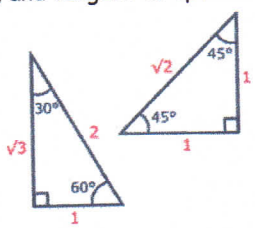
UNIT #6: Trigonometric Transformations

Special Triangles

Learning Goal:

I will learn how to find the exact values of the sine, cosine, and tangent of special triangles.



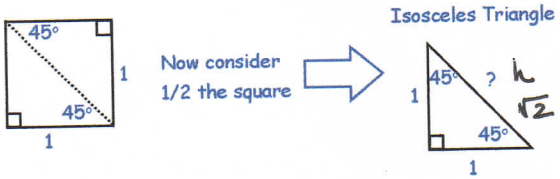
Lesson: Special Triangles

Some angles occur so frequently that their trig ratios should be known exactly (i.e., no decimals, no rounding). The angles are 30°, 45°, 60°, and 90° (and their multiples).

There are **TWO** special **RIGHT TRIANGLES** used to calculate the **EXACT** trig ratios.

Special Triangle #1 45°-45°-90° (1, 1, $\sqrt{2}$)

Triangle #1 comes from a square with side lengths 1.



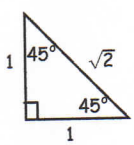
$h^2 = 1^2 + 1^2$

$h^2 = 2$

$h = \sqrt{2}$

SUMMARY:

The 45°-45°-90° (1, 1, $\sqrt{2}$) Triangle



$\sin 45° = \frac{o}{h} = \frac{1}{\sqrt{2}}$

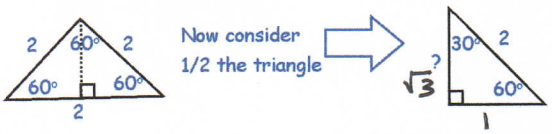
$\cos 45° = \frac{a}{h} = \frac{1}{\sqrt{2}}$

$\tan 45° = \frac{o}{a} = \frac{1}{1} = 1$

Exact values

Special Triangle #2 30°-60°-90° (1, $\sqrt{2}$, $\sqrt{3}$)

Triangle #2 comes from an equilateral triangle with side length 2.



$2^2 = 1^2 + a^2$

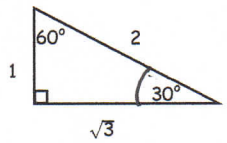
$4 = 1 + a^2$

$3 = a^2$

$\sqrt{3} = a$

SUMMARY:

The 30°-60°-90° (1, $\sqrt{2}$, $\sqrt{3}$) Triangle

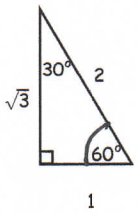


$\cos 30° = \frac{\sqrt{3}}{2}$

$\sin 30° = \frac{1}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

} exact values

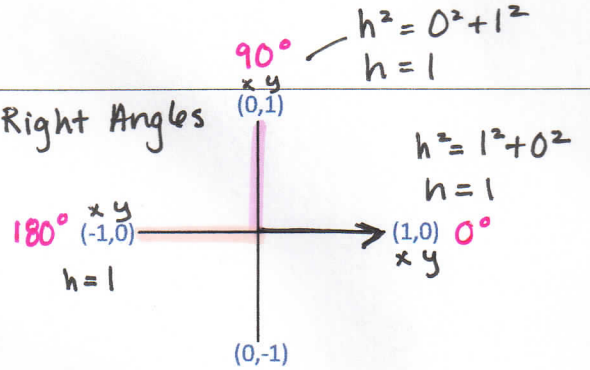


$\cos 60° = \frac{1}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\tan 60° = \frac{\sqrt{3}}{2}$

Right Angles



$h^2 = 0^2 + 1^2$

$h = 1$

$h^2 = 1^2 + 0^2$

$h = 1$

$\cos 0° = \frac{1}{1} = 1$	$\cos 90° = \frac{0}{1} = 0$	$\cos 180° = \frac{-1}{1} = -1$
$\sin 0° = \frac{0}{1} = 0$	$\sin 90° = \frac{1}{1} = 1$	$\sin 180° = \frac{0}{1} = 0$
$\tan 0° = \frac{0}{1} = 0$	$\tan 90° = \frac{1}{0} =$ undefined	$\tan 180° = \frac{0}{-1} = 0$

Example 1:

Evaluate using exact values and express your answer in reduced radical form.

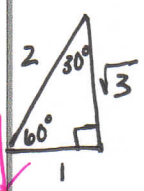
a) $\sin 45° = \frac{o}{h}$

$= \frac{1}{\sqrt{2}}$ ✓

b) $\cos^2 60°$

$(\cos 60°)^2 = \left(\frac{a}{h}\right)^2$

$= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$



c) $\cos 45° + \sin 30°$

$\frac{1}{\sqrt{2}} + \frac{1}{2}$

$= \frac{2(1) + \sqrt{2}(1)}{2\sqrt{2}}$

$= \frac{2 + \sqrt{2}}{2\sqrt{2}}$ ✓

d) $2\tan 60° - 3\tan 30°$

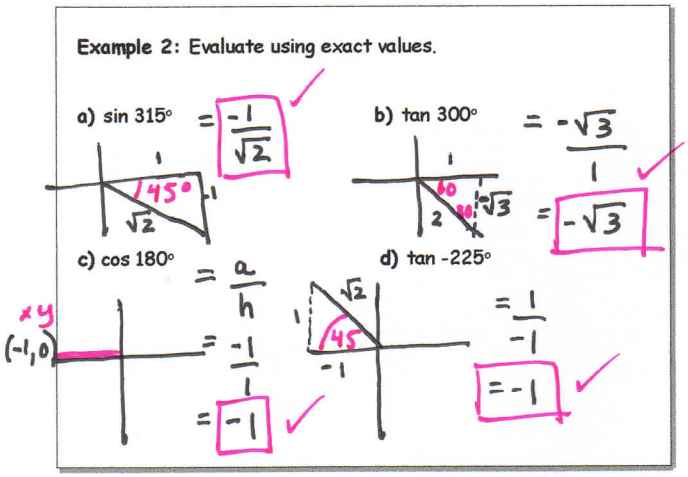
$2\left(\frac{\sqrt{3}}{1}\right) - 3\left(\frac{1}{\sqrt{3}}\right)$

$= 2\sqrt{3} - \frac{3}{\sqrt{3}}$

$= \frac{2\sqrt{3}(\sqrt{3}) - 3}{\sqrt{3}}$

$= \frac{2(3) - 3}{\sqrt{3}}$

$= \frac{6 - 3}{\sqrt{3}} = \frac{3}{\sqrt{3}}$ ✓ 2



UNIT 6: Trignometric Functions

Special Triangles

Learning Goal:

I will learn how to find the exact values of the sine, cosine, and tangent of special triangles.

Success Criteria:

To be successful, I must be able to...

- identify the two special triangles (45-45-90 and 30-60-90)
- evaluate for sine, cosine, and tangent using exact values from the special triangles

Practice Work

p. 348 #5, 18a - d

p. 354 #3a - d, 4