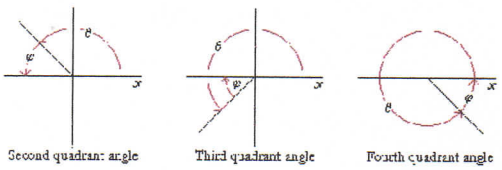
UNIT #6: Trigonometric Transformations

Related Acute Angles

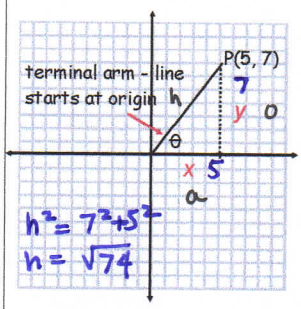
Learning Goal:

I will learn how to calculate the related angle from a given angle.



Second quadrant angle | Third quadrant angle | Fourth quadrant angle

Lesson: Related Acute Angles



$h^2 = 7^2 + 5^2$

$h = \sqrt{74}$

RECALL:

Trig ratios of any angle

$\sin\theta = \frac{o}{h} = \frac{y}{r} = \frac{7}{\sqrt{74}} = 0.81$

$\cos\theta = \frac{5}{\sqrt{74}} = 0.58$

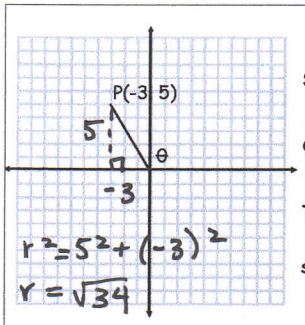
$\tan\theta = \frac{7}{5} = 1.4$

All ratios are **positive**

For any angle θ in standard position with point P(x, y) on the terminal arm:

$$r = \sqrt{x^2 + y^2}$$

$$\sin\theta = \frac{y}{r}, \cos\theta = \frac{x}{r}, \tan\theta = \frac{y}{x}$$



$r^2 = 5^2 + (-3)^2$

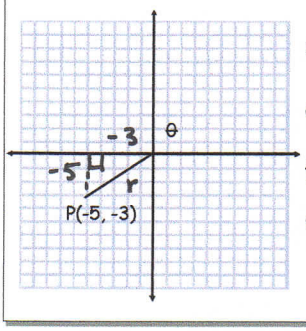
$r = \sqrt{34}$

$\sin\theta = \frac{5}{\sqrt{34}} = 0.86$

$\cos\theta = \frac{-3}{\sqrt{34}} = -0.51$

$\tan\theta = \frac{5}{-3} = -1.6$

sin (and csc) are **positive**



$\sin\theta = \frac{-5}{\sqrt{34}} = -0.86$

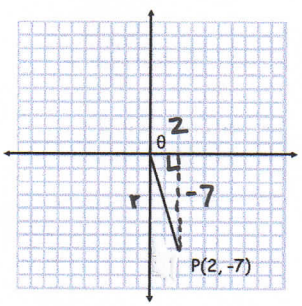
$\cos\theta = \frac{-3}{\sqrt{34}} = 0.51$

$\tan\theta = \frac{-5}{-3} = 1.67$

tan (and cot) are **positive**

$r^2 = (-5)^2 + (-3)^2$

$r = \sqrt{34}$



$r^2 = (2)^2 + (-7)^2$

$r^2 = 4 + 49$

$r = \sqrt{53}$

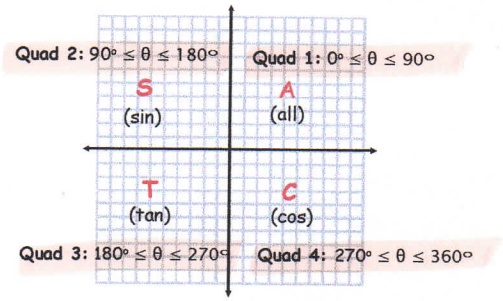
$\sin\theta = \frac{-7}{\sqrt{53}} = -0.96$

$\cos\theta = \frac{2}{\sqrt{53}} = 0.27$

$\tan\theta = \frac{-7}{2} = -3.5$

cos (and sec) are **positive**

RECALL: CAST Rule



The letters indicate which trig ratios are **positive** in that quadrant.

If θ is an acute angle ($0 \le \theta \le 90$), then the value of the trig ratios are related by the expressions in the quadrants, and the sign of the trig ratio is obtained from the CAST rule.

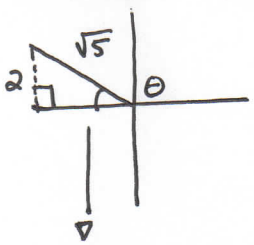
S	A
180 − θ	θ
T	**C**
θ − 180	360 − θ

Example 1:
If $\sin\theta = 2/\sqrt{5}$, and θ is in quad 2, find the other two trig ratios.

$\sin\theta = \frac{2}{\sqrt{5}} = \frac{o}{h}$

$\cos\theta = \frac{\sqrt{3}}{\sqrt{5}}$

$\tan\theta = \frac{2}{\sqrt{3}}$



$(\sqrt{5})^2 = 2^2 + a^2$

$5 - 2 = a^2$

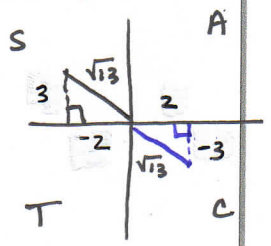
$\sqrt{3} = a$

Example 2:
$\tan\theta = -3/2$, $0 \le \theta \le 360$, find the other trig ratios.

$\tan\theta = \frac{-3}{2} = \frac{o}{a}$

$\sin\theta = \pm\frac{3}{\sqrt{13}}$

$\cos\theta = \pm\frac{2}{\sqrt{13}}$



$r^2 = a^2 + b^2$

$r^2 = (-3)^2 + (2)^2$

$\boxed{r = \sqrt{13}}$

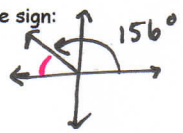
Related Acute Angle: is between the terminal arm and the x-axis

Related Acute Angles:

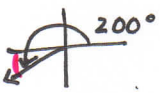
Q1 sin 40° = 0.6428
Q2 sin 140° = 0.6428
Q3 sin 220° = −0.6428
Q4 sin 320° = −0.6428

State the related acute angle and the sign:

Q2 cos 156° $180° - 156°$
$= -\cos 24°$



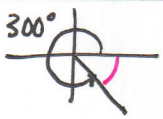
Q3 tan 200° $200° - 180°$
$= \tan 20°$



csc 300°

Q4 $360° - 300°$
$= -\csc 60°$

neg. b/c sin is neg in Q4

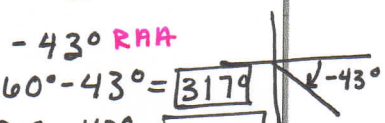


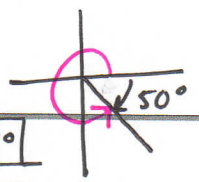
Examples: Find two angles for

sin θ = 0.9129 Q1 = 66° RAA OR $180° - 66°$
+'ve ∴ Q1 OR Q2 = 114° Q2

sin θ = −0.6820
= −43° RAA
−'ve ∴ Q3 OR Q4
Q4 $360° - 43° = 317°$
cot θ = −0.8391
−'ve ∴ Q2 OR Q4 Q3 $180° + 43° = 223°$

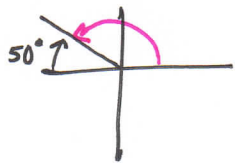
$\cot\theta = \tan^{-1}\left(\frac{-1}{0.8391}\right)$
$= -50°$ RAA





Q4 $360° - 50° = 310°$

Q2 $180° - 50° = 130°$ OR



UNIT 6: Trignometric Functions

Related Acute Angles

Learning Goal:

I will learn how to calculate the related angle from a given angle.

Success Criteria:

To be successful, I must be able to...

- Use CAST to identify the related angle and the sign of the function
- Find angles between 0 and 360 degrees that have the same trigonometric function value

Practice Work

Related Angles Worksheet