Unit 1: Transformations and Congruence

Unit Overview: Students discover the roles of postulates, definitions, theorems, and precise geometric notation in the construction of formal geometric proofs. They begin their formal explorations in geometry by interpreting and using the axioms and undefined terms that lay the foundation of the discipline. Students will explore and construct parallel and perpendicular lines. They will examine the relationship between geometric transformations and algebraic representations of parallel and perpendicular lines in preparation for the proof of the slope criteria. Students extend their understanding of geometric evidence and valid arguments and apply their understanding of rigid motion transformations to formally prove theorems about angle pairs, parallel lines, and perpendicular lines. They build on their understanding of congruence transformations to prove triangle congruence theorems (focus on MP.1, MP.3, and MP.5)

Concept 1: Explore the Building Blocks of Geometry

Concept Overview: Students learn the roles of postulates, definitions, theorems, and precise geometric notation in the construction of formal geometric proofs. They begin their formal explorations in geometry by interpreting and using the axioms and undefined terms that lay the foundation of the discipline. Students will explore important definitions and postulates to prepare for the development of formal arguments to establish geometric facts. They investigate evidence of vocabulary, postulates, and theorems in real-world scenarios and mathematical diagrams to prepare for geometric modeling.

Unit 1, Concept 1 Standards
- HSG-CO.A.1
- HSG-CO.C.9
- HSG-MG.A.1

Concept 2: Explore Parallel and Perpendicular Lines

Concept Overview: Students explore and construct parallel and perpendicular lines. They examine the relationship between geometric transformations and algebraic representations of parallel and perpendicular lines in preparation for the proof of the slope criteria.

Unit 1, Concept 2 Standards
- HSG-CO.A.1
- HSG-CO.D.12
- HSG-GPE.B.5

Concept 3: Prove Theorems about Lines and Angles

Concept Overview: Students extend their understanding of geometric evidence and valid arguments and apply their understanding of rigid motion transformations to formally prove theorems about angle pairs, parallel lines, and perpendicular lines.

Unit 1, Concept 3 Standards
- HSG-CO.C.9
- HSG-GPE.B.4
- HSG-GPE.B.5

Concept 4: Prove Congruence Theorems

Concept Overview: Students prove theorems about triangles including the triangle sum theorem, exterior angle theorem, and isosceles and equilateral triangles theorems. They build on their understanding of congruence transformations to prove triangle congruence theorems.

Unit 1, Concept 4 Standards
- HSG-CO.B.8
- HSG-CO.C.10
- HSG-SRT.B.5

Unit 2: Similarity

Unit Overview: Students broaden their understanding of dilations of figures by identifying a dilation as a non-rigid transformation. They will use their previous work with transformations to experimentally verify the properties of dilations as functions. Students will use and expand their knowledge of similarity to decide if two given figures are similar and justify their reasoning by examining all corresponding pairs of angles and sides. They use triangle similarity criteria to help them prove theorems about triangles. They build on what they have learned about similarity and apply it to three-dimensional figures. Students will develop an informal definition of similarity in three dimensions and use it to determine whether three-dimensional figures are similar. They also apply properties of similar figures in three dimensions to solve problems, including problems involving surface area and volume. (focus on MP.3. MP.5, and MP.7).

Concept 1: Explore Similarity and Dilation

Concept Overview: Students expand upon their understanding of dilations of figures by identifying a dilation as a non-rigid transformation. They use their previous work with transformations to experimentally verify the properties of dilations as functions. Students establish the definition of a similarity transformation as a composition of rigid motion transformations followed by a dilation.
They will use and expand their knowledge of similarity to decide if two given figures are similar and justify their reasoning by examining all corresponding pairs of angles and sides.

Unit 2, Concept 1 Standards
- HSG-SRT.A.1
- HSG-SRT.A.1a
- HSG-SRT.A.1b
- HSG-SRT.A.2

Concept 2: Prove Similarity Theorems

Concept Overview: Students apply their knowledge of transformations and the properties of similar figures to establish triangle similarity criteria. They use triangle similarity criteria to help them prove theorems about triangles. Students also use their understanding of similarity and congruence of triangles to
solve real-world and mathematical problems as they continue in their development to prepare and solve design problems.

Unit 2, Concept 2 Standards
- HSG-CO.C.10
- HSG-MG.A.3
- HSG-SRT.A.3
- HSG-SRT.B.4
- HSG-SRT.B.5

Concept 3: Apply Similarity Theorems

Concept Overview: Students build on what they have learned about similarity and apply it to three-dimensional figures. They develop an informal definition of similarity in three dimensions and use it to determine whether three-dimensional figures are similar. Students also apply properties of similar figures in three dimensions to solve problems, including problems involving surface area and volume.

Unit 2, Concept 3 Standards
- HSG-SRT.A.2
- HSG-SRT.B.5

Unit 3: Rational Exponents and Polynomials

Unit Overview: Students extend the properties of rational and irrational numbers, as well as integer exponents, to that of rational exponents. They rewrite radical expressions, many of which are irrational, using rational exponents. Students build on their earlier work with linear expressions and integer exponents as they begin to explore more complex polynomial expressions. They interpret different parts of polynomials in context and begin to see expressions as sums, products, and factors instead of different entities. They extend their knowledge of properties of rational exponents and use these properties to simplify variable expressions. Students further explore algebraic expressions that can be expressed as products of factors. They will rely on patterns observed in previous work to identify factors, leading to the examination of the structure of quadratic equations. Students will discover different methods for factoring quadratic expressions, including factoring by graphing, using geometric area models, and factoring algebraically (focus on MP.1, MP.7, and MP.8).

Concept 1: Investigate Rational Exponents

Concept Overview: Students extend the properties of rational and irrational numbers, as well as integer exponents, to that of rational exponents. They rewrite radical expressions, many of which are irrational, using rational exponents. Students employ the properties of exponents to write equivalent expressions, providing insight into the structure of the expression.

Unit 3, Concept 1 Standards
- HSN-RN.A.1
- HSN-RN.A.2
- HSN-RN.B.3
- HSA-SSE.B.3c

Concept 2: Perform Operations on Polynomial

Concept Overview: Students build on their earlier work with linear expressions and integer exponents as they explore more complex polynomial expressions. They interpret different parts of polynomials in context and begin to see expressions as sums, products, and factors instead of different entities. They add, subtract, and multiply polynomials to create equivalent expressions that allow them to interpret different forms of quadratic functions in later investigations. They extend their knowledge of properties of rational exponents and use these properties to simplify variable expressions.

Unit 3, Concept 2 Standards
- HSA-APR.A.1
- HSA-SSE.A.1a
- HSA-SSE.A.1b
- HSN-RN.A.2

Concept 3: Factor Polynomials

Concept Overview: Students further explore algebraic expressions that can be expressed as products of factors. They will rely on patterns observed in previous work to identify factors, leading to the examination of the structure of quadratic equations. Students will discover different methods for factoring quadratic expressions, including factoring by graphing, using geometric area models, and factoring algebraically.

Unit 3, Concept 3 Standards
- HSA-SSE.A.1a
- HSA-SSE.A.2
- HSA-SSE.B.3a

Unit 4: Quadratic Equations

Unit Overview: Students build on previous knowledge of functions and equations as they solve quadratic equations by factoring, taking square roots, completing the square, and graphing. They also use the properties of rational and irrational numbers to solve quadratic equations with rational or irrational solutions. Students use their new learning to develop the graph of the related function and determine some of the key features. They derive the quadratic formula based on their understanding of solving quadratic equations by completing the square, realizing the different structures of the related equations. They also make use of properties of rational and irrational numbers as they solve quadratic equations with rational or irrational solutions. In addition, students continue their exploration of quadratic functions and key features of the functions' graphs (focus on MP.3, MP.7, and MP.8).

Concept 1: Solve Quadratics

Concept Overview: Students use their background knowledge of functions and equations as they solve quadratic equations by factoring, taking square roots, completing the square, and graphing. They also use the properties of rational and irrational numbers to solve quadratic equations with rational or irrational solutions. In addition, students begin to investigate some of the properties of quadratic functions, reflecting on how the roots of equations determine the *x*-intercepts on the graph of the function. Students use their new learning to develop the graph of the related function and determine some of the key features.

Unit 4, Concept 1 Standards
- HSA-APR.B.3
- HSA-REI.B.4
- HSA-REI.B.4a
- HSA-REI.B.4b
- HSF-IF.C.8
- HSF-IF.C.8a
- HSN-RN.B.3

Concept 2: Analyze Quadratic Equations

Concept Overview: Students derive the quadratic formula based on their understanding of solving quadratic equations by completing the square, realizing the different structures of the related equations. Then, they apply the quadratic formula to solve quadratic equations and identify the type and number of real solutions given by the formula, extending their understanding to interpret other key features of the related graph. Students also make use of properties of rational and irrational numbers as they solve quadratic equations with rational or irrational solutions. In addition, students continue their exploration of quadratic functions and key features of the functions' graphs.

Unit 4, Concept 2 Standards

- HSF-IF.C.7a
- HSF-IF.C.8
- HSF-IF.C.8a
- HSA-REI.B.4a
- HSA-REI.B.4b
- HSA-REI.B.7
- HSN-RN.B.3
- HSA-APR.B.3

Unit 5: Quadratic Functions

Unit Overview: Students further analyze and compare quadratic, exponential, and linear behaviors numerically, algebraically, and graphically. They apply their understanding of completing the square to rewrite quadratic functions into vertex form. Students graph quadratic functions and interpret intercepts, maxima, and minima. They discover a new subset of real numbers and how they behave in the system of numbers. They recognize the need for the expansion of the system through their study of quadratic equations and solutions that are not represented as real numbers in the quadratic formula or *x*-intercepts on the graph. For the first time, students discover a number that cannot be plotted on the real number line. Although complex numbers require a new model, students discover that operations on complex numbers are a natural extension of real number operations (focus on MP.2, MP.3, and MP.7).

Concept 1: Analyze Graphs of Quadratic Functions

Concept Overview: Students further analyze and compare quadratic, exponential, and linear behaviors numerically, algebraically, and graphically. They apply their understanding of completing the square to rewrite quadratic functions into vertex form. Students interpret quadratic functions in both standard form and vertex form, identifying the key features by analyzing the characteristics of the functions. Students graph quadratic functions and interpret intercepts, maxima, and minima.

Unit 5, Concept 1 Standards

- HSA-CED.A.1
- HSF-IF.B.5
- HSF-LE.A.3
- HSF-BF.B.3
- HSF-IF.B.4
- HSF-IF.C.7a
- HSF-IF.C.7c
- HSF-IF.C.8a
- HSF-IF.C.9
- HSF-BF.A.1

Concept 2: Determine Complex Quadratic Roots

Concept Overview: Students discover a new subset of real numbers and how they behave in the system of numbers. They recognize the need for this expansion of the system through their study of quadratic equations and solutions that are not represented as real numbers in the quadratic formula or *x*-intercepts on the graph. For the first time, students discover a number that cannot be plotted on the real number line. Although complex numbers require a new model, they discover that operations on complex numbers are a natural extension of real number operations. The commutative, associative, and distributive properties guide early explorations in complex number operations.

Unit 5, Concept 2 Standards
- HSN-CN.A.1
- HSN-CN.A.2
- HSN-CN.C.7

Unit 6: Power and Inverse Functions

Unit Overview: Students discover that power functions have many similarities to polynomial functions. They analyze the key features and use power functions with direct variation to solve problems. Students develop the ideas and notation for composite functions and apply this understanding to the relationship between functions and their inverses. They explore inverses of functions, identifying graphical and algebraic representations by analyzing ordered pairs, domains, and ranges. They will gain insight about the relationship between functions and their inverses while investigating how to restrict the domain of a quadratic function and create an inverse function (focus on MP.2, MP.4, and MP.8).

Concept 1: Compare Power and Inverse Functions

Concept Overview: Students learn that power functions have many similarities to polynomial functions. They analyze the key features and use power functions with direct variation to solve problems. In addition, students transform power functions and compare the graphs to the equations.

Unit 6, Concept 1 Standards
- HSF-BF.B.3
- HSF-IF.B.4
- HSF-IF.B.5
- HSF-IF.C.7c

Concept 2: Explore Inverse Functions

Concept Overview: Students develop the ideas and notation for composite functions and apply this understanding to the relationship between functions and their inverses. Students explore inverses of functions, identifying graphical and algebraic representations by analyzing ordered pairs, domains, and ranges. They garner insight about the relationship between functions and their inverses while investigating how to restrict the domain of a quadratic function and create an inverse function.

Unit 6, Concept 2 Standards
- HSF-BF.A.1c(+)
- HSF-BF.B.4
- HSF-BF.B.4a
- HSF-BF.B.4c(+)
- HSG-CO.A.5
- HSG-CO.B.6

Unit 7: Trigonometry and Circles

Unit Overview: Students develop an understanding of how the ratios of sides in similar right triangles lead to the definitions of trigonometric ratios of angles. They use these ratios with the Pythagorean theorem to solve real-world problems. They also learn the relationship between the sine and cosine of complementary angles and apply this relationship to solve problems. Students explore relationships between central angles, arc measure, inscribed angles, and circumscribed angles and identify and describe relationships among inscribed angles, radii, and chords. They will derive the equation of the circle in the coordinate plane. Given the equation of a circle, they will be able to complete the square to determine the center and radius of the circle and graph the circle in the coordinate plane. Students will apply their understanding of the equation of a circle to determine whether a given point lies on the circle (focus on MP.3, MP.5, and MP.8).

Concept 1: Investigate Right Triangle Trigonometry

Concept Overview: Students develop an understanding of how the ratios of sides in similar right triangles lead to the definitions of trigonometric ratios of angles. They use these ratios with the Pythagorean theorem to solve real-world problems. They learn the relationship between the sine and cosine of complementary angles and apply this relationship to solve problems. Students extend their understanding of trigonometric relationships to derive the formula $A = \frac{1}{2} ab \sin C$ for the area of a triangle.

Unit 7, Concept 1 Standards
- HSG-SRT.C.6
- HSG-SRT.C.7
- HSG-SRT.C.8
- HSG-SRT.D.9(+)

Concept 2: Investigate Circles and Parts of Circles

Concept Overview: Students explore relationships between central angles, arc measure, inscribed angles, and circumscribed angles and identify and describe relationships among inscribed angles, radii, and chords. In addition, students construct tangents from a point outside the circle. They will also prove that all circles are similar.

Unit 7, Concept 2 Standards
- HSG-C.A.1
- HSG-C.A.2
- HSG-C.A.4(+)

Concept 3: Investigate and Interpret Circle Equations

Concept Overview: Students will derive the equation of the circle in the coordinate plane. Given the equation of a circle, they will be able to complete the square to determine the center and radius of the circle and graph the circle in the coordinate plane. Students will apply their understanding of the equation of a circle to determine whether a given point lies on the circle. They will use special right triangles to determine geometrically the values of sine and cosine to develop the unit circle and prove the Pythagorean identity. Students will identify and interpret key features of the sine and cosine functions, including period and amplitude, and explore transformations of the two trigonometric functions.

Unit 7, Concept 3 Standards
- HSF-BF.B.3
- HSG-GPE.A.1
- HSG-GPE.B.4
- HSF-IF.B.5
- HSF-IF.B.4
- HSF-IF.C.7e
- HSF-TF.A.3 (+)
- HSF-TF.C.8

Unit 8: Area and Volume

Unit Overview: Students develop a limit argument that proves the formulas for circumference and area of a circle and use the formulas to solve real-world problems involving area and circumference. They will extend their understanding of cross-sections to include two-dimensional slices of real-world three-dimensional objects and identify three-dimensional objects generated by rotation of two-dimensional objects. They use transformations of two-dimensional geometric shapes to describe and model real-world three-dimensional objects. Students explore the cross sections and characteristics of figures through informal arguments for volume of cylinders, spheres, and other solid figures. They will use geometric shapes to describe and model real-world objects and apply their formulas for volume and surface area to solve problems (focus on MP.2, MP.4, and MP.5).

Concept 1: Investigate and Apply Area and Circumference Formulas

Concept Overview: Students create a limit argument that proves the formulas for circumference and area of a circle and use the formulas to solve real-world problems involving area and circumference. They also develop formulas for arc length and the area of a sector and define the radian measure of an angle.

Unit 8, Concept 1 Standards
- HSG-C.B.5
- HSG-GMD.A.1

Concept 2: Investigate Cross Sections and Rotations

Concept Overview: Students extend their understanding of cross-sections to include two-dimensional slices of real-world three-dimensional objects and identify three-dimensional objects generated by rotation of two-dimensional objects. They use transformations of two-dimensional geometric shapes to describe and model real-world three-dimensional objects. Students apply their knowledge of cross-sections to solve design problems.

Unit 8, Concept 2 Standards
- HSG-GMD.B.4
- HSG-MG.A.1
- HSG-MG.A.3

Concept 3: Develop and Apply Volume Formulas

Concept Overview: Students explore the cross sections and characteristics of figures through informal arguments for volume of cylinders, spheres, and other solid figures. They will use geometric shapes to describe and model real-world objects and apply their formulas for volume and surface area to solve problems.

Unit 8, Concept 3 Standards
- HSG-GMD.A.1
- HSG-GMD.A.2(+)
- HSG-GMD.A.3
- HSG-MG.A.1
- HSG-MG.A.2
- HSG-MG.A.3

Unit 9: Polygons

Unit Overview: Students explore points of concurrency of a triangle and prove that the medians of a triangle meet at a point of concurrency. They use points of concurrency to construct circumscribed and inscribed triangles. Students will understand how to use constructions to inscribe a polygon in a circle as well as explore and prove properties of angles in inscribed polygons. They extend properties of parallelograms to other quadrilaterals and develop a classification of quadrilaterals based on the inclusive definition of quadrilaterals. Students algebraically use coordinates to prove geometric theorems involving quadrilaterals (focus on MP.3, MP.4, and MP.7).

Concept 1: Investigate Concurrency in Triangles

Concept Overview: Students investigate points of concurrency of a triangle and prove that the medians of a triangle meet at a point of concurrency. They use points of concurrency to construct circumscribed and inscribed triangles.

Unit 9, Concept 1 Standards
- HSG-C.A.3
- HSG-CO.C.10

Concept 2: Construct and Explore Polygons

Concept Overview: Students will understand how to use constructions to inscribe a polygon in a circle as well as explore and prove properties of angles in inscribed polygons.

Unit 9, Concept 2 Standards
- HSG-CO.D.13
- HSG-C.A.3

Concept 3: Prove and Apply Theorems about Quadrilaterals

Concept Overview: Students prove theorems about parallelograms. They extend properties of parallelograms to other quadrilaterals and develop a classification of quadrilaterals based on the inclusive definition of quadrilaterals. Students algebraically use coordinates to prove geometric theorems involving quadrilaterals.

Unit 9, Concept 3 Standards
- HSG-CO.C.11
- HSG-GPE.B.4

Unit 10: Probability

Unit Overview: Students will describe a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events. They understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities. They apply this understanding to the Monty Hall problem. They also construct and interpret two-way frequency tables of data and use the two-way tables to decide if events are independent and to approximate conditional probabilities. Students formalize their understanding of conditional probability and independence. They find conditional probabilities and use the addition and multiplication rules and interpret the probabilities. They learn to analyze decisions and strategies using probability concepts and use probability to make fair decisions. Students use counting methods to solve problems. They will increase their understanding of probability and how to calculate probability as they use permutations and combinations to calculate probability (focus on MP.2, MP.7, and MP.8).

Concept 1: Explore Conditional Probability

Concept Overview: Students learn to describe a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events. They understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities. In addition, students develop an informal understanding of conditional probability, recognizing that conditional probability is dependent on a primary event that has already occurred. They apply this understanding to the Monty Hall problem. They also construct and interpret two-way frequency tables of data and then use the two-way tables to decide if events are independent and to approximate conditional probabilities.

Unit 10, Concept 1 Standards

- HSS-CP.A.1
- HSS-CP.A.2
- HSS-CP.A.3
- HSS-CP.A.4
- HSS-CP.A.5

Concept 2: Apply the Rules of Probability

Concept Overview: Students formalize their understanding of conditional probability and independence. They find conditional probabilities and use the addition and multiplication rules and interpret the probabilities. They learn to analyze decisions and strategies using probability concepts and use probability to make fair decisions.

Unit 10, Concept 2 Standards

- HSS-CP.B.6
- HSS-CP.B.7
- HSS-CP.B.8(+)
- HSS-MD.B.6(+)
- HSS-MD.B.7(+)

Concept 3: Examine Permutations and Combinations

Concept Overview: Students use counting methods to solve problems. In addition, they increase their understanding of probability and how to calculate probability as they use permutations and combinations to calculate probability.

Unit 10, Concept 3 Standards
- HSS-CP.B.9(+)

Unit 11: Piecewise Functions

Unit Overview: Students will depend on their previous work with writing linear equations to write and define piecewise functions. They will use their knowledge of key features and interpretations of graphs to explore other nonlinear function families, including piecewise, absolute value, and step functions (focus on MP.2, MP.4, MP.8).

Concept 1: Create and Analyze Piecewise Functions

Concept Overview: Students will use their previous work with writing linear equations to write and define piecewise functions. They will use their knowledge of key features and interpretations of graphs to explore other nonlinear function families, including piecewise, absolute value, and step functions. Students will extend their knowledge of interpreting nonlinear functions to compare properties of functions represented in a different way: algebraically, graphically, numerically in tables, and by verbal descriptions.

Unit 11, Concept 1 Standards
- HSA-CED.A.2
- HSF-BF.B.3
- HSF-IF.B.5
- HSF-IF.C.7b
- HSF-IF.C.9