Unit 1: Absolute Value and Compound Inequalities

Unit Overview: Students will investigate compound inequalities in one variable and their relationship to absolute value equations and inequalities. They expand their ability to distinguish between situations with zero, one, and many solutions and apply this ability to compound inequalities and absolute value equations and inequalities (focus on MP.3, MP.4, and MP.8).

Concept 1: Solve Absolute Value Equations and Inequalities

Concept Overview: Students will investigate compound inequalities in one variable and their relationship to absolute value equations and inequalities. They will translate real-world constraints into compound inequalities and absolute value equations and inequalities, and they will solve and graph the equations. Students expand their ability to distinguish between situations with zero, one, and many solutions and apply this ability to compound inequalities and absolute value equations and inequalities. They recall that solutions can be verified by substituting them into the original equation and use this strategy to find extraneous solutions.

> **Unit 1, Concept 1 Standards**
> - HSA-CED.A.3
> - HSA-REI.B.3

Unit 2: Radical Functions and Transformations

Unit Overview: Students extend their understanding of rational and irrational numbers to square and cube root functions. They use approximations of rational and irrational numbers to graph square root and cube root functions. Students explore the effects of vertical and horizontal transformations to the square root and cube root functions and describe the features of the transformed functions. They also relate the importance of restricted domain and range of the functions to its graph and to the context of the problem. They graph radical functions, explore transformations of these functions, and compare their properties. Students continue to apply the laws of exponents as they work with radical functions and equations. Students explore the effects of translations, reflections, stretches, and compressions on the graphs of functions. They predict the effects of changes in a function rule on the graph of a function and sketch the graphs of transformations of functions (focus on MP.2, MP.7, and MP.8).

Concept 1: Investigate Square Root and Cube Root Functions

Concept Overview: Students extend their knowledge of rational and irrational numbers to square and cube root functions. They will use approximations of rational and irrational numbers to graph square root and cube root functions. Students explore the effects of vertical and horizontal transformations to the square root and cube root functions and describe the features of the transformed functions. They also relate the importance of restricted domain and range of the functions to its graph and to the context of the problem.

> **Unit 2, Concept 1 Standards**
> - HSF-BF.B.3
> - HSF-IF.B.4
> - HSF-IF.B.5
> - HSF-IF.C.7b

Concept 2: Analyze Radical Functions

Concept Overview: Students build on their understanding of inverses as they find inverses of quadratic and cubic functions. They graph radical functions, explore transformations of these functions, and compare their properties. Students continue to apply the laws of exponents as they work with radical functions and equations. In addition, they write and solve radical equations in one variable, including those with extraneous solutions.

Unit 2, Concept 2 Standards
- HSA-CED.A.1
- HSA- REI.A.2
- HSF-BF.A.1b
- HSF-BF.B.3
- HSF-BF.4a
- HSF-IF.C.7b
- HSF-IF.C.9
- HSN-RN.A.1
- HSN-RN.A.2

Concept 3: Explore Function Transformations

Concept Overview: Students explore the effects of translations, reflections, stretches, and compressions on the graphs of functions. They predict the effects of changes in a function rule on the graph of a function and sketch the graphs of transformations of functions.

Unit 2, Concept 3 Standards
- HSF-BF.A.1b
- HSF-BF.B.3
- HSF-IF.C.7b
- HSF-IF.C.9

Unit 3: Linear and Exponential Models

Unit Overview: Students learn to write recursive and explicit formulas for arithmetic and geometric sequences and use these formulas to solve problems. They write and apply the formulas for the sums of finite arithmetic and geometric series and for infinite geometric series when the sum exists. Students use exponential equations to model real-world problems including population and compounding interest, building their understanding of exponential parameters in context. They investigate exponential patterns within the compound interest formula, compounding the interest in smaller and smaller intervals to develop the number *e* (focus on MP.2, MP.7, and MP.8).

Concept 1: Explore Recursive Functions

Concept Overview: Students learn to write recursive and explicit formulas for arithmetic and geometric sequences and use these formulas to solve problems. They relate arithmetic sequences to linear functions and geometric sequences to exponential functions. Students also write and apply the formulas for the sums of finite arithmetic and geometric series and for infinite geometric series when the sum exists.

Unit 3, Concept 1 Standards
- HSA-SSE.B.4
- HSF-BF.A.1a
- HSF-BF.A.1
- HSF-BF.A.2
- HSF-IF.A.3
- HSF-LE.A.2

Concept 2: Model Exponential Growth and Decay

Concept Overview: Students use exponential equations to model real-world problems including population and compounding interest, building their understanding of exponential parameters in context. They investigate exponential patterns within the compound interest formula, compounding the interest in smaller and smaller intervals to develop the number *e*. Students explore different methods of evaluating an equation, including analysis of representative graphs of the expressions within the equations.

Unit 3, Concept 2 Standards
- HSA-CED.A.1
- HSA-REI.D.11
- HSF-BF.A.1b
- HSF-IF.C.8
- HSF-IF.C.8b
- HSF-LE.A.1c
- HSF-LE.A.2
- HSF-LE.B.5

Unit 4: Logarithmic Functions

Unit Overview: Students learn to graph exponential and logarithmic functions and note the key features of the graphs. They learn to solve exponential equations by using properties of logarithms. Students discover how to use the inverse relationship between exponential and logarithmic functions to solve real-world problems, interpreting appropriate parameters for these functions in terms of their contexts. They express logarithmic equations as equivalent exponential functions and evaluate, using technology when appropriate (focus on MP.1, MP.2, and MP.8).

Concept 1: Discover and Analyze Logarithms

Concept Overview: Students learn to graph exponential and logarithmic functions and note the key features of the graphs. They also explore the inverse relationship between exponents and logarithms and use this understanding to solve problems. They learn to solve exponential equations by using properties of logarithms.

Unit 4, Concept 1 Standards
- HSF-BF.B.3
- HSF-BF.B.4a
- HSF-BF.B.4c(+)
- HSF-BF.B.5(+)
- HSF-IF.B.4
- HSF-IF.C.7e

Concept 2: Apply Logarithmic Functions

Concept Overview: Students discover how to use the inverse relationship between exponential and logarithmic functions to solve real-world problems, interpreting appropriate parameters for these functions in terms of their contexts. They express logarithmic equations as equivalent exponential functions and evaluate, using technology when appropriate.

Unit 4, Concept 2 Standards
- HSF-BF.A.1a
- HSF-BF.B.5(+)
- HSF-LE.A.4

Unit 5: Nonlinear Systems

Unit Overview: Students build on their knowledge of systems of equations to include solving systems consisting of a linear and quadratic equation. They will extend their understanding of solving a linear system by graphing to encompass simple polynomial, rational, absolute value, exponential, and logarithmic functions (focus on MP.1, MP.2, and MP.4).

Concept 1: Solve Nonlinear Systems

Concept Overview: Students extend their understanding of systems of equations to include solving systems consisting of a linear and quadratic equation and will extend their understanding of solving a linear system by graphing to encompass simple polynomial, rational, absolute value, exponential, and logarithmic functions.

Unit 5, Concept 1 Standards
• HSA-REI.C.7   • HSA-REI.D.11

Unit 6: Trigonometric Functions

Unit Overview: Students discover the relationships between arc lengths on the unit circle and radian measures of angles. They investigate the unit circle and discover how it is used to find the trigonometric values for all real numbers. Students invoke the Pythagorean theorem for ordered pairs on the unit circle to prove the Pythagorean identity, which they use to find trigonometric quantities. They analyze the key features of trigonometric graphs, including period, midline, and amplitude and compare transformations for these functions (focus on MP.2, MP.7, and MP.8).

Concept 1: Explore Angle Measure

Concept Overview: Students explore the relationships between arc lengths on the unit circle and radian measures of angles. They discover how to convert between radians and degrees and how to calculate arc lengths for any given angle measure and radius.

Unit 6, Concept 1 Standards
• HSF-TF.A.1

Concept 2: Represent Trigonometric Functions

Concept Overview: Students investigate the unit circle and discover how it is used to find the trigonometric values for all real numbers. Students invoke the Pythagorean theorem for ordered pairs on the unit circle to prove the Pythagorean identity, which they use to find trigonometric quantities. They analyze the key features of trigonometric graphs, including period, midline, and amplitude and compare transformations for these functions.

Unit 6, Concept 2 Standards
• HSF-BF.B.3   • HSF-IF.B.4   • HSF-IF.C.7e   • HSF-TF.C.8   • HSF-TF.A.2   • HSF-TF.A.3(+)   • HSS-ID.B.6

Unit 7: Trigonometric Applications

Unit Overview: Students will use trigonometric functions to model real-world, periodic phenomena. They relate the features of graphs of the trigonometric functions to characteristics of real-world scenarios and write equations describing real-world phenomena. Students state and prove the law of sines and the law of cosines. They apply the two laws to solve problems, investigating when to use the law of sines and when to use the law of cosines to solve a triangle. Real-world scenarios are used to develop these laws, challenging students to apply geometric methods to satisfy physical constraints and to choose a level of accuracy appropriate to limitations on measurement (focus on MP.2, MP.7, and MP.8).

Concept 1: Apply Trigonometric Relationships

Concept Overview: Students learn to use trigonometric functions to model real-world, periodic phenomena. They relate the features of graphs of the trigonometric functions to characteristics of real-world scenarios and write equations describing real-world phenomena. They analyze real-world data in various forms and convert from one form to another when appropriate.

Unit 7, Concept 1 Standards
- HSF-IF.B.4
- HSF-IF.B.5
- HSF-IF.C.7e
- HSF-IF.C.9
- HSF-TF.B.5

Concept 2: Investigate Laws of Sines and Cosines

Concept Overview: Students state and prove the law of sines and the law of cosines. They apply the two laws to solve problems, investigating when to use the law of sines and when to use the law of cosines to solve a triangle. Students investigate the conditions when the law of sines generates a unique value and when it generates fewer than or more than one unique value. Real-world scenarios are used to develop these laws, challenging students to apply geometric methods to satisfy physical constraints and to choose a level of accuracy appropriate to limitations on measurement.

Unit 7, Concept 2 Standards
- HSG-SRT.D.10(+)
- HSG-SRT.D.11(+)
- HSG-MG.A.3
- HSN-Q.A.3

Unit 8: Data Modeling

Unit Overview: Students build upon their understanding of sampling by exploring and evaluating a variety of sampling methods. They use simulations to develop an understanding of variability in sampling and learn to calculate the margin of error for data collected from a sample. Students also focus on the characteristics of a normal distribution. They use statistical pictures and numerical measures to analyze normally distributed data. They investigate the empirical rule and its role in describing normal distributions. They also use z-scores, in the context of real-world problem solving, to analyze individual data points (focus on MP.3, MP.4, and MP.5).

Concept 1: Explore Normal Distributions

Concept Overview: Students focus on the characteristics of a normal distribution. They use statistical pictures and numerical measures to analyze normally distributed data. Students investigate the empirical rule and its role in describing normal distributions. They also use z-scores, in the context of real-world problem-solving, to analyze individual data points.

Unit 8, Concept 2 Standards
• HSS-ID.A.4
• HSS-MD.B.6(+)
• HSS-MD.B.7(+)

Concept 2: Collect, Analyze, and Interpret Statistical Data

Concept Overview: Students expand their understanding of sampling by exploring and evaluating a variety of sampling methods. They use simulations to develop an understanding of variability in sampling and learn to calculate the margin of error for data collected from a sample. Students investigate the use of experiments in testing hypotheses and analyze the ability of experiments to establish causal relationships.

Unit 8, Concept 1 Standards
• HSS-IC.A.1
• HSS-IC.A.2
• HSS-IC.B.3
• HSS-IC.B.4
• HSS-IC.B.5
• HSS-IC.B.6

Unit 9: Polynomials

Unit Overview: Students build on their earlier work with quadratic and other types of expressions as they write polynomials and complex numbers in different forms, perform operations with polynomials and complex numbers, and interpret parts of polynomial expressions. They apply factoring techniques learned when working with quadratic expressions to help them factor polynomials and identify zeros of polynomial functions. Students expand on previous work with quadratic functions as they develop an understanding of the relationship between the zeros of a polynomial function and the factors of its function rule. They apply the rational root theorem, polynomial long division, synthetic division, and factoring to identify rational roots and the quadratic formula to identify irrational and complex roots. Students extend their knowledge of complex numbers to include complex conjugates and they simplify rational expressions by applying properties of rational and complex numbers. They will sketch graphs of polynomial functions using characteristics identified from the equations of the functions. In addition, students write polynomial equations and develop polynomial functions to model and solve real-world problems (focus on MP.6, MP.7, and MP.8).

Concept 1: Operate With Polynomials

Concept Overview: Students build on their earlier work with quadratic expressions and other types of expressions as they write polynomials and complex numbers in different forms, perform operations with polynomials and complex numbers, and interpret parts of polynomial expressions. They develop an understanding of the relationship between the zeros of a polynomial function and the factors of its function rule. Students also apply factoring techniques learned when working with quadratic expressions to help them factor polynomials and identify zeros of polynomial functions. In addition, students learn how they can use Pascal's triangle to help them expand powers of binomials.

Unit 9, Concept 1 Standards
- HSA-APR.A.1
- HSA-APR.C.5(+)
- HSA-APR.B.2
- HSA-APR.B.3
- HSA-APR.C.4
- HSA-SSE.A.1b
- HSA-SSE.A.2
- HSN-CN.A.2
- HSN-CN.B.4(+)

Concept 2: Explore Polynomial Factors

Concept Overview: Students expand on previous work with quadratic functions as they develop an understanding of the relationship between the zeros of a polynomial function and the factors of its function rule. They also write polynomials in different forms to identify roots of the related equations. Students apply the rational root theorem, polynomial long division, synthetic division, and factoring to identify rational roots and the quadratic formula to identify irrational and complex roots. Students extend their knowledge of complex numbers to include complex conjugates and they simplify rational expressions by applying properties of rational and complex numbers.

Unit 9, Concept 2 Standards
- HSA-APR.B.2
- HSA-SSE.B.3
- HSN-CN.A.3(+)

Concept 3: Analyze Polynomial Functions

Concept Overview: Students use their background regarding other types of functions as they identify and interpret key features of polynomial functions. They also sketch graphs of polynomial functions using characteristics identified from the equations of the functions. In addition, students write polynomial equations and develop polynomial functions to model and solve real-world problems.

Unit 9, Concept 3 Standards
- HSA-APR.B.3
- HSA-CED.A.1
- HSF-IF.B.6
- HSS-ID.B.6a
- HSF-IF.B.4
- HSF-IF.C.7c

Unit 10: Rational Expressions and Equations

Unit Overview: Students build on previously learned mathematics concepts, enabling them to explore how operations with rational expressions are analogous to operations with rational numbers. They use their knowledge of writing rational numbers in different forms to help them write rational expressions in different forms. Students use their knowledge of rational expressions as they write rational equations and inequalities in one variable in order to model real-world situations (focus on MP.1, MP.2, and MP.7).

Concept 1: Develop Rational Expressions

Concept Overview: Students build on previous mathematics concepts learned, enabling them to explore how operations with rational expressions are analogous to operations with rational numbers. They use their knowledge of writing rational numbers in different forms to help them write rational expressions in different forms. In addition, students will apply their understanding of factoring to identify and generate equivalent rational expressions.

Unit 10, Concept 1 Standards
• HSA-APR.D.6
• HSA-APR.D.7(+)
• HSA-SSE.A.2

Concept 2: Solve Rational Equations

Concept Overview: Students use their knowledge of rational expressions as they write rational equations and inequalities in one variable in order to model real-world situations. They will apply what they have learned about algebraic properties to solve rational equations and inequalities, including those that model inverse variation or give rise to extraneous solutions. Students then extend their understanding of direct and inverse variation to explore situations involving combined variation. In addition, they rearrange formulas involving combined variation to highlight a quantity of interest.

Unit 10, Concept 2 Standards
• HSA-CED.A.1
• HSA-CED.A.3
• HSA-CED.A.4
• HSA-REI.A.2
• HSA-SSE.A.2

Unit 11: Rational Functions

Unit Overview: Students apply what they have learned about functions and rational expressions as they explore rational functions. They graph rational functions and identify key features of the graphs, including asymptotes and end behavior. Students will solve rational equations graphically or by using tables to find approximate solutions. They learn to write simple rational functions and use the functions to investigate related one-variable equations and inequalities. Using the zero(s) and the asymptote(s) of the functions, students can determine solutions for the original inequalities (focus on MP.2, MP.4, and MP.7).

Concept 1: Represent Rational Functions

Concept Overview: Students use what they have learned about functions and rational expressions as they explore rational functions. They graph rational functions and identify key features of the graphs, including asymptotes and end behavior. They determine and interpret the mathematical and reasonable domains and ranges of rational functions. Students also solve rational equations graphically or by using tables to find approximate solutions.

Unit 11, Concept 1 Standards
• HSA-REI.D.10
• HSF-IF.B.4
• HSF-IF.B.5
• HSF-IF.C.7d(+)

Concept 2: Compare Rational Functions

Concept Overview: Students learn to write simple rational functions and use the functions to investigate related one-variable equations and inequalities. They analyze one-variable rational inequalities by writing functions for the expressions on either side of the inequalities. Using the zero(s) and the asymptote(s) of the functions, students determine solutions for the original inequalities.

Unit 11, Concept 2 Standards
• HSA-CED.A.1
• HSA-CED.A.2
• HSA-REI.D.11
• HSF-BF.B.4a
• HSF-IF.B.5

Unit 12: Conics

Unit Overview: Students expand upon their previous learning with equations in two variables by graphing the equations of circles and ellipses and identifying key features of the graphs. They discover the relationship between the points on an ellipse and the foci of the ellipse, and they write the equations of ellipses given key information. Students graph the equations of hyperbolas and identifying key features of the graphs. Students learn how to recognize and analyze an equation of a parabola. They derive the equation of a parabola using the graph, distance formula, and algebraic manipulation through several tests and the general form. This equation will be applied in algebraic and real-world situations during the investigations. Students investigate the relationships among different conic sections. They extend their understanding of individual conics, discovering how to discern between a variety of algebraic representations and their characteristics (focus on MP.4, MP.5, and MP.7).

Concept 1: Analyze Graphs and Equations of Circles and Ellipses

Concept Overview: Students deepen their past learning with equations in two variables by graphing the equations of circles and ellipses and identifying key features of the graphs. They write the equations of circles given key information and complete the square to determine key information about circles given their equations. Students also discover the relationship between the points on an ellipse and the foci of the ellipse, and they write the equations of ellipses given key information.

Unit 12, Concept 1 Standards
• HSG-GPE.A.1
• HSG-GPE.A.2
• HSG-GPE.A.3(+)

Concept 2: Analyze Graphs and Equations of Hyperboles

Concept Overview: Students build on their previous work with equations in two variables by graphing the equations of hyperbolas and identifying key features of the graphs. They also explore the relationship between the points on a hyperbola and the foci of the hyperbola, and they write the equations of hyperbolas given key information.

Unit 12, Concept 2 Standards
- HSG-GPE.A.3(+)

Concept 3: Analyze Graphs and Equations of Parabolas

Concept Overview: Students learn how to recognize and analyze an equation of a parabola. Key features are determined from the equation and validated through graphing. Then, students will derive the equation of a parabola using the graph, distance formula, and algebraic manipulation through several tests and the general form. This equation will be applied in algebraic and real-world situations during the investigations.

Unit 12, Concept 3 Standards
- HSG-GPE.A.2

Concept 4: Compare Conic Equations

Concept Overview: Students investigate the relationships among different conic sections. They extend their understanding of individual conics, discovering how to discern between a variety of algebraic representations and their characteristics. Students will also explore solving systems of equations that include conic sections.

Unit 12, Concept 4 Standards
- HSA-REI.C.7
- HSA-REI.D.11
- HSG-GPE.A.1
- HSG-GPE.A.2
- HSG-GPE.A.3(+)