

Unit 1: Recursive, Explicit, and Inverse Functions

Unit Overview: Students will write recursive and explicit formulas for arithmetic and geometric sequences, and they will use these formulas to solve problems. They will relate arithmetic sequences to linear functions and geometric sequences to exponential functions. They will develop the ideas and notation for composite functions and apply this to the relationship between functions and their inverses. Students will investigate inverses of functions, identifying graphical and algebraic representations by analyzing ordered pairs, domains, and ranges. They will explore the effects of translations, reflections, stretches, and compressions on the graphs of functions. Students will predict the effects of changes in a function rule on the graph of a function and sketch the graphs of transformations of functions. (Focus on MP.2, MP.7, MP.8)

Concept 1: Explore Recursive Functions

Concept Overview: Students discover how to write recursive and explicit formulas for arithmetic and geometric sequences and use these formulas to solve problems. They relate arithmetic sequences to linear functions and geometric sequences to exponential functions. Students also write and apply the formulas for the sums of finite arithmetic and geometric series and for infinite geometric series when the sum exists.

Unit 1, Concept 1 Standards

- HSF-BF.A.1
- HSF-BF.A.1a
- HSF-BF.A.2
- HSF-IF.A.3
- HSF-LE.A.2
- HSA-SSE.B.4

Concept 2: Explore Inverse Functions

Concept Overview: Students develop the ideas and notation for composite functions and apply this understanding to the relationship between functions and their inverses. They explore inverses of functions, identifying graphical and algebraic representations by analyzing ordered pairs, domains, and ranges. Students garner insight about the relationship between functions and their inverses while investigating how to restrict the domain of a quadratic function to create an inverse function.

Unit 1, Concept 2 Standards

- HSF-BF.B.4
- HSF-BF.B.4a
- HSF-BF.B.4c
- HSF-BF.A.1c

Concept 3: Explore Function Transformations

Concept Overview: Students explore the effects of translations, reflections, stretches, and compressions on the graphs of functions. They predict the effects of changes in a function rule on the graph of a function and sketch the graphs of transformations of functions.

Unit 1, Concept 3 Standards

- HSF-BF.B.3
- HSF-IF.C.7b
- HSF-BF.A.1b

Unit 2: Exponents and Logarithms

Unit Overview: Students will expand upon their knowledge of exponential functions, equations, and graphs. They will learn to solve exponential equations by inspection and analyze graphs of exponential functions. Students will interpret the expressions or parameters for an exponential function in terms of the situation that it models. They will apply their understanding of arithmetic and geometric sequences and series to build and compare linear and exponential function models. They will use exponential equations to model real-world problems including population and compounding interest, building their understanding of exponential parameters in context. Students will study exponential patterns within the compound interest formula, compounding the interest in smaller and smaller intervals to develop the number e . They will graph exponential and logarithmic functions and note the key features of the graphs. Students will learn to use the inverse relationship between exponential and logarithmic functions to solve real-world problems, interpreting appropriate parameters for these functions in terms of their contexts. (Focus on MP.1, MP.2, MP.8)

Concept 1: Represent Exponential Functions

Concept Overview: Students build on their understanding of exponential functions, equations, and graphs. They learn to solve exponential equations by inspection and analyze graphs of exponential functions. Students study the exponential function family, understanding how a change in parameters affects the graph of a function. They graph simple cases by hand and more complex cases using technology. They learn to interpret the expressions or parameters for an exponential function in terms of the situation that it models. Students apply their understanding of arithmetic and geometric sequences and series to build and compare linear and exponential function models.

Unit 2, Concept 1 Standards

- HSA-REI.D.11
- HSF-IF.C.7
- HSF-IF.C.7e
- HSF-BF.A.2
- HSF-LE.B.5
- HSF-IF.C.9

Concept 2: Model Exponential Growth and Decay

Concept Overview: Students use exponential equations to model real-world problems including population and compounding interest, building their understanding of exponential parameters in context. They investigate exponential patterns within the compound interest formula, compounding the interest in smaller and smaller intervals to develop the number e .

Unit 2, Concept 2 Standards

- HSA-CED.A.1
- HSF-LE.A.1c
- HSF-LE.B.5
- HSF-BF.A.1b
- HSF-IF.C.8
- HSF-IF.C.8b
- HAS-REI.D.11
- HSF-LE.A.2

Concept 3: Discover and Analyze Logarithms

Concept Overview: Students learn to graph exponential and logarithmic functions and note the key features of the graphs. They explore the inverse relationship between exponents and logarithms and use this understanding to solve problems. They also learn to solve exponential equations by using properties of logarithms.

Unit 2, Concept 3 Standards

- HSF-IF.C.7e
- HSF-BF.B.4a
- HSF-BF.B.5
- HSF-BF.B.4c

Concept 4: Apply Logarithmic Functions

Concept Overview: Students will use the inverse relationship between exponential and logarithmic functions to solve real-world problems, interpreting appropriate parameters for these functions in terms of their contexts. They express logarithmic equations as equivalent exponential functions and evaluate, using technology when appropriate.

Unit 2, Concept 4 Standards

- HSF-BF.A.1a
- HSF-BF.B.5
- HSF-LE.A.4
- HSF-LE.B.5

Unit 3: Real and Complex Solutions

Unit Overview: Students will improve upon their understanding of inverses as they find inverses of quadratic and cubic functions. They will graph radical functions, explore transformations of these functions, and compare their properties. Students will continue to apply the laws of exponents as they work with radical functions and equations. They will explore a new subset of real numbers and how they behave in the system of numbers. Students will recognize the need for this expansion of the system through their study of quadratic equations and solutions that are not represented as real numbers in the quadratic formula or x -intercepts on the graph. (Focus on MP.3, MP.7, MP.8)

Concept 1: Analyze Radical Functions

Concept Overview: Students build on their understanding of inverses as they find inverses of quadratic and cubic functions. They graph radical functions, explore transformations of these functions, and compare their properties. Students continue to apply the laws of exponents as they work with radical functions and equations. In addition, they write and solve radical equations in one variable, including those with extraneous solutions.

Unit 3, Concept 1 Standards

- HSN-RN.A.1
- HSN-RN.A.2
- HSA-CED.A.1
- HSA-REI.A.2
- HSF-IF.C.7b
- HSF-IF.C.9
- HSF-BF.B.3
- HSF-BF.B.4a
- HSF-BF.A.1b

Concept 2: Determine Complex Quadratic Roots

Concept Overview: Students discover a new subset of real numbers and how they behave in the system of numbers. They recognize the need for this expansion of the system through their study of quadratic equations and solutions that are not represented as real numbers in the quadratic formula or x -intercepts on the graph. For the first time, students discover a number that cannot be plotted on the real number line. Although complex numbers require a new model, students discover that operations on complex numbers are a natural extension of real number operations. The commutative, associative, and distributive properties guide early explorations in complex number operations.

Unit 3, Concept 2 Standards

- HSN-CN.A.1
- HSN-CN.A.2
- HSN-CN.C.7

Unit 4: Conic Sections

Unit Overview: Students will use prior knowledge to graph the equations of circles and ellipses, and they will identify key features of the graphs. They will discover the relationship between the points on an ellipse and the foci of the ellipse, and they will write the equations of ellipses given key information. Students will build on their earlier work with equations in two variables by graphing the equations of hyperbolas and identifying key features of the graphs. Students will learn how to recognize and analyze an equation of a parabola. Key features will be determined from the equation and validated through graphing. Students will derive the equation of a parabola using the graph, distance formula, and algebraic manipulation through several tests and the general form. They will investigate the relationships among different conic sections and extend their understanding of individual conics, discovering how to discern between a variety of algebraic representations and their characteristics. (Focus on MP.4, MP.5, MP.7)

Concept 1: Analyze Graphs and Equations of Circles and Ellipses

Concept Overview: Students build on their past learning with equations in two variables by graphing the equations of circles and ellipses and identifying key features of the graphs. They write the equations of circles given key information, and they complete the square to determine key information about circles given their equations. Students also discover the relationship between the points on an ellipse and the foci of the ellipse, and they write the equations of ellipses given key information.

Unit 4, Concept 1 Standards

- HSG.GPE.A.1
- HSG.GPE.A.2
- HSG.GPE.A.3

Concept 2: Analyze Graphs and Equations of Hyperbolas

Concept Overview: Students use what they learned in their earlier work with equations in two variables by graphing the equations of hyperbolas and identifying key features of the graphs. They also explore the relationship between the points on a hyperbola and the foci of the hyperbola, and they write the equations of hyperbolas given key information.

Unit 4, Concept 2 Standards

- HSG-GPE.A.3

Concept 3: Analyze Graphs and Equations of Parabolas

Concept Overview: Students learn how to recognize and analyze an equation of a parabola. Key features are determined from the equation and validated through graphing. Then, students derive the equation of a parabola using the graph, distance formula, and algebraic manipulation through several tests and the general form. This equation is applied in algebraic and real-world situations during the investigations.

Unit 4, Concept 3 Standards

- HSG-GPE.A.2

Concept 4: Compare Conic Equations

Concept Overview: Students investigate the relationships among different conic sections. They extend their understanding of individual conics, now discovering how to discern between a variety of algebraic representations and their characteristics. Students also explore solving systems of equations that include conic sections.

Unit 4, Concept 4 Standards

- HSA-REI.C.7
- HSA-REI.D.11
- HSG-GPE.A.1
- HSG-GPE.A.2
- HSG-GPE.A.3

Unit 5: Multivariate Equations and Inequalities

Unit Overview: Students will focus on solving systems of inequalities by various methods, including graphing. They will write equations and inequalities modeling constraints and use the graphs of the inequalities to find optimal solutions. Students will deepen their understanding of systems of equations by including the solving of systems consisting of linear and quadratic equations, and they will extend their understanding of solving a linear system by graphing to encompass simple polynomial, rational, absolute value, exponential, and logarithmic functions. (Focus on MP.1, MP.2, MP.4)

Concept 1: Investigate Linear Systems

Concept Overview: Students extend their ability to solve systems of inequalities by various methods, including graphing. They apply their knowledge to real-world situations that require a greater degree of scrutiny than they have applied previously; students write equations and inequalities modeling constraints and use the graphs of the inequalities to find optimal solutions.

Unit 5, Concept 1 Standards

- HSA-CED.A.2
- HSA-CED.A.3
- HSA-REI.C.6
- HSN-Q.A.2
- HSN-Q.A.3

Concept 2: Solve Nonlinear Systems

Concept Overview: Student build on their understanding of systems of equations to include solving systems consisting of linear and quadratic equations, and they extend their understanding of solving a linear system by graphing to encompass simple polynomial, rational, absolute value, exponential, and logarithmic functions.

Unit 5, Concept 2 Standards

- HSA-REI.C.7
- HSA-REI.D.11

Unit 6: Polynomial Expressions and Equations

Unit Overview: Students will write polynomials in different forms, perform operations with polynomials, and interpret parts of polynomial expressions. They will develop an understanding of the relationship between the zeroes of a polynomial function and the factors of its function rule. They will write polynomials in different forms to identify roots of the related equations. Students will apply the rational root theorem, polynomial long division, synthetic division, and factoring to identify rational roots and the quadratic formula to identify irrational and complex roots. They will build on their earlier work with other types of functions as they identify and interpret key features of polynomial functions. They will sketch graphs of polynomial functions using characteristics identified from the equations of the functions. (Focus on MP.6, MP.7, MP.8)

Concept 1: Operate with Polynomials

Concept Overview: Students use their prior knowledge of quadratic and other types of expressions as they write polynomials in different forms, perform operations with polynomials, and interpret parts of polynomial expressions. They develop an understanding of the relationship between the zeroes of a polynomial function and the factors of its function rule. Students also apply factoring techniques learned when working with quadratic expressions to help them factor polynomials and identify zeroes of polynomial functions. In addition, students learn how they can use Pascal's triangle to help them expand powers of binomials.

Unit 6, Concept 1 Standards

- HSA-SSE.A.1b
- HSA-SSE.A.2
- HSA-APR.A.1
- HSA-APR.B.2
- HSA-APR.B.3
- HSA-APR.C.4
- HSA-APR.C.5

Concept 2: Explore Polynomial Factors

Concept Overview: Students build on their work with quadratic functions as they develop an understanding of the relationship between the zeroes of a polynomial function and the factors of its function rule. They write polynomials in different forms to identify roots of the related equations. Students apply the rational root theorem, polynomial long division, synthetic division, and factoring to identify rational roots and the quadratic formula to identify irrational and complex roots.

Unit 6, Concept 2 Standards

- HSA-SSE.B.3
- HSA-APR.B.2

Concept 3: Analyze Polynomial Functions

Concept Overview: Students expand on their earlier work with other types of functions as they identify and interpret key features of polynomial functions. They also sketch graphs of polynomial functions using characteristics identified from the equations of the functions. In addition, students write polynomial equations and develop polynomial functions to model and solve real-world problems.

Unit 6, Concept 3 Standards

- HSA-APR.B.3
- HSA-CED.A.1
- HSF-IF.B.4
- HSF-IF.B.6
- HSF-IF.C.7c
- HSS-ID.B.6a

Unit 7: Rational Expressions and Equations

Unit Overview: Students will use mathematics concepts learned previously to enable them to explore how operations with rational expressions are analogous to operations with rational numbers. They will use their knowledge of writing rational numbers in different forms to help them write rational expressions. In addition, students will apply their understanding of factoring to identify and generate equivalent rational expressions. They will build on their earlier work with rational expressions as they write rational equations and inequalities in one variable in order to model real-world situations. In addition, they will apply what they have learned about algebraic properties to solve rational equations and inequalities, including those that give rise to extraneous solutions. (Focus on MP.1, MP.2, MP.7)

Concept 1: Develop Rational Expressions

Concept Overview: Students build on mathematics concepts learned previously, enabling them to explore how operations with rational expressions are analogous to operations with rational numbers. They use their knowledge of writing rational numbers in different forms to help them write rational expressions in different forms. In addition, students apply their understanding of factoring to identify and generate equivalent rational expressions.

Unit 7, Concept 1 Standards

- HSA.SSE.A.2
- HSA.APR.D.6
- HSA.APR.D.7

Concept 2: Solve Rational Equations

Concept Overview: Students use their earlier work with rational expressions as they write rational equations and inequalities in one variable in order to model real-world situations. In addition, they apply what they have learned about algebraic properties to solve rational equations and inequalities, including those that give rise to extraneous solutions.

Unit 7, Concept 2 Standards

- HSA-CED.A.1
- HSA-REI.A.2

Unit 8: Rational Functions

Unit Overview: Students will use what they have learned about functions and rational expressions as they explore rational functions. They will graph rational functions and identify key features of the graphs. They will determine and interpret the mathematical and reasonable domains and ranges of rational functions. Students will learn to write simple rational functions and use the functions to investigate related one-variable equations and inequalities. They will analyze one-variable rational inequalities by writing functions for the expressions on either side of the inequalities. (Focus on MP.2, MP.4, MP.7)

Concept 1: Represent Rational Functions

Concept Overview: Students apply what they have learned about functions and rational expressions as they explore rational functions. They graph rational functions and identify key features of the graphs. They determine and interpret the mathematical and reasonable domains and ranges of rational functions. They also solve rational equations graphically or by using tables to find approximate solutions.

Unit 8, Concept 1 Standards

- HSF-IF.C.7d
- HSF-IF.B.5
- HAS-REI.D.10

Concept 2: Compare Rational Functions

Concept Overview: Students learn to write simple rational functions and use the functions to investigate related one-variable equations and inequalities. Students analyze one-variable rational inequalities by writing functions for the expressions on either side of the inequalities. Using the zero(s) and the asymptote(s) of the functions, students determine solutions for the original inequalities.

Unit 8, Concept 2 Standards

- HSA-CED.A.1
- HSA-REI.D.11

Unit 9: Trigonometry

Unit Overview: Students will focus on the relationships between arc lengths on the unit circle and radian measures of angles. They will learn how to convert between radians and degrees, and they will calculate arc lengths for any given angle measure and radius. Students will learn about the unit circle and how it is used to find the trigonometric values for all real numbers. They will use the Pythagorean theorem for ordered pairs on the unit circle to prove the Pythagorean identity, which they will use to find trigonometric quantities. Students will analyze the key features of trigonometric graphs, including period, midline, and amplitude, and they will compare transformations for these functions. They will use trigonometric functions to model real-world, periodic phenomenon. Students will relate the

features of graphs of the trigonometric functions to characteristics of real-world scenarios and write equations describing real-world phenomena. (Focus on MP.2, MP.7, MP.8)

Concept 1: Explore Angle Measures

Concept Overview: Students learn the relationships between arc lengths on the unit circle and radian measures of angles. They learn how to convert between radians and degrees and calculate arc lengths for any given angle measure and radius.

Unit 9, Concept 1 Standards

- HSF-TF.A.1

Concept 2: Represent Trigonometric Functions

Concept Overview: Students focus on the unit circle and how it is used to find the trigonometric values for all real numbers. Students invoke the Pythagorean theorem for ordered pairs on the unit circle to prove the Pythagorean identity, which they use to find trigonometric quantities. Students analyze the key features of trigonometric graphs, including period, midline, and amplitude, and they compare transformations for these functions.

Unit 9, Concept 2 Standards

- HSF-IF-C.7e
- HSF-TF.A.2
- HSF-TF.A.3
- HSF-TF.C.8

Concept 3: Apply Trigonometric Relationships

Concept Overview: Students develop their knowledge of trigonometric functions to model real-world, periodic phenomenon. They relate the features of graphs of the trigonometric functions to characteristics of real-world scenarios and write equations describing real-world phenomena. They analyze real-world data in various forms and convert from one form to another when appropriate.

Unit 9, Concept 3 Standards

- HSF-IF.C.9
- HSF-TF.B.5

Unit 10: Probability

Unit Overview: Students will describe a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events. They will develop an informal understanding of conditional probability, recognizing that conditional probability is dependent on a primary event that has already occurred. Students will also construct and interpret two-way frequency tables of data and then use the two-way tables to decide if events are independent and to approximate conditional probabilities. They will formalize their understanding of conditional probability and independence. Students will find conditional probabilities, use the addition and multiplication rules, and interpret the probabilities. They will learn to analyze decisions and strategies using probability concepts and use probability to make fair decisions. (Focus on MP.1, MP.2, MP.4)

Concept 1: Explore Conditional Probability

Concept Overview: Students learn to describe a sample space using characteristics of the outcomes or as unions, intersections, or complements of other events. They understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities. In addition, students develop an informal understanding of conditional probability, recognizing that conditional probability is dependent on a primary event that has already occurred. They apply this understanding to the Monty Hall problem. They also construct and interpret two-way frequency tables of data and then use the two-way tables to decide if events are independent and to approximate conditional probabilities.

Unit 10, Concept 1 Standards

- HSS-CP.A.1
- HSS-CP.A.2
- HSS-CP.A.3
- HSS-CP.A.4
- HSS-CP.A.5

Concept 2: Apply the Rules of Probability

Concept Overview: Students formalize their understanding of conditional probability and independence. They find conditional probabilities, use the addition and multiplication rules, and interpret the probabilities. They learn to analyze decisions and strategies using probability concepts and use probability to make fair decisions.

Unit 10, Concept 2 Standards

- HSS-CP.B.6
- HSS-CP.B.7
- HSS-CP.B.8
- HSS-MD.B.6
- HSS-MD.B.7

Unit 11: Data Modeling

Unit Overview: Students will expand their understanding of sampling by exploring and evaluating a variety of sampling methods. They will investigate the use of experiments in testing hypotheses and analyze the ability of experiments to establish causal relationships. Students will focus on the characteristics of a normal distribution. They will use statistical pictures and numerical measures to analyze normally distributed data. Students will investigate the empirical rule and its role in describing normal distributions. (Focus on MP.3, MP.4, MP.5)

Concept 1: Collect, Analyze, and Interpret Statistical Data

Concept Overview: Students build upon their understanding of sampling by exploring and evaluating a variety of sampling methods. Students use simulations to develop an understanding of variability in sampling and learn to calculate the margin of error for data collected from a sample. Students investigate the use of experiments in testing hypotheses and analyze the ability of experiments to establish causal relationships.

Unit 11, Concept 1 Standards

- HSS-IC.A.1
- HSS-IC.A.2
- HSS-IC.B.3
- HSS-IC.B.4
- HSS-IC.B.5
- HSS-IC.B.6

Concept 2: Explore Normal Distributions

Concept Overview: Students focus on the characteristics of a normal distribution. They use statistical pictures and numerical measures to analyze normally distributed data. Students investigate the empirical rule and its role in describing normal distributions. They also use z-scores, in the context of real world problem solving, to analyze individual data points.

Unit 11, Concept 2 Standards

- HSS-ID.A.4