WARM UP

Find all possible angles between -360 and 360 degrees.
$\sec \theta = \frac{-\sqrt{5}}{2}$

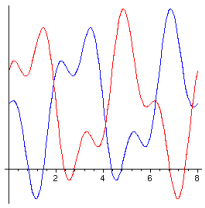
Find the exact value of $\sin 150^{\circ}$ without a calculator.

UNIT #6: Trigonometric Transformations

Translations of Periodic Functions

Learning Goal:

I will learn how to graph the translations a sine and cosine function.



Lesson: Translations of Periodic Functions

General form of a Trigonometric Function:

$y = a \sin k(x - c) + d$

a: amplitude
k: horizontal stretch or compression
c: phase shift
d: vertical shift

Vertical Translations

To sketch $y = a \sin x + c$, shift the graph up c units if c is positive and down c units if c is negative.

Example 1: $y = 2\sin x - 1$.

Transformations:
- vertical stretch BAFO 2
- vertical shift down 1 unit

Amplitude - 2

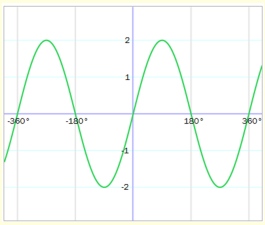
Period - 360°

Example 1 Continued: y = 2sin x - 1

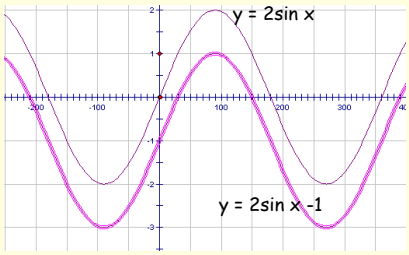
Method - First sketch the graph without the vertical shift by using the 5 - point method, then apply the shift.

Since the period is 360°, the five points occur at:
x = 0°, 360°, 180°, 90°, 270°.

Max of 2 at **x = 90°**
Min of -2 at **x = 270°**
0 at **x = 0°, 180°, 360°**



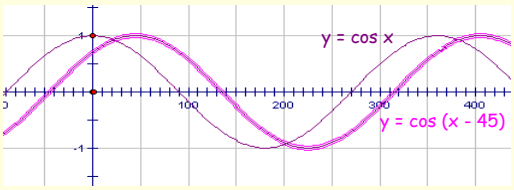
Now shift the graph of y = 2sin x down 1 unit.



Horizontal Translations

To sketch **y = a cos(x-d)** the graph is shifted right **d** units if **d** is positive, and left **d** units if **d** is negative. The shift **d** is called a phase shift.

Example: y = cos (x - 45)
First sketch y = cos x, then shift 45° to the right.



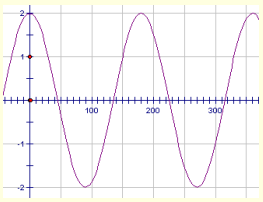
Example: y = 2cos 2 (x + 90°).

Transformations:
- vertical stretch BAFO 2
- horizontal compression BAFO 1/2
- phase shift 90° to the left

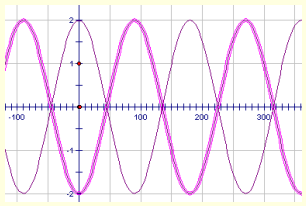
Amplitude - 2

Period - 360/2 = 180°

Without the phase shift, the 5 points occur at:
x = 0°, 180°, 90°, 45°, 135°



y = 2 cos 2(x)



y = 2 cos 2(x + 90°)

Combinations of Transformations

Transformations are applied in the following order:

1. Expansions and compressions
2. Reflections
3. Translations

Example $y = 5\cos(\frac{1}{3}x - 60) + 2$, $-180° \leq x \leq 180°$.

First factor the coefficient of x to better see the shifting:

$y = 5\cos\frac{1}{3}(x - 180) + 2$

Transformations:
- vertical stretch BAFO 5
- horizontal stretch BAFO 3
- phase shift 180° to the right
- vertical shift 2 units up

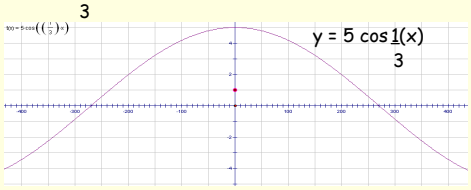
Period - 360/(1/3) = 1080°

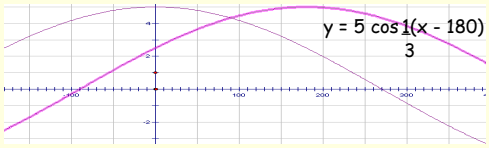
Amplitude - 5

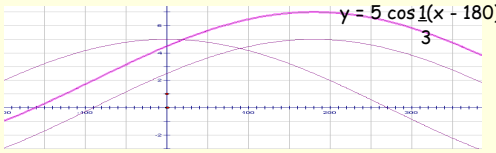
5 main points without the translations:
x = 0°, 1080°, 540°, 270°, 810°

Restrictions only allow us to plot the following main points:
0°, 270°, -270°

$y = 5\cos\frac{1}{3}(x - 180) + 2$







RECALL!

Changing Radian Measure to Degrees:

Multiply the number of radians by $(180/\pi)^\circ$

Eg. $3\pi/4$

UNIT 6: Trignometric Functions

Translations of Periodic Functions

Learning Goal:

I will learn how to graph the translations a sine and cosine function.

Success Criteria:

To be successful, I must be able to...

- graph the translations of a sine and cosine function by identifying 5 key points
- identify the translations from a sine and cosine graph and state its equation

Practice Work

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