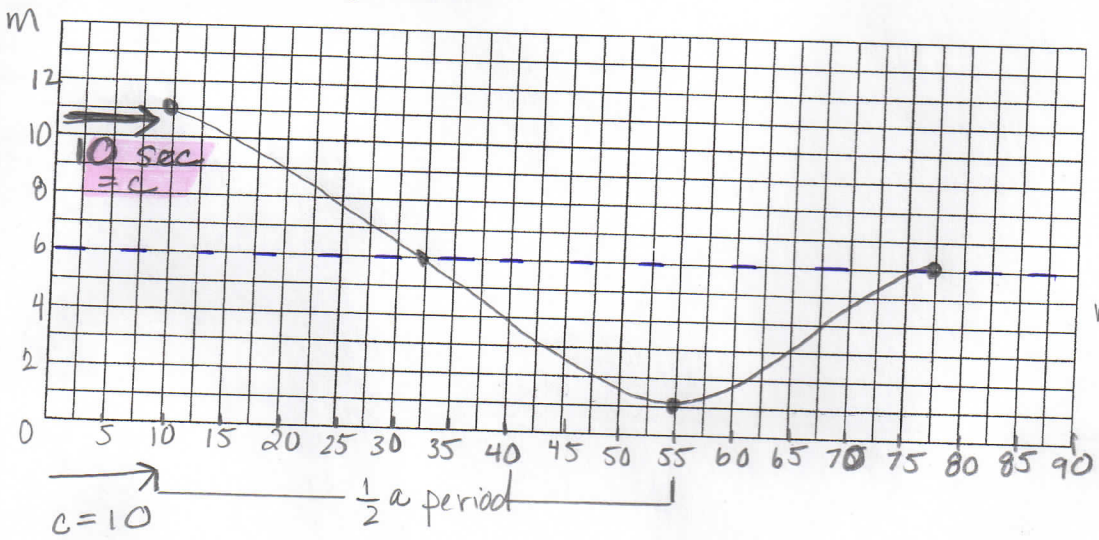
$d = \text{midline} = \frac{17+1}{2} = 9$ ($d = \frac{\text{max}+\text{min}}{2}$)

$y = 8 \sin 30t + 9$

$\text{Period} = \frac{360}{K}$

$12 = \frac{360}{K}$ $\quad K = 30$

4. John is riding a ferris wheel. He reaches the maximum height of 11 m at 10 s and the minimum height of 1 m at 55 s.
a) Write an equation.
b) Sketch the graph.
c) What will be his height after 78 s.



Max = 11m

Min = 1 m

$a = \frac{11-1}{2} = 5$

$d = \text{midline} = \frac{11+1}{2} = 6$ ($d = \frac{\text{max}+\text{min}}{2}$)

$\text{Period} = \frac{360}{K}$

$\frac{1}{2}\text{Period} = 45 \text{ sec}$

$\text{Period} = 90 \text{ sec}$

$90 = \frac{360}{K}$

$K = 4$

$y = 5 \cos 4(x-10) + 6$

c) Let $x = 78$.

$y = 5 \cos 4(78-10) + 6$

$y = 5 \cos 272 + 6$

$y = 6.17 \text{ m}$