

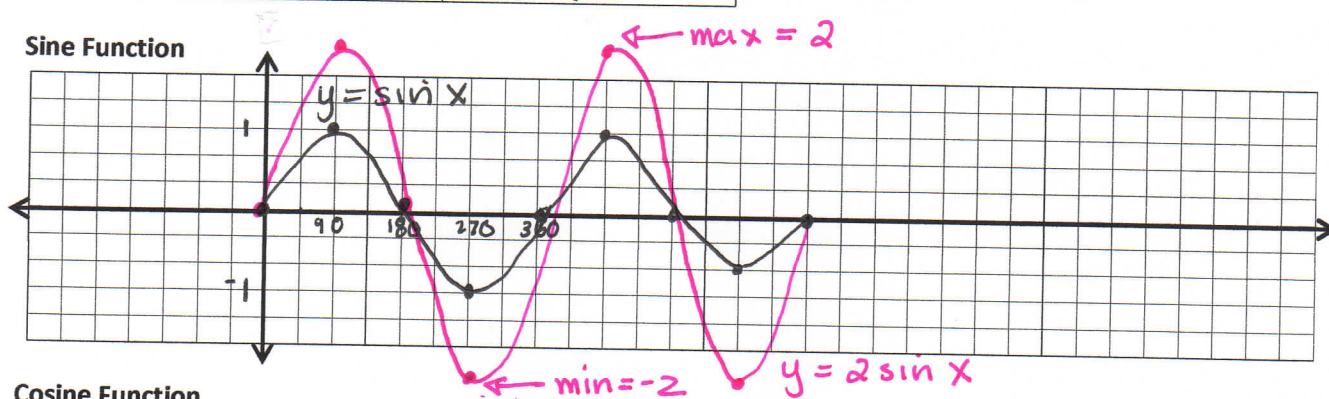
# Trig Transformations: Stretches/Compressions of Functions

## Graphing Parent Functions:

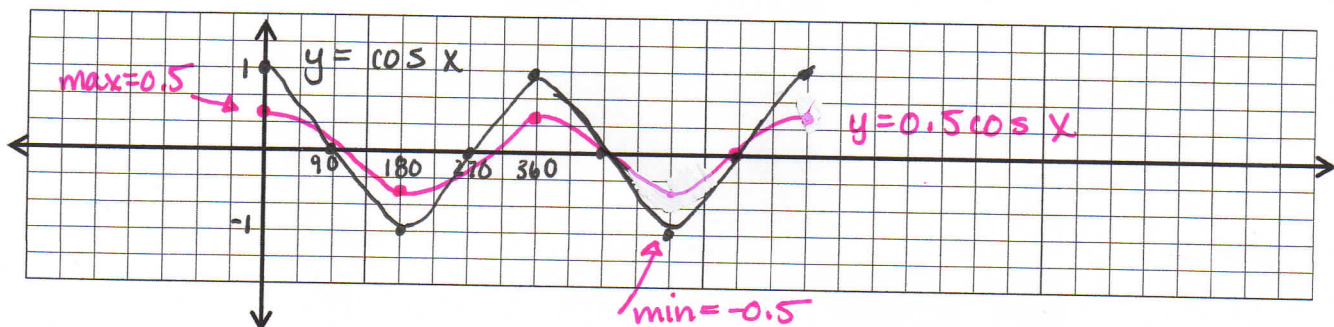
The simplest way to sketch the parent function for sine or cosine is to use 5 key points at  $90^\circ$  intervals ( $0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ$ ).

x-value	sin x	cos x
$0^\circ$	0	1
$90^\circ$	1	0
$180^\circ$	0	-1
$270^\circ$	-1	0
$360^\circ$	0	1

### Sine Function



### Cosine Function



## Vertical Stretches & Compressions:

For the functions  $f(x) = a \sin x$  and  $f(x) = a \cos x$  the graphs are stretched in the y direction if  $a > 1$  OR  $a < -1$  and compressed in the y direction if  $-1 < a < 1$ .

### Example 1: $2 \sin x$

Transformation:

vertical stretch by factor of 2

$\times 2$

x-value	y = sin x	2 sin x
$0^\circ$	0	0
$90^\circ$	1	2
$180^\circ$	0	0
$270^\circ$	-1	-2
$360^\circ$	0	0

+ Graph above \*

### Example 2: $0.5 \cos x$

Transformation:

vertical compression by factor of 0.5

$\times .5$

x-value	cos x	0.5 cos x
$0^\circ$	1	.5
$90^\circ$	0	0
$180^\circ$	-1	-.5
$270^\circ$	0	0
$360^\circ$	1	.5

# Trig Transformations: Stretches/Compressions of Functions

## Horizontal Stretches and Compressions

Functions of the form  $y = \sin Kx$

Period =  $\frac{360^\circ}{K}$  ↗  
K value

One complete cycle occurs in the period. Five key points divide the cycle into four sections.

**Example 1:**  $f(x) = \sin 3x$  ↖ K value

Transformation:  
horizontally compressed by  $\frac{1}{3}$

Period =  $\frac{360}{3} = 120^\circ$   
↑  
K value

Five key points:

Zeros: occur at  $0^\circ$  and  $120^\circ$  and half way between at  $60^\circ$ .

Maximum of 1 occurs at  $30^\circ$  ( $90^\circ \div 3$ )

Minimum of -1 occurs at  $90^\circ$  ( $270^\circ \div 3$ )

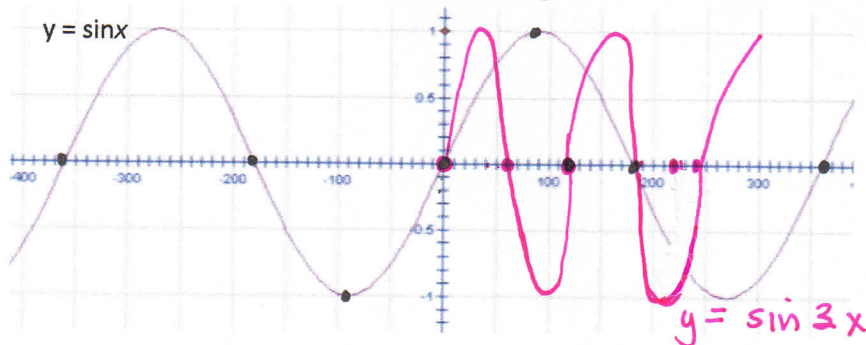
OR

### Table of Values

Keep the y-values the same from the parent  $\sin x$  graph and multiply x-values by  $\frac{1}{3}$ .

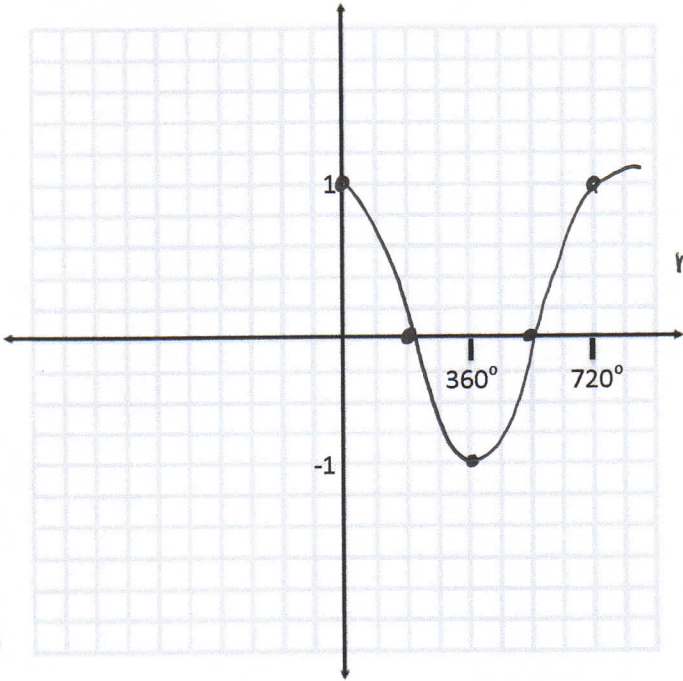
y = sin x	x-value	Multiply x by 1/3
0	$0^\circ$	$0^\circ$
1	$90^\circ$	$30^\circ$
0	$180^\circ$	$60^\circ$
-1	$270^\circ$	$90^\circ$
0	$360^\circ$	$120^\circ$

↘ divide by 3. Graph below!



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EXAMPLE 2:  $f(x) = \cos \frac{1}{2}x$  *← k value.*



Period:  $\frac{360^\circ}{\frac{1}{2}} = 360^\circ \times \frac{2}{1} = 720^\circ$

Zeros: Parent:  $90^\circ$  and  $270^\circ$   
 mult. by 2.  $\rightarrow 180^\circ$  and  $540^\circ$

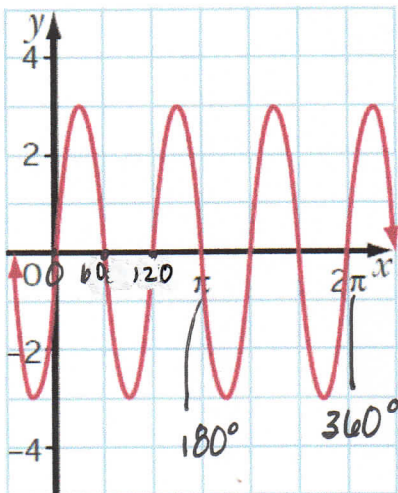
Maximum: 1 at  $0^\circ$  and  $720^\circ$   
 (beginning and end of cycle).

Minimum: -1 at  $360^\circ$   
 (halfway b/w period).

**TOV**  
 mult. x values by 2.

y	x
1	$0^\circ$
0	$180^\circ$
-1	$360^\circ$
0	$540^\circ$
1	$720^\circ$

EXAMPLE 3: Determine the equation for the sine function.



$y = a \sin kx$

$k = \frac{360}{\text{Period}}$

$k = \frac{360}{120}$

$k = 3$

Period =  $120^\circ$

max value = 3

min value = -3

$\therefore a = 3$

equation  $y = 3 \sin(3x)$