THE SENSITIVITY ANALYSIS OF THE FHA TECHNIQUE OF HOUSING MARKET ANALYSIS: THE EFFECT OF RATIOS AND VARIABLES, AND THEIR PERTURBATIONS ON FAMILY AND ELDERLY DEMAND ESTIMATES

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Abstract

This paper applies the sensitivity analysis to the Federal Housing Administration's (FHA) techniques for reviewing family and elderly housing market conditions through the perturbations of its fundamental parameters. Models are presented for estimating the demand on the family and the elderly housing, and empirical illustrations for the practitioners of a housing market analysis are indicated. We elevate the FHA technique from a descriptive level within an engineering paradigm that did not change for over half a century to an evolutionary statistical model. Housing market analysts will find the model a useful supplement to their regular operations in terms of the diagnostic checks for their estimates.

Introduction

In their celebrated article, Klein and Kosobud (1961, 178) had pioneered the use of both "fundamental parameters" and "simple ratios" in economic theory. Housing economists had been following the FHA techniques by focusing on such parameters and ratios in their housing demand models for a long time. We find that both these studies emphasized caution in the use of ratios. In the former, the authors stated that ". . . stability or plainly systematic variation in ratios must be found in order to enhance their usefulness." In the latter, the guide warned that while participation rates based on labor force or employment data ". . . are both useful in estimating and projecting population they have some shortcomings . . . primarily from the unavailability and limitations of pertinent locality data and from the adjustments that may be required to reflect the level and trends of unemployment" (FHA 1970, 33).

As an example of the use of ratios, the FHA guide, most likely, has outdone any other framework in optimizing the number of ratios to be used in a housing analysis. Besides the participation rates, it prescribed the use of household sizes, institutional populations, demolition rates, and the rate of construction for the single and multi-family units. The guide requires inputs for at least two census years, a current period, and a forecast period, and it uses long-term natural growth rates for population, households, labor force, employment, and construction permits. These rates are kept at a high level, perhaps with an eye for simulating "golden" growth scenarios where a combination of variables would move simultaneously in the same trend and cycle. At a local level, it is apparent that market analysts would be time-constrained to perform "stability" and "variations" studies for these parameters. Hence they would be prevailed upon to accept these studies on faith as uncontroversial stable ratios and parameters. A better way for practitioners in the field to overcome such problems would be to use a less time-consuming method that could enhance their model's estimation and predictive performance. This paper provides one such method, namely, the simulation of stable parameters and normed variables within the FHA model for family and elderly market analysis.

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The Models

The FHA guide subdivides housing markets into family, elderly, and military, in terms of their relative price, rent, size and other measurements. Both the family and the elderly markets have a rental component. Traditionally, the family market has rentals for both, the family and the independent living adult population between the ages of 45 to 62. The elderly rental component, which we presume to target (persons 62 years of age and over) are mostly housed in either independent living, Retirement Service Centers (RSCs), Assisted Living (ALF), Board and Care (BC), Nursing Homes (NH), and Alzheimer's (ALZ) facilities. The population housed in RSCs is characterized by an "at risk" criterion, i.e., persons 62 and over who can perform less than three activities of daily living (ADLs) or instrumental activities of daily living (IADLs) with difficulty. The population housed in the ALFs, BCs, NHs, and ALZs are primarily 62 years of age and over who have varying levels of care dependency, who can perform three or more ADLs or IADLs with difficulty.[1]

A major premise that we investigate is that while the FHA technique had a long gestation period, it has not evolved statistically. For instance, the family market is based on the premise that either rising labor force or an increase in population translates quantitatively into household growth, which in turn is a measure of demand for the new housing units. The quantity is then put into an income growth stream to estimate the effective demand.[2] For the elderly, we may rely on the living arrangements of the one-person households for the demand estimates, and use a survival methodology to age that population. We will look at some of the controversy in the aging cohort process below, particularly relating to Mankiw and Weil's (1989) work. A novel aspect of this paper is that it perturbs the assumptions of natural, psychological, or institutional drivers in order to assess market conditions. For the family market, we will conduct such an estimation in terms of the quantitative level. In this case the income level's influence is evaluated usually in a subsequent process of the analysis. We will also suggest a new approach to distribute family demand by rents and bedroom types in Appendix 1. For the elderly market, we will qualitatively estimate the age-income-qualified population that is integral to the processes of estimating the needs. Also, we offer new ways on how to combine ADL and IADL factors with the help of the new technology of belief functions in Appendix 2. The rest of this paper is divided into two major parts, addressing the family and the elderly markets, respectively.

I. Family Market

The first step in characterizing the FHA model is to postulate that the market demand and supply of housing units are quantity vectors that depend on some predetermined variables. The FHA handbook states that housing is ". . . a commodity in the physical sense; it is identified with and measured in terms of the dwelling units or housing units" (FHA 1970, 9). While the handbook failed to identify the structure of the housing market in the economic sense (monopolistic and/or oligopolistic competition), it allowed lots of theoretical and practical leeway for analysts to implement a study. For instance, besides the physical differentiation, it includes some differentiating characteristics and excludes others. The excluded characteristics are "service" and "rights" while housing type, quality, location, and sub-market determination such as tenure, price, rent, size, and other variables are taken into account. Since price is treated as an exogenously determined variable, it appears that the housing market is characterized by price-taking and product differentiation. In such a market we expect to find concessions, service packages, and other non-price variables dominating price competition.

A second step in determining the FHA model is to incorporate some market clearing operations. Assuming that the market is competitive, and that it is not in equilibrium, it is sufficient to show that there is an equilibrium price, P_e, such that if P_i is any other price, then, excess demand, $z(p) \rightarrow 0$ as $P_i \rightarrow P_e$. The market mechanism or an auctioneer yields that outcome, raising the price if $z(p) > 0$, lowering or not raising the price if $z(p) < 0$, and would never make price negative, or vary it when $z(p) = 0$ (Arrow and Hahn 1971). If one does not view the market as a competitive one, then similar adjustments can be envisioned. For instance, in the non-price area the vacancy rate, excess construction, concessions and other variables would be foremost in play. Equilibrium can then be restored with entry and exit operations.[3] A variation of the model may be obtained by arguing that if normal vacancy is a constant and supply is fixed for simplification, then the market vacancy rate and the rents are inversely related. This is more of an empirical than a theoretical proposition. Following Smith (1974, 479) let the vacancy level, be VL = (S)upply – (D)emand. Dividing through by (S)upply, and making (D)emand a function of (R)ents, then the changes in rents will be inversely related to the vacancy rates, that is, $VL/S = [1 - (D= f(R))]/S$.

The next step in the family model requires more complex dynamic analysis. For instance, if the price in the housing market is sticky, then a disequilibrium price can persist for a long period of time. Also, during fixed price episodes, the vacancy level may not equal the natural vacancy rate. Smith (1972, 228) reasoned that disequilibrium is more likely in the rental than the owner market. In the rental market, price reduction in vacant units is difficult to maintain and not passed on to other occupied rental units. Therefore, rents will be sticky downwards because the necessity to lower rents across the board in a price reduction attempt may adversely affect revenues. Another instance of complex dynamics occurs under the market conditions of excess supply or demand. If demand exceeds supply, then the buyers will go unsatisfied, and if supply exceeds demand, suppliers will go unsatisfied. Thus, it will be reasonable to argue that if the observed quantity is either D for demand or S for supply, it follows that the observed quantity will be $Q = \min(D,S)$ according to Fair and Jaffee (1972, 501). The analysis can be extended to the normal behavior in the mortgage market if we replace price by the mortgage rate. Then, we will not only be concerned with observing the traditional flexible adjustment to equilibrium in the market, but we should also take into account the state of the credit market equilibrium or disequilibrium. Sometimes excess credit will have an effect only in the subsequent period when the rate of interest adjusts, but will have no effect in the current period. On the other hand, tight credit will have an immediate effect (Ricks 1972, 226).

Another complex dynamic situation would be to evaluate the optimal stock for the owner or the renter. It is worth explaining the term optimal because some analysts perceive the housing market as having multiple equlibria. For example, it might appear that a market with a normal vacancy rate of 5 percent is in an equilibrium state, and the one with a vacancy rate of nearly 10 percent is in a disequilibrium state. This would imply that a market with a vacancy rate in the interval of 5 to 9.99 percent is in equilibrium. Under these circumstances, the housing market will have multiple equilibria. The FHA technique seeks an optimal equilibrium from a generalized welfare function point of view. Such an optimal point of view is obtained if we start from the FHA model that states that the ". . . aggregate of the individual scales of preference links dwelling units, types, quality, and location in an area." This statement however, disregards Arrow's impossibility theorem, which states that one cannot have such an aggregation without a dictator solving conflict situations. But modern investigators also take a pragmatic route around the impossibility theorem. Following Anas and Arnott (1991), let the consumer be endowed with an intertemporal utility function, $U = \beta U(\bullet)$. Let k = 1,.....,K be housing types of sub markets or bundles of housing. Let S^t_k be apartments in k, and assume that the consumer can choose and rent one apartment per year, given that all apartments are in the choice set. Furthermore, let the consumer have a contemporaneous budget where each year's income is spent on that year's consumption, and let the constraints include: 1) a budgetary allowance; 2) credit availability; 3) FHA's participation in the supply of adequate and affordable housing; 4) fiscal policy incentives (taxes in particular) and 5) market conditions, such as changes in rents that are inverse to demand. Then, the problem at hand is to find the optimal stock of rental housing and the optimal path of consumption of renters by solving this discrete maximization problem for the given constraints. A simple example of how these problems can be solved for the above assumptions can be illustrated by using the Cobb-Douglas utility function, $U_a = x_1^{0.25} x_2^{0.5}$ for one state of the economy, and $U_a = x_1^{0.25} x_2^{0.25}$ for another state of the economy. The first and second periods of endowment can be $e = (e^{a1}, e^{a2})$. Other considerations may include market tightness or softness, rational expectations that assume a probability distribution over the possible states of the economy, an intertemporal dependent utility function, and the inclusion of price and concession signals that the players may observe. We now focus on the specific cases in which the FHA model can include several of those phenomena through a simulation of changes in its parameters and variables.

Table 1 displays some typical assumptions we make in estimating quantitative demand for the

TABLE 1
Assumptions for the Modeling of Family Quantitative Demand

Ratios	Statistical Properties
Participation Rates:	
PR1990(Z)	normal((driver1990/POP1990),.2)
PR2000(Z)	normal((driver2000/pop2000),.2)
PRCURRENT(Z)	(((pr2000(z) – pr1990(z))/10) * yearsincecensus) + pr2000(z)
PRFORECAST(Z)	(((prcurrent(z) – pr2000(z))/10) * yrsfor) + prcurrent(z);
Driver :	
DRIVERTREND	= (((DRIVERCURRENT – DRIVER2000)) / YEARSINCECENSUS) * (YRSFOR)) + DEMAND DRIVERCURRENT
DRIVERFORECAST	= DRIVERTREND OR ANY OTHER FORECAST
Household Size:	
HHSIZE1990(z)	normal((HHPOP1990/HH1990"),.2)
HHSIZE2000(z)	normal((HHPOP2000/HH2000),.2)
HHSIZECURRent(z)	(((HHSIZE2000(z) – HHSIZE1990(z))/10) * yearsincecensus) + HHSIZE2000(z)
HHSIZEforeCAST(z)	(((HHSIZEcurrent(z) – HSIZE2000(z))/10) * yrsfor) + HHSIZEcurrent(z)
NonhouseholdPop:	
NONHHPOP1990	SCALAR = POP1990 – HHPOP1990
NONHHPOP2000	SCALAR = POP2000 – HHPOP2000
NONHHPOPCURRRENT	(((NONHHPOP2000 – NONHHPOP1990)/10) * yearsincecensus) + NONHHPOP2000
NONHHPOPFORECAST	(((NONHHPOPCURRENT – NONHHPOP2000)/YEARSINCECENSUS) * YRSFOR) + NONHHPOPCURRENT
Population:	
POPCURRENT(Z) =	NONHHPOPCURRENT + POPINHHCURRENT(Z);
POPCURRUSINGPRATE(Z)	(DEMAND("DRIVERCURRENT")/PRCURRENT(z))
POPCURR	SCALAR = POPCURR OR POPCURRPRRATE OR ANY ESTIMATE
POPFORECAST(Z)	DRIVERFORECAST/PRFORECAST(Z)
POPFORECASTTREND	(((POPCURR – POP2000))/YEARSINCECENSUS) * YRSFOR) + POPCURR
POPFORE	= POPFORETREND OR ANY OTHER FORECAST
Pop. In Households:	
POPINHHCURRENT(Z)	HHSIZECURRENT(Z) * CURRENTHOUSEHOLDSTOTAL
Household Forecast:	
HHFORECAST(Z)	= ((POPFORE – NONHHPOPFORECAST))/HHSIZEFORECAST(Z)
Parameters	
Building Permits	
SFCONSTPERIOD(Z)	UNIFORM(2,3)
MFCONSTPERIOD(Z)	UNIFORM(6,9)
MF2OWNERSH(Z)	NORMAL(.5,.01)
MF34OWNERSH(Z)	NORMAL(.25,.01)
Inventory:	
LOSTINVENTORY(Z)	UNIFORM(.002,.005)
Vacancies:	
CURRENTOWNERVACANCY(Z)	NORMAL(.03,.001)
CURRENTRENTERVACANCY(Z)	NORMAL(.095,.05)
NORMOWNVAC(Z)	NORMAL(.03,.01);
NORMRENVAC(Z)	NORMAL(.06,.01);
UCOWNERSCURRENT	SCALAR
UCRENTERSCURRENT	SCALAR

PR = participation rate, UC = under construction, Vac = vacancy, HH = households, SF = singlefamily, MF = multifamily, FORE = Forecast, Normal = Normal Distribution, Norm = The norm for normal market, and YRS = years.

owners and the renters.[4] Here we are using only uniform and normal distributions to perturb the stable parameters. The uniform distribution is a natural choice for those variables that have boundary values. For example, it takes two to three months to build a single-family home, and in the case of demolition, we typically do not know the actual number of units that are demolished but estimate it to be in the 0.002 to 0.005 interval of the inventory. The normal or t-distribution would be a natural choice for those estimated parameters and the variables that are associated with standard errors, such as the numerous census variables. In estimating the participation rate, for instance, we use the ratio of the census employment or labor force to the census population.[5] The first column of Table 1 presents the typical assumptions of the FHA technique under the headings of participation rates, driver, household size, non-household population, population, permits, and vacancies. The second column of Table 1 shows how we can specify a statistical distribution, which can be either normal or uniform, to the assumption or parameter described in column one. The number of replications is indicated by the variable "z." For instance, the first entry in the table indicates that the 1990 participation rate (PR1990) is defined as a two-parameter family distribution with a mean value equal to the 1990 census ratio of the driver to the population of the housing market area and a standard error of 0.2. The single family construction period (SFCONSTPERIOD) is defined as a uniform distribution that takes on a time period between two to three months. In general the variables are self-explanatory but where necessary the cues are noted in the footnote following the table.

In the statistical parlance we are engaged in an experiment. We draw equilikely values for our parameters and variables through the use of a normal or a uniform distribution for the experiment on theoretical grounds. The experimenter can follow other procedures such as historical data, "bootstrap" methods that select historical data at random, or the development of an approximate distribution from data at hand. But the FHA technique suggests looking at the ". . . trend in labor force and nonagricultural employment" (FHA 1970, 265) as it relates to local job situations. This can be achieved either through "extrapolation," "ratio," "school enrollment" or "housing units" methods. In Table 1, the years 1990 and 2000 PR ratios are set by the Census for those years. However, the current and forecast PR ratios are extrapolated from the Census trends. The contribution we make to this method is that the ratios are perturbed before extrapolation via the normal distribution.

After specifying the distribution, we also need to be aware of the distribution of the resulting parameter estimates for the purpose of setting up confidence intervals. The experiment involves filling 18 boxes with a random transaction before a point estimate of demand is obtained. The experiment is then replicated for, say, a number of times equal "Z = TOTALSIMULATION," which could be 15,000 times. Table 2 indicates how annual rental quantitative demand (annrent) may be estimated. The idea is to find an equation that relates all the variables and parameters to be replicated.

II. The Elderly Model

The elderly demand for housing posits at least four market segments. Studies of the elderly utility functions recognize the independent (in), intergenerational (to), and nursing home segments (nh). We need to expand that categorization in two ways. One way will include the assisted living (al), Alzheimer's (az), and the continuum (co) segments, and the other will include different states of the economy. The first introduces another choice set into the elderly compound utility function. The second proposition is concerned with at least two states of the economy, $\Omega = (\omega 1, \omega 2)$, corresponding to equilibrium or disequilibrium of the overall housing market. The states can be influenced by the overall position of the economy—recession or boom, as well as by the softness or tightness in the housing market. The conditions of these states determine the choices of the elderly.

Following Hoeger et al., (1996) and Anas and Arnott (1991), the elderly utility function can be represented as[6]:

$$\Sigma_{(j,k)} U(j,k) = U(U^{to}, U^{in}, U^{az}, U^{nh}, U^{al}, \cup U^{k}) \quad (1)$$

where:

to = living in an intergenerational setting (together), az = Alzheimer's facility, nh = nursing home, al = assisted living, U = utility, and $\cup$ = continuum of care which is a union of all the others allowing for some ks to be zero.

TABLE 2
Flowchart for the Estimate of Annual Rental Demand (Annrent)

Description	= Rentave	
Rental Demand	= SUM(Z,annrent(Z))/totalsim	Average Rental demand for total simulation
	= Sum(z,quantddrent(z) – excessrent-const(z) – Excessrenvac)	Less excess construction and vacancy from rental demand.
	= Sum(z, quantddrent(z) – (desiredcon-rent(z) – ucrenterscurrrent) – min(sum(z,excessrenvac(z)/totalsim),0)	Insert formula for excess construction and vacancies.
	= Sum(z, [totalhhgwth – owner-hhgwth(z)] + losrent(z) – (Desired-conrent(z) – Ucrenterscurrrent) – min(sum(z,excessrenvac(z)/totalsim),0)	Substitute totalhhgwth – ownerhhgwth(z) for Rental-hhgwtoh.
	= Sum(z, + [totalhhgwth – {total-hhgwth(z) * ((currownhh(z) – occown2000)/ (currtotalhh(z) – occ-total2000))}]	Substitute totalhhgwth(z) * ((currownhh(z) – occown2000)/(currtotalhh(z) – occtotal2000))
	= losrent(z) – (desiredconrent(z) – ucrenterscurrrent) – min(sum(z,excess-renvac(z)/totalsim),0)	for ownerhhgwth

Where: Sum = Summation sign, min = minimum, gwt = growth, hh = households, Uc = under construction, ave = average, sim = simulation, quant = quantity, con = construction, los = lost, vac = vacant.

k = 1,.....,K represents the housing types or market segments—in, az, nh, al, co.

j = agents making the decision = 1, 2 . . . = individual, family, government, and social service.

To get a feel for the equilibrium conditions, let the consumer be endowed with an intertemporal utility function. Let S_k be apartments in k. Let each consumer choose and rent one apartment per year, given that all apartments are in the choice set. Give each consumer a contemporaneous budget where each year's income is spent on that year's consumption. However, the statement about budget constraints needs several modifications. First, each market can take on its own constraints according to Hoeger et al., (1996). Next, we can add 1. A budget constraint, 2. A credit availability constraint, 3. An institutional constraint where FHA participates in the supply of adequate and affordable housing, 4. Fiscal policy constraints (taxes in particular), and 5. An operation constraint such as one that shows that a change in rent is inverse to demand. Then, we wish to find the optimal stock of rental housing and the optimal path of consumption for renters by solving this discrete maximization subject to constraints.

A first step in the direction of understanding the decision of elderly would be, for instance, to make $\Omega = \omega 1$, and let a Cobb-Douglas utility function for agent "one" in k = "assisted living" be $U_{\{1,a\}} = x_1^{\{.25\}} x_2^{\{.5\}}$. If $\Omega = \omega 2$, the utility function may be $U_{\{1,a\}} = x_1^{\{.25\}} x_2^{\{.25\}}$.

Next, we need to identify the players and their initial endowment. For instance, player "one's" endowment of goods 1 and 2 may be e = (e {1.g1}, e {1.g2}). It is usual to fix a player's first period endowment, and make its second period endowment a function of the state variable. For instance, agent "one" may have a period 1 endowment where $e^{\{1.g1\}} = 3$ and a period 2 endowment of $e^{\{1.g2\}}(s)$ (see McAllister 1990, 341).

Now we consider the state variables, say state one represents equilibrium; state two, disequilibrium. The rental market can also be characterized as tight or soft as indicated by a threshold vacancy level. If rational expectations are assumed, a probability distribution of the two states should be contemplated. The end process will yield a state and intertemporal dependent utility function, such as was entertained by Anderson and Sonnenschein (1982). Since price is considered within a joint distribution with the exogenous states and signals, no explicit price function is needed. The function takes non-negative real numbers from the joint price the state and signal distributions in the domain to a real number in the range:

$$U\ (j, k): R^{N+} \times \Omega \rightarrow R \qquad (2)$$

We also specify the observations of the players. Let S1 be player's "one" observation. Then S1 (ω1) will be taken as an indication that player "one" observed an equilibrium state. In that state of equilibrium, the information set, S1 = s1 (ω1) exists where $\omega 1 \varepsilon \Omega$ is used by player "one" to maximize its utility.

A solution to a naive agent problem will be to maximize a state, intertemporal, information dependent utility function subject to the constraints. A more sophisticated solution would be to take into account information contained in price and other non-price variables. There are also situations where a solution may not exist (see Kreps 1977 for an elementary example of nonexistence). Anderson and Sonnenschein (1982) do not assume that the agents know the joint distribution of price, state and information. They determine these variables empirically, and they use a fixed-point solution method to estimate equilibrium values.

In estimating the growth of elderly demand for housing and care, the literature agrees on the common circumstance that demand is most probably age rather than employment driven. We can age the population by survival rates, which are rates that describe the percent of an age group that will survive in a later period. We can make an assessment of mortality between the two census years. The difference between the two censuses that are not explained by mortality is attributed to the net-immigration effect.

We can then try to predict demand from the aged population following the dictum that "My parents and I are one." It resembles very much the approach of the General Equilibrium theorist who builds up large-scale economies by classifying people into types based on the similarity or differences in their preferences and endowments (Debreu and Scarf 1963). This approach allows comparative static analysis between 24 million persons in the 20–30 cohort in the 1960's with the 44.6 millions in the same age group during the 1980's, as reported by the U.S. Census Bureau. The possible predictions within such a framework can be illustrated as: two elderly persons with the same endowment but different preferences would require different types of elderly housing. Similarly, two persons with the same preferences but different endowments would require different types of housing. The number of people in a certain age group, along with information on their preferences and endowments, is an indication of the demand for certain types of housing.

The traditional model put forth by Mankiw and Weil (1989) created quite a stir in the literature. In naïve form, their model specified supply and demand equations both as a function of price, and by allowing for an age drift variable in addition to price in the demand equation. While a student of econometrics will quickly spot that the demand equation is under-identified, here the authors' objective is not the identification. Rather, they are interested in the reduced form when solving for price, which enables the heroic prediction. Due to the Baby-Bust in the 1970's housing prices declined by 47 percent. While this prediction was criticized by a number of authors (Engelhardt and Poterba 1991; Hamilton 1991), we are not directly concerned with it other than with the possibility of looking at demand from the perspective of age. While Mankiw and Weil had not mentioned the elderly, we can argue by a simple extension of their argument, that their age proposition would also hold for the elderly. For instance, the front end of the baby boomers of 1946 will be 65 in 2011, and the tail end of the baby boomers in 1964 will be 65 in 2029, and to that pertains the story of the demand for elderly housing predicted from age alone. The opposing view is more evolutionary. It does not take the static look at cohorts over time as the traditional view does. Rather, it considers changes in taste and preferences of the child, as it approximates the age of the parents, and as it is influenced by other variables than age, such as income, family support, and wealth broadly defined. The influences of these variables have been controversial. What we propose is a simulation of the elderly market, pulling from various heuristic studies proposed in the literature. In the next section we will show how demand can be simulated from various points of view in the literature.

II. A Heuristic Study for Elderly Demand Estimates

Many practitioners of empirical studies try to find practical applications of the above model. The mortgage insurers for instance, including FHA, want to predict demand for assisted living facilities. The lenders want to assess the market risk of a project, or other market conditions for additional units within a price range. To enable a feel for demand they take an engineering approach to demand that has guided economic theory on the one hand, and is based on generally available data, on the other hand. This section develops a basic framework to be applied to some markets across the U.S.

II.A.1. Hypothesis I—Importance of the Headship Rates:

Normally each household is represented by a person who heads it. In early age cohorts, individuals choose to remain in their current housing arrangement to head a household. This could mean that their consumption pattern is explained either by the independent (in) or togetherness (to) hypothesis above. In later cohorts, they must move into an elderly facility. It is generally assumed that they would choose an assisted living or board and care if they have need for three or more ADL, or a nursing home if their health deteriorates, or an ALZ facility for dementia related illnesses. Montgomery (1996, 174) examined the first half of this hypothesis. She found that only per capita income and lagged headship rates were significant in explaining headship rates for the 65 plus population. We may use the equation that the formation of headship rates for the 65 plus is:

$$\begin{aligned} \text{Headship} = {} & -0.083 - 0.026 \text{ Rental Price} + \\ & 15.07 \text{ Per Capita Income} + \\ & 0.693 \text{ Headship}(t-1) \end{aligned} \tag{3}$$

Headship rates are important because one-person head of households are most likely to demand assisted care facilities. It tells us that we should analyze the income distribution of this target population for the major share of demand.

II.A.2. Hypothesis II—Income-Equity Hypothesis:

The elderly receive income from pensions, social security, equity, and family. Most demand for assisted living occurs after they have made their decision to retire, which is based on their utility function that depends on factors such as income, wealth, state of health, and cost of home maintenance which includes insurance, heat and utilities for owners, and shelter plus utility and heat for renters.

Several income hypotheses are involved in the demand for elderly care—absolute, relative, life-cycle, permanent income, and full-income. The absolute income hypothesis utilizes current income only as a base for consumption and is based on the work of J. M. Keynes, the founder of macroeconomics. The relative income hypothesis assumes a peak level income in the past and explains well the consumption pattern of individuals over the swings in the economy. The life-cycle theory explains elderly consumption patterns over the age cohorts (Feinstein and McFadden 1987, 11). The permanent income hypothesis holds that consumption is made from lifetime resources. Both the life-cycle and permanent hypotheses assume that consumers would consume uniformly over time. The full income hypothesis assumption is more akin to intergenerational consumption decision-making. For example, Fuchs thinks "full income" is "the sum of personal income and health care expenditures not paid from personal income" (Fuchs 2000, 61). It also treats government expenditures as an insurance policy.

Problems arise in terms of how to estimate the present value of all income over all time periods. The Nobel laureate Milton Friedman (1957), for instance, suggests a complicated polynomial equation in his permanent income hypothesis to accomplish such a calculation. We fall back on borrowing against equity that assumes a capital market that allows consumers to smooth expenditures over their lifetime transitory and permanent income (Dornbusch et al., 1998, Ch. 13). But the idea that consumers will draw down their equity is not without problems. Households may prefer to bequeath their equity to children—a large part of the savings in the U.S. comes from bequests. The market also segments into cases where they draw down equity between current and permanent income (Megbolugbe et al., 1999, 290).

There are several methods for estimating growth in income. It can be measured by a trend line: Fit = 0.0447 – 0.000535*T (Montgomery 1996, 190). HUD estimates median income growth annually, which some practitioners use as a low-end estimate of elderly growth because it includes family income as well. Because we have seen elderly income growing faster than family or household income between census years, a practical consideration is to consider whether the elderly income has doubled or tripled for some cohorts since the last census was taken. This methodology is realistic because such multiple increases have been observed for the local areas between actual census dates, but it is not found to be a common practice. The simulation model is properly chosen in this instance because there is an equally likely probability for the income of an estimated demand to fall between the low end set of HUD adjustment factors, and the upper end set of its multiples.

II.A.3. Hypothesis III—Cohort-Linked of Income Eligible Living Alone Population with ADLs factors:

Demand is derived demographically, from an index of age expected consumption. This model was advanced by Mankiw and Weil (1989), and modified by Pitkin and Myers (1994). Demand is the sum product of specific age consumption rates (c_a) times the population in that age group (P_{at}). For example, demand for 62 plus elderly for ALF can be specified as:

$$\text{Demand (ALF)} = c(62\text{–}65) \times \text{Pop}(62\text{–}65) + c(65\text{–}70) \times \text{Pop}(65\text{–}70) \text{ etc.} \tag{4}$$

The c(•) is a dependent function of the need (ADLs and IADLs) factors. The population would be represented through the one-person income eligible persons in the specific age group. Conceptualizing on the need factors has been the weakest link in the estimation process. A number of practitioners look at an NCHS study of ADLs, which used the 1986 National Health Interview Survey (NHIS) data. NCHS reported two sets of factors of persons performing three or more ADLs with difficulty. For all living-arrangements, the study found that 6.3 percent of the 65–74, 13.7 percent of the 75–84, and 24.3 percent of the 85 plus cohort perform 3 or more ADLs with difficulty (NCHS 1993, 29). Further, these three or more ADL percent responses were broken down to the living-alone arrangements for only two cohorts. For the 65–74 cohort, males living-alone was 4.2 percent, and females living-alone was 5.3 percent. These numbers are close to the 6.3 percent for all living-arrangements noted above. For the 75 and over cohort, the male percent was 13.6, and the female percent was 14.5. These figures are close to the 13.7 percent for all living arrangements. It is only in the 85 plus cohort that the use of the living-alone percent of 13.7 would make very conservative estimates (NCHS 1993, 38). Although the survey was taken in 1993, the data was for 1986, and is still used because the NHIS decided to have a supplemental survey that year for the elderly. A newer study by Hawes et al., (1999, Tables B4, B5), *National Study of Assisted Living for the Frail Elderly* (NSA) based on a universe of 11,459 ALFs, with 611,300 beds, and 521,500 residents reported that 23.6 percent of the residents received assistance with three or more ADLs. If the two surveys can be compared, the 23.6 percent is close to the 23.3 percent of the NCHS survey.

A naïve approach in the application of those statistics would be to treat them as incident rates. This might refer to every ten out of a 1,000 persons needing Alzheimer's or dementia care. The FHA handbook sanctions this incident rate method, which counts the number of persons in a population that have the incident present. However, the solution to this problem is not straightforward. The NSA study struggled to put together the data from heterogeneous sources in constituting the universe. Even if we are willing to accept its results as statistically sound, we will still run into major problems. One problem with the incident factor is that it is not available, say, at the per thousand population, so that we cannot follow the estimation pattern of listing our eligible population and then applying the incident factor as suggested by the FHA guide for NCHS (FHA 1970, 282–283). For instance, The NCHS incident rate of 24.3 for the 85 plus residents is representative of 126,725 (521,500x .243) persons. Market analysts are dealing with the areas such as counties, cities, 5-mile centroids, latitude and longitude, or an amalgam of census tracts or blocks that come in with very small income-eligible one-person populations. In some new growth areas, the one-person eligible population is only prospective.

On account of this problem with incident factors as discussed above, a practitioner may seek some local criteria to base the C(•) factors. For most local markets, the 1990 Census provides limitations relating to self-care, mobility, and their levels cojoined by cohorts. One way to adopt these limitation factors to the ALF estimate would be to merge NCHS and NSA information with the local mobility and self care data. The traditional approach to such a problem looks for answers from a Bayesian statistical point of view where information for single need factors is easily combined. In such a setting, it is difficult to distinguish a belief from a value. For instance, in real estate analysis a common practice is to write off overbuilding as "speculative" investment, without an attempt to assess the beliefs and values of the investor. We have decided to distinguish belief from the values by the use of a technique called "belief function." This model is developed in a Transferable Belief Model (TBM) in Appendix 2. TBM offers a non-probabilistic way to transfer information from one set of beliefs to another. If the estimate and the forecast of demand has been carried out to a level of an effective demand, then the "degree of belief" about the need factors can become a way of choosing them to complete the demand estimate. The literature on TBM is vast, and its computing algorithm is complex, which may explain why it is not used more widely. It is clear that the "belief function" technique uses mathematical probability in a more general way than Bayesian theory. It uses numerical degrees of support based on the evidence, and then it combines them based on this evidence. It is a tool for dealing with imperfect information, imprecision and uncertainty, without using probability (see Appendix 2).

For this paper, we would draw the NCHS factors from a normal distribution using the NSA standard error of 2.6 at the 95 CL, i.e., normal (0.243, 0.026) for the 85 plus cohort, and normal (0.137, 0.26) for the 75–79, and 80–84 cohorts, and only introduce TBM at this stage. Yet the application of NCHS factors runs into some difficulties. The source of the problem comes from the well-known Arrow impossibility theorem to the effect that if any two agents were asked to rank three or more factors, then a conflict in the ranking will arise. To illustrate, suppose we have local (A), belief function (B), and census, (C) factors to rank for a specific cohort. Then, any one agent will have 6 profiles (from 3 choose 1), and any two agents will have 36 (6x6) profiles to rank. Of those 36 about 6 profiles will not be a problem to rank i.e., both agents will come out consistent in their ranking. But the others will need some arbitrary dictator, who in HUD is usually a headquarters person to settle the issue. It is the hope that if we can find some degree of belief to choose or combine factors, the need for a dictator would be obviated. The TBM introduced in this paper is therefore a step in this direction. Meanwhile, to illustrate the simulation, we use a pragmatic method to move the factor down to the local need ratio if it exceeds the national need ratio. For example, the 85 plus population would be adjusted down by the ratio of the minimum of {(local care percentage 85 plus) / (national care percentage normal (0.493, .01), or the local care percentage 85 plus}. For the purpose of illustration below we shall use the NCHS factors for all living arrangements, given that they are not far apart for the living alone arrangements in two of three cohorts, along with the NSA standard error, with adaptation to local mobility and self-care factors to the local level.

Similar procedures can be applied to the ALZs and NHs markets. For the dementia care case several practitioners looked at an old Congressional report entitled "Losing a Million Minds." The report made some projections of "Cases of Severe Dementia" between 1980–2040 based on about 7 percent of the 75–84, and 25 percent of the 85 plus population (U.S. Congress 1987, Table 1–4, 16). A more recent study by Herbert et al., (1995) based on a stratified sample of 642 persons in the East Boston, Massachusetts area found: 2 percent for persons 75–79 (95% CL: 1.3 to 2.7), 3.3 percent for persons 80–84 (95% CL: 2.2 to 4.4), and 8.4 percent (95% CL: 3.7 to 13.1) for the 85 plus group. Yet, the A. Samuel Milbank Study funded by AAHSA (August 2000) reported that the rate of moderate to severe dementia is 8 percent for the 75–79, and 16 percent for those aged 85 and over. For the purpose of illustrating the ALZ demand, we shall use Herbert's factors. This study is derived with an approximate 2.6 standard error.

III. Computation of Family Demand

In this section, we provide a set-up for estimating housing market conditions for the owner and the renter in the family market. The data is taken from a market trial county in the State of Nevada. The family model is given in a spreadsheet layout in the FHA technique guide (FHA 1970, 264). The essential view of the spreadsheet calculation is to forecast household growth. Starting from the top of the sheet, the drivers (employment or labor force) are changed via stable factors such as labor participation rates, household size, and institutional population to come up with a forecast for households. Starting from the bottom of the sheet, housing construction permits are added to the housing inventory and changed via stable assumptions based on how units in structures are categorized into owner or renter households, assumptions about rates of demolition, and vacancy rates to arrive at an estimate of current households. The difference between the forecast and the current number of households is a measure of the market conditions for housing units. These are further changed into the annual owner and renter demand with the assumptions about tenure changes, excess construction and vacancies. The final stage of translating demand into price or rent levels is exogenously performed by updating the income and the rent distributions for the market area. In the simulation of demand for the Trial County in Nevada, Table 3 displays the statistical specifications for the parameters, norms, and constants in the model.

The 27 listed equations are defined in Table 1, and applied to obtain the estimate of demand for the Trial County in the State of Nevada. Starting from the bottom of the list going upward, we will illustrate how to fulfill the requirements of FHA spreadsheet estimations for the current population. Item 25 uses the probability that the estimate will lie between a current estimate of the population based on a participation rate methodology and a current estimate of the population derived from updating the components of the population given in item 21, using assumptions from items 12 to 21. The resulting growth scenarios for the top half of the spreadsheet is as shown in Table 4.

Some cautions in interpreting the above estimates are called for. The 1990 and 2000 data are primarily from the Census. The current estimate column is derived from items 1 to 11 above, and the forecast column utilizes items 12 to 21. It is important to note that there are as many snapshots of the current and forecast estimates as there are replications. In the empirical section below, we made 20,000 such estimates, and then computed the annual demand based on another FHA report “Work Sheet for Estimating Net Quantitative Demand” (FHA 1970, 271).

TABLE 3

Parametric Specification Variables in a Simulated Rental Demand Model Trial County, Nevada

```
 1. SFCONSTPERIOD(Z)=UNIFORM(2,3);
 2. MFCONSTPERIOD(Z)=UNIFORM(6,9);
 3. MF2OWNERSH(Z)=NORMAL(.5,.001);
 4. MF34OWNERSH(Z)=NORMAL(.25,.001);
 5. Lostinventory(z)=uniform(.003,.005);
 6. Currentownervacancy(z)=NORMAL(.03,.001);
 7. Currentrentervacancy(z)=NORMAL(.095,.005);
 8. Nnormownvac(z)=normal(.03,.01);
 9. Normrenvac(z)=normal(.06,.01);
10. Scalar ucownerscurrent /4816/;
11. Scalar ucrenterscurrent /3805/;
12. Pr1990(z)=normal(demand(“driver1990”)/(demand(“POP1990”)),.001);
13. Pr2000(z)=normal(demand(“driver2000”)/(demand(“pop2000”)),.001);
14. Prcurrent(z)=(((pr2000(z)–pr1990(z))/10)*yearsincecensus)+pr2000(z);
15. PrforeCAST(z)=(((prcurrent(z)–pr2000(z))/10)*yrsfor)+prcurrent(z);
16. DRIVERTREND=(((DEMAND(“DRIVERCURRENT”)–DEMAND(“DRIVER2000”))
17. HHSIZE1990(z)=normal(demand(“HHPOP1990”)/(demand(“HH1990”)),.001);
18. HHSIZE2000(z)=normal(demand(“HHPOP2000”)/(demand(“HH2000”)),.001);
19. HHSIZECURRent(z)=(((HHSIZE2000(z)–HHSIZE1990(z))/10)*yearsincecensus)+HHSIZE2000(z);
20. HHSIZEforeCAST(z)=(((HHSIZEcurrent(z)–HHSIZE2000(z))/10)*yrsfor)+HHSIZEcurrent(z);
21. POPCURRENT(Z)=NONHHPOPCURRENT+POPINHHCURRENT(Z);
22. POPCURR=SUM(Z,POPINHHCURRENT(Z))/totalsim;
23. POPCURRUSINGPRATE(Z)=(DEMAND(“DRIVERCURRENT”)/prCURRENT(z))24.
25. Pocurrprob(z)=uniform(popcurrprate,popcurr);
26. Pocurrprobave=sum(z,popcurrprob(z))/totalsim;
27 POPCURR=popcurrprobave;
```

TABLE 4
Simulated Estimates and Growth for Rental Demand—Trial County, Nevada

	1990	2000	Current	Forecast
Driver	394,100	660,000	680,100	700,715
Part. Rate	0.53	0.48	0.47	0.47
Population	741,459	1,375,765	1,531,321	1,690,865
Pop. Using PR.			1,448,151	
Pop. In HH.	729,567	1,356,350	1,613,260	
Non. HH. Pop.	11,892	18,415	20,882	22,387
HH. Size	2.62	2.65	2.65	2.65
Households	278,025	512,253	608,215	628,813

IV. Computation of the Elderly Model

In this section, we present some assumptions and specifications for the simulation analysis of this model. Here the population and income parameters are singled out for specific treatments.

IV.A Population Growth

We now turn to a more systematic discussion on the estimation side of the elderly market. The first step is to benchmark and forecast the one-person elderly population. As the Census 2000 special tabulation of these data are not yet available, and given the lack of one person data from the 1980 census to be trended with the 1990 census for which the one person data is available, we can use a survival technique to age the elderly population of 1990 to date, and consider that growth rate as the lower end of the one person growth rate. The upper-end of the population growth interval is set by the market analysts.

IV.B Income Growth

Generally, the income for one-person housing is not available for the specific cohort levels we may want to investigate. However, from the STF4 data we can get an aggregate category for persons 75 plus by income distribution levels in dollars: 0–5,000, 6,000–10,000, ...100,000 and over. From the 1990 *Census Population and Housing: Special Tabulation on Aging* (CD90-A0-A9-1), we can obtain the one-person households by cohort levels for 75–79, 80–84, and 85 plus. For owners, the percent of one person households at income level 0–5,000 from the STF data, times the percent of 75–79 of "one persons" households from the CD data would yield the percent of 1 person (75–79) at the income level 0–5,000 households. To convert it to a number, we would then multiply by the STF4 total number of 75 plus owner households.

It is important to note that the estimate for the income growth is not yet updated, nor is it interpolated for the limiting d-factors explained below. We now discuss some controversy around the updating of income distributions for the elderly. The consideration of income and affordability illustrates well the Keynesian concerns of liquidity and timidity on demand constrained by income. The FHA handbook recognizes in discussing NH that we are not only concerned with income distribution changes, but also with the information on ". . . resources of the patient, family contributions, public welfare payments, and charitable contributions, as well as some other factors" (FHA 1970, 227). Some of the factors the modern literature focuses on are: 1) differential income-reproduction or fertility rates, 2) Intergenerational income mobility, where all income groups have the possibility of upward (downward) mobility into other income groups (Chu and Koo 1990, 1125), and 3) High population growth which causes inequality in income distribution, and high reproduction resulting in high population growth. Many of these characteristics can be put together in the form of a two period Overlapping Generation Model. There are two states of the population—old and young. Let income in the two periods be Y_t and Y_{t+1}. Savings (St) is current income (Yt) less current consumption (Ct). If a "young" desires children (F), then bequest to children will be s/F. It follows that income growth at Yt + 1 = f(s/F, luck = a) = f (Yr,a), implying a transition M(Y,X) function that a person with the income in group "i" will occupy the income group "j" in the latter period (Granger 1999, 12). For the estimation, we specify the equation:

$$\text{Income Qualified } (X_A) = [(\text{Nown}(d1) + \text{Nren}(d2)] \tag{5}$$

where:

A = agents 75–79, 80–85, and 85 plus households

Nown = Census1990 STF 4 one-person owner households 75 plus without selected conditions.

Nren = Census1990 STF 4 one-person renter household 75 plus without selected conditions.

(d1) = Cummulative probability density function evaluated at d1.

(d2) = Cummulative probability density function evaluated at d2.

$$(d1) = \left[\frac{(Beg\ cost\ /\ RY)*12}{owninc fact_A(z)}\right] / SH_A sh(z)$$

$$(d2) = \left[\frac{(Beg\ cost\ /\ RY)*12}{reninc fact_A(z)}\right] / SH_A sh(z)$$

SH_A = Share of the one-person market by the different cohorts.

RY = Rent to income ratio the agents will spend for services.

Ownincfact = Owner income growth factor.

Renincfact = Renter income growth factor.

Table 5 indicates the assumptions for updating the STF 4 income distribution. The growth rate for renters and owners is assumed proportional. Any beginning cost (Beg cost), which is the starting price for the units, can be entered. To estimate the price elasticity of demand, the model should be estimated twice for the two beginning costs, and then the arc elasticity format can be applied.

V. Empirical Results.

In this section we will give empirical results for some of the simulations. Figures 1 and 2 present results for the elderly model—ALFs and ALZs, for the Trial County, California market area. Figures 3 and 4 provide the results for the family model—owner and renter for the Trial County, Nevada market area. The results are based on the assumptions outlined in Table 1 for the family model, and in Table 5 for the elderly model. We have selected factors for the two models to keep the economy in a corridor of its long-term growth level. Broadly speaking, adjusting for shocks, such as tax shelter regulations, liquidity of the market, or the 9/11 phenomena would require an ad hoc adjustment of perturbed norms and ratios to bring them down to a short-term level.

The first observation we make is that Figures 1 to 3 follow a normal distribution, and Figure 4 is between a uniform and normal distribution. In the latter, it appears that the 20,000 replications were not enough to converge to a normal distribution, resulting in a flat maturity range for the rental market. Because the experimental design we used involved drawing several times from the uniform and normal distributions, we would expect that the result would indicate a combination of them, unless the sample size was large enough to allow convergence. The convergence for Figure 4 however, came at a high cost: the sample size had to be increased geometrically in order to reflect a significant gain. For the case in point, the sample size was increased to 500,000 estimates with little gain in the convergence.

The mean estimates displayed in the four charts represent demand for the types of housing in the areas. For the elderly demand in Trial County, CA, Figures 1 and 2 show mean estimates of 412 (STD 42.5) ALFs and 120 (STD 19.4) ALZ beds as the gross potential demand at a representative price of \$2,650 per month. Two cautionary points should be followed here. One point is that the estimates should be further reduced to reflect comparable beds already built or planned for the area. Another point is that the estimates will vary as the charges vary due to the law of demand. Therefore, increasing charges would result in decreasing demand. We can get price and quantity changes to estimate the elasticity for the market, which can be applied for all charge levels without resimulating the model. The analogous estimate for the family market as shown in Figures 3 and 4 are mean estimates of 17,640 (STD 720.3) units for the owner market, and 10,124 (STD 450.9) units for the renter market.

Conclusions

This paper indicates how the old FHA techniques of estimating housing market conditions for the elderly and family can be modeled to evaluate modern market conditions. It presents models that capture the essence of the FHA guide, and applies them

TABLE 5
Illustration of Elderly Estimates
Trial County, California.
13 years since the 1990 Census

Own./Rent Pop.:	Growth Factors 1990 Census to Current Lo. End Factor	Growth Factor Potential to Forecast Period Up. End Factor	Distribution Specification	Source
1 Person 75–79	1.215	Double	U(1.215,2)	Cohort Forecast
1 Person 80–84	1.333	Double	U(1.333,2)	And Analyst's
1.Person 85+	1.633	Triple	U(1.633,3)	Assumptions
Income Update:				
Market Share:	Lo. End Pct.	Standard Error		
SH75–79	0.9	0.1	N(0.9,0.1)	Analyst's
SH80–84	0.7	0.1	N(0.7, 0.1)	Assumptions
SH85 Plus	0.75	0.1	N(0.75,, 0.10)	
Own. Inc. Fac.:	HUD Inc. Adj Factor:	Up. End Factor		
75–79	1.568	Double	U(1.568,2)	HUD and Analyst
80–84	1.568	Triple	U(1.568,3)	Income
85 Plus	1.568	Triple	U(1.568,3)	Adjustment
Rent. Inc.Fac.:				
75–79	1.568	Double	U(1.568,3)	(Ibid.)
80–84	1.568	Triple	U(1.568,3)	
85 Plus	1.568	Triple	U(1.568,3)	
RY Ratio:	0.8	0.1	N(0.8, 0.1)	
3+ ADLs:	Survey Data	Standard Error		
ALFs:				
NCHS75–84	0.137	0.026	N(0.137,0.026)	(NCHS, 1993)
NCHS85+	0.234	0.026	N(0.234,0.026)	
Mobility and Self-care	Percent of all three groups			
Nation75–79	0.222	0.026	N(0.222,0.26)	STP14 Data
Nation80–84	0.319	0.026	N(0.319,0.026)	
Nation85+	0.498	0.026	N(0.498,9.026)	
Alzheimer's				
Jama75–79	0.02	0.003	N(0.02,0.003)	(Herbert et al., 1993).
Jama80–85	0.033	0.005	N(0.033,0.005)	
Jama85Plus	0.84	0.0188	N(0.084,0.0188)	

in two markets. For the elderly market in Trial County, CA the model yields demand estimates for the ALF and ALZ markets, and for the family market in Trial County, NV the model yields demand estimates for the owner and rental markets.

The novel aspects of this paper are that: 1) The simulation techniques help the builder to move away from having to build a model on untested and non-verifiable claims that its parameters and norms are stable, 2) The FHA paradigm can be extended and articulated within the domain of modern research such as NCHS and Herbert et al., studies on the applied side, and modern literature on the theory side, 3) The model can make both short and long-term estimates with ease by allowing specification of the parameters in the distributions. In fact, the adjustment for shocks can be easily realized by correlating the distributions with dynamically declining weights to indicate how the market converges after a shock such as 9/11 in 2001, and 4). For the first time the model puts in the public domain a harmonized way to integrate many fragmented efforts that can easily be adopted into any computerized software that allows the drawing of random numbers from statistical distributions. Finally, these models can be easily adapted for submarkets for either different housing types such as the NH or independent living, or for sub-areas for which data is available.

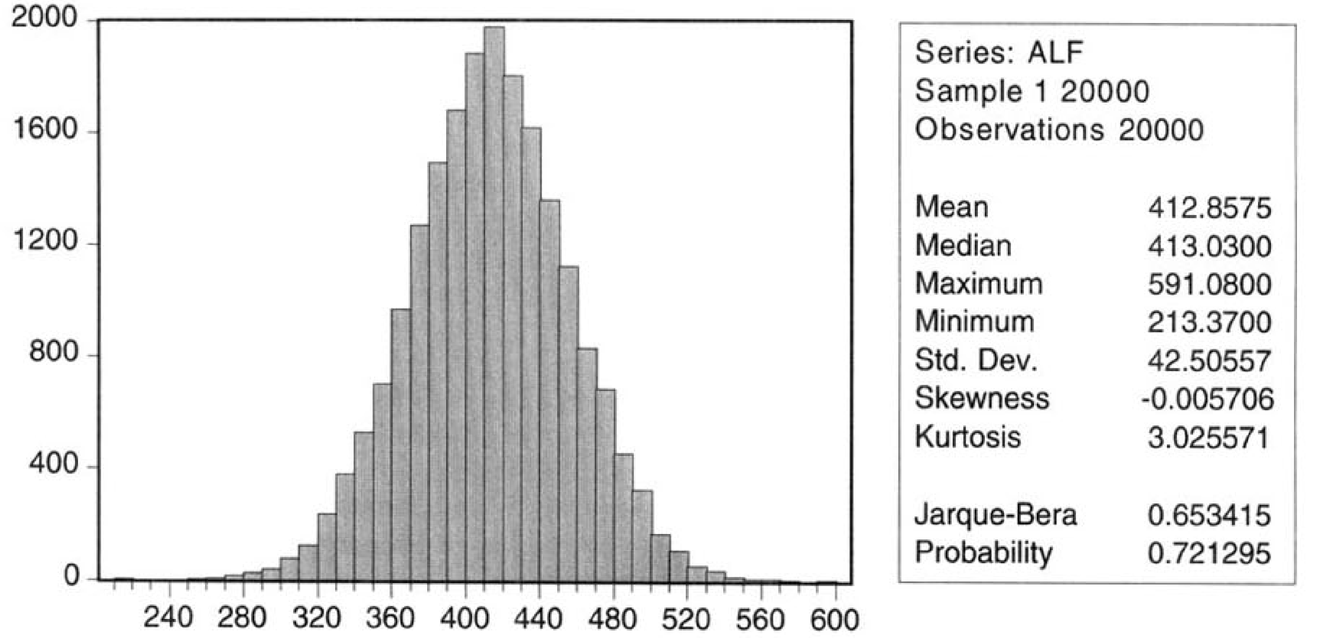

FIGURE 1. Histogram for ALF (20,000 Replications)

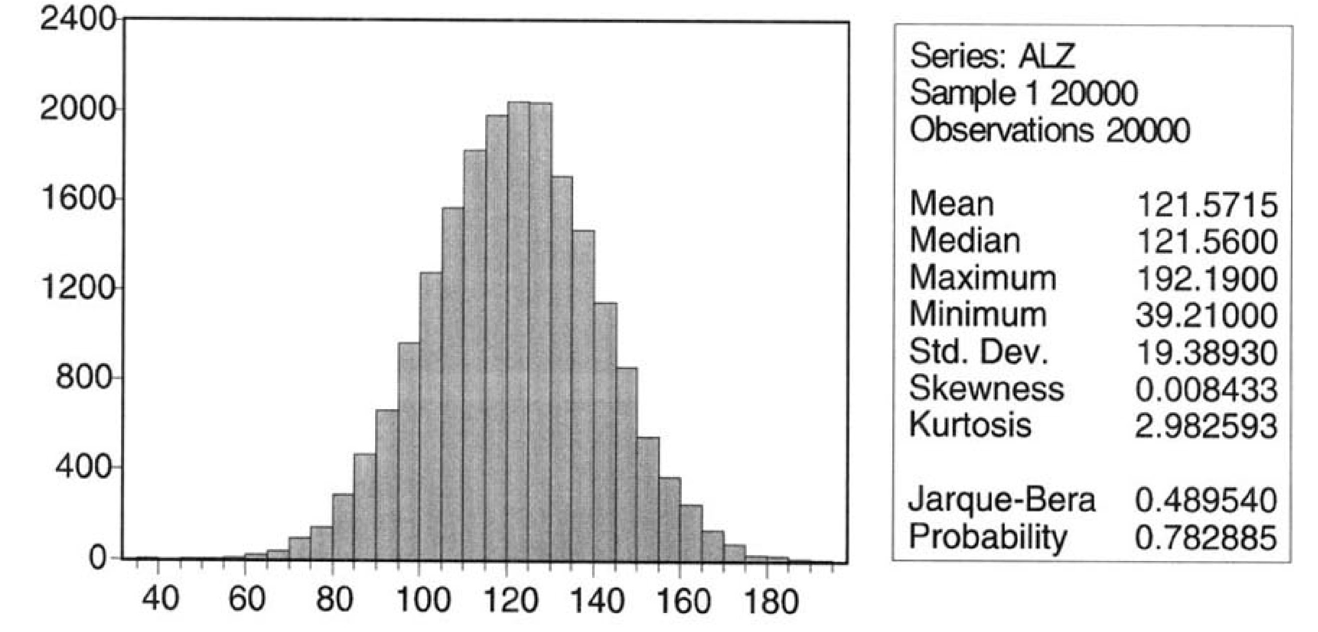

FIGURE 2. Histogram for ALZ (20,000 Replications)

APPENDIX 1

Qualitative Demand for Rental Housing: A First Step in Updating Rental Qualitative Demand Factors

In this appendix, we develop factors that can be used to distribute housing demand for renters and owners categorized by their income levels. On the rental side, the FHA handbook sets about to develop a demand schedule, which plots demand by gross rents. The schedule starts with a ". . . minimum gross rent at which acceptable rental accommodations currently can be produced locally without subsidy" (FHA 1970, 175) . Such a minimum rent (Rm) is set by local cost conditions, which functionally can be specified as follows: R_m = f(cost of land, labor, and materials).

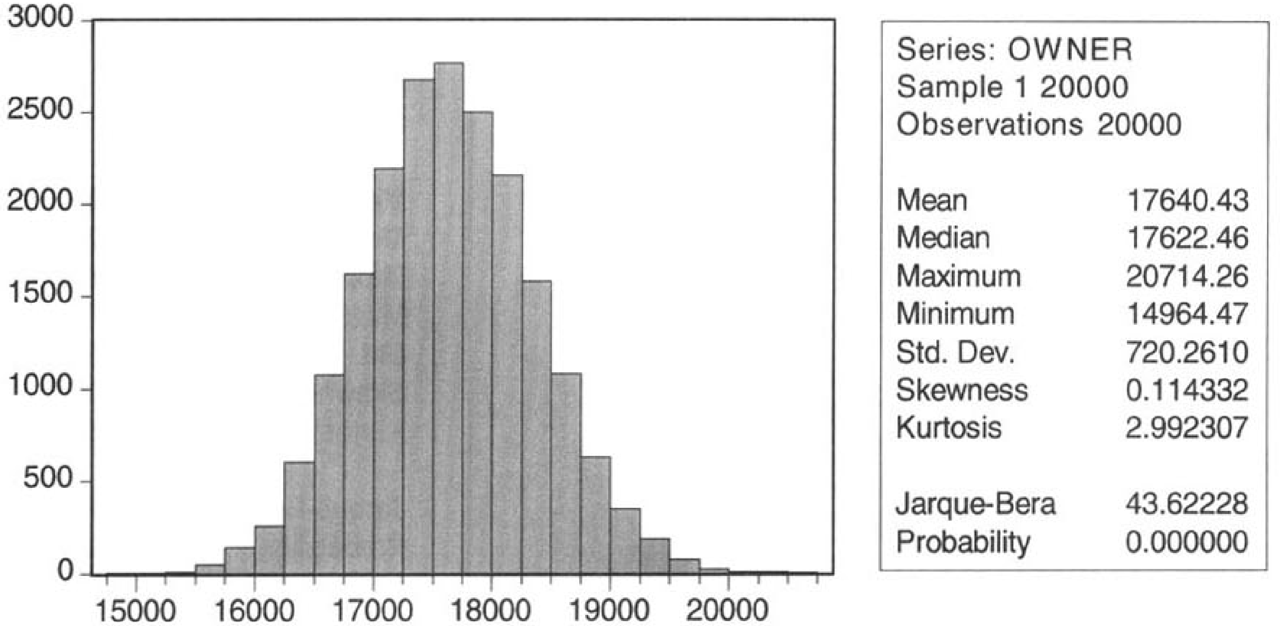

FIGURE 3. Histogram Owner (20,000 Replications)

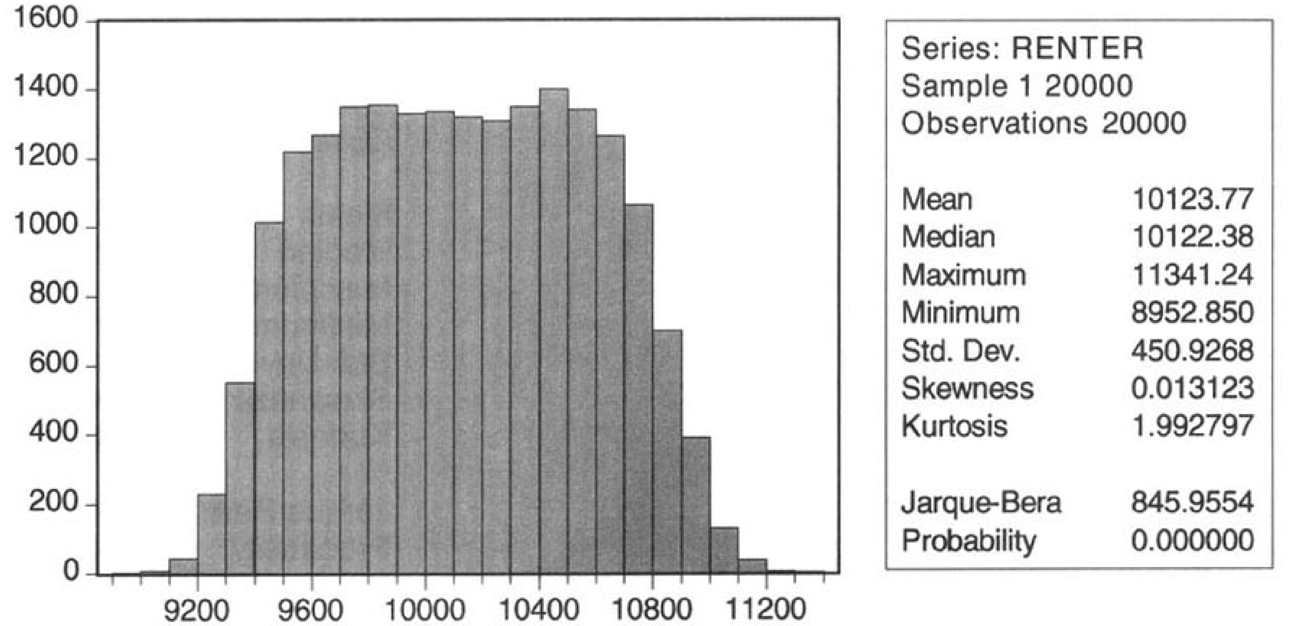

FIGURE 4. Histogram Renter (20,000 Replications)

Since the literature on this specification of qualitative demand is sparse and very specialized we should first make clear that the market for housing is a stock concept, while the market for housing starts is a flow concept. We are distributing our forecast demand by factors, and therefore we are concerned to some extent with the housing starts, although HUD economists would want to emphasize that we are forecasting a market potential, that is, the full capacity of the market.

Second, there is controversy on the cost variable. Labor costs can be specified in terms of real wages in construction, but Swan insists on some measure of productivity as well (Swan 1973, 965). Also, other measures such as tight construction markets, tight labor markets, as well as the state of the business cycle need to be taken into account. Controversy abounds as to what should be considered the cost of labor, and how income is drawn into the process. The FHA handbook tells us that people of the same income class can commit to "widely varying monthly payments," and that units with similar rents "will be marketed among families widely diverse in terms of income." Armed with this background information, we proceed to update the qualitative demand factors for rental housing from their values in the FHA handbook (FHA 1970 revised edition).

Method

The methodology we use on the renter side is a regression analysis. Starting with Muth's celebrated study that regresses the rate of new construction on income, a number of studies have concentrated in establishing responsiveness of demand for new units consequent to a change in rents. Muth found that the responsiveness according to cross-section studies is almost unitary (Muth 1960). There are some who have questioned this approach. For instance, Lee argued that the estimate had an upward bias because of multi-correlation between the interest rate and the income variable (Lee 1964).

Models based on Census data used 1940 as a basis year. As Schiff (1946, 104–105) pointed out Table 3 of the *Population and Housing, Families, Income and Rent* provided a ". . . triple cross tabulation of frequency distributions of families by family size, by income brackets, and by rents classes" for four regions of the U.S. with 16 subgroups. Another study by Leeuw (1971, 8) used 1960 Census data to calculate price and income elasticity. On the rental side, Leeuw ran regressions of the median gross rental expenditures of median income, and rental cost. The regressions were performed for different household sizes by different race groups. On the owner side, he ran regressions of the median housing expenses on a normal median income and the general price level.

The literature has not answered our question of how to obtain current factors in order to distribute demand estimates for the rental units by rent levels. The literature only focused on the elasticity of the rent and income, and on identifying other independent variables. The rest of this paper attempts to find the answer to the original query. The next section identifies the most generally available data sources where a researcher may get data needed for such an analysis. We also propose a statistical model that would be appropriate for the investigation. This section's analysis remains at the national level in order to be coherent with the FHA handbook. In other sections, we will attempt to disaggregate the findings to a more regional or local level.

Data Sources and Analysis

The C-25 Current Construction Reports of HUD, *Characteristics of New Housing* have consistently provided data on some selected characteristics of "New Rental Apartments." Since 1987, it has provided *Asking Rents* (by landlords) by bedroom types in its supplement survey, but it is missing data for several years, especially that for efficiency units (units without a bedroom). Asking rent includes such features as swimming pool, off-street parking, air-conditioning, and dishwasher. To get gross rents, we must add fuel and utility services that are not paid to the landlord as per normal contract rent.

Two sources of inflation must be considered in our analysis. Table A1 shows trend increases in *Asked Rents*. It illustrates that the annual rate of inflation in asked rent by bedroom types between

TABLE A1
Annual Inflation of Asked Rents & Fuel and Utility Costs by Bedroom Types
1987–1999

Period	Efficiency	One-Bedroom	Two-Bedrooms	Three-Bedrooms
Asked Rents:				
1987–1999	5.9	4.6		
1988–1999			4.5	4.9
Costs:				
CPI Index	Electricity per 500 KWH	Natural Gas 100 Therm	Fuel Oil #2 per Gallon	
1987–1999	40.94–45.6	55.77–65.94	0.85–0.90	
Annual %	–8.30%	–1.50%	–0.50%	

1987–1999 was about 5 percent. The underlying factors seem to have a uniform influence across the bedroom types. Considering that the Reagan Administration brought inflation under control in the early 1980's, the increases are substantial. Generally, such increases are sourced to either excess demand, increases in fixed costs, or higher costs for utilities paid by landlords, which should be distinguished from the utilities paid by the tenants.

Strictly speaking, we should be looking at the increases in gross rents; i.e., contract rents adjusted for fuel costs and utility services. Considering the problems we have had with oil and gas prices subsequent to OPEC policies during the period when the sample was taken, we should expect that the weight given by renters to the functional use of fuel (such as lighting and appliances, cooking, water and space heating) should alter rents even more than the forces that vary contract rents. An annual representation of the CPI index for electricity, gas, and fuel oil is indicated below. Except for OPEC shock periods, market forces acting on contract rents or owners paid utilities appear to rise at a faster annual rate.

Specification

We rely on the law of demand derived from a rental tenant utility function for our analysis. *A priori,* rents and quantity should be inversely related, and income elasticity should be positive. The relationships among the bedroom types represent special statistical problems. The unit sizes are related to the extent that they all belong to a project, are all managed together, or they share the same fixed costs. This would mean that we should employ a system model in the Seemingly Unrelated Regression (SUR) type. Ordinary Least Squares (OLS) regression would not be appropriate even though the equation can be made to be exactly identified. We have selected the following model for the estimate.

$$\begin{aligned}\text{Log(zerounits)} &= c(1) + C(2)*\log(\text{zerorent}) + \\ &c(3)*\log(\text{mfi}) + c(4)*\log(\text{smd}) + \\ &c(5)*\log(\text{vacfive}) + c(6)*\log(\text{time}) \\ &+ [ar(1) = c(7)]\end{aligned} \tag{A1}$$

$$\begin{aligned}\text{Log(oneunits)} &= c(10) + c(11)*\log(\text{onerent}) + \\ &c(12)*\log(\text{mfi}) + c(13)*\log(\text{smd}) + \\ &c(14)*\log(\text{vacfive}) + c(15)*\log(\text{time}) + \\ &[ar(1) = c(16)]\end{aligned} \tag{A2}$$

$$\begin{aligned}\text{Log(twounits)} &= c(18) + c(19)*\log(\text{tworent}) + \\ &c(20)*\log(\text{afford}) + \\ &c(21)*\log(\text{smd}) + \\ &c(22)*\log(\text{vacfive}) + c(23)*\log(\text{time}) + \\ &[ar(1) = c(24)]\end{aligned} \tag{A3}$$

$$\begin{aligned}\text{Log(threeunits)} &= c(26) + c(27)*\log(\text{threerent}) + \\ &c(28)*\log(\text{mfi}) + c(29)*\log(\text{smd}) + \\ &c(30)*\log(\text{vacfive}) + c(31)*\log(\text{time}) + \\ &[ar(1) = c(32)]\end{aligned} \tag{A4}$$

where

Zerounits = Efficiency units, twounits = two bedroom units etc. *(Characteristics of New Housing)*
Log = Log arithmetic values
C(*) = Coefficients to be estimated
MFI = Median family income *(US Housing Market Conditions)*
AFFORD = Index of affordability *(US Housing Market Conditions)*
SMD = Mortgage amount on the construction of New Rental Units *(US Housing Market Conditions)*
Vacfive = Vacancies in apartments in 5+ structure units *(Statistical Abstract of the US).*
Time = A time trend variable.
AR = Autoregressive correction for serial correlation.

Equations A1–A4 regress zero, one, two, and three bedroom units, respectively on several independent variables. On *a prior*i grounds we expect the rent coefficient to be negative, and that of income to be positive. However the sign of the SMD variable is harder to predict. Traditionally, the SMD variable has been used in order to measure the influence of the "wealth effect" on demand. Swan (1973, 966) argued that a negative sign has implications for a stock adjustment mechanism at work. It could also act as a proxy for the "left-out stock of houses." Vacfive function captures the states of the market—normal, soft, or tight. If a soft market is predominant, conventional wisdom would dissuade construction of new units, unless builders would want to do so for speculative purposes.

Results

Table A2 displays the results of our regression analysis. It is generated via a three-stage least square (3SLS) model with Fuel#2 per gallon, Electric 500 KWH, and Gas 100 Term CPI index among the non-rental variables as instrumental variables for the fitted equations. We used the utility cost index as an instrumental variable that makes a surrogate adjustment for the variation in the asked rents. This is a direct result of the variation in utility and fuel factors. Also, due to the small sample size no higher autoregressive adjustment of the first order correction was performed.

Two observations should be noted. The rent data is not brought up to the gross level fully: at least the utility costs should be added in. The income variable in Eq.A3 did not perform well, so an affordability index was used instead. This is probably why the rent coefficient in that equation is not significant. We are hoping that a better utility and cost data will improve the results.

The mortgage variable is consistently positive and significant. The market condition variable (vacfive) is 50 percent significant, in that two of four equations show significant results with positive signs, and high R squares.

A new table reflecting the above results can now be constructed to replace, or update, Exhibit B in the FHA handbook, as shown on the next page

In order to use Exhibit B, we proceed in a manner as explained in the FHA (1970, 276). Assume that your study so far indicates that demand is 500 units annually distributed as—25 0-BR, 200 1BR, 240 2-BR, and 35 3-BR (where BR refers to bedrooms). Then the entry at the row label "10" and column label "0-BR" is 0.67, or 67 percent. It means that 67 percent of 25 units, or approximately 17 units, will be the demand at that rent level for that bedroom type. The other cells have similar interpretations.

TABLE A2
SUR Regression of Units on Rents and other Variables.

Description	Efficiencies (EQ. A1)	One Bedroom (EQ. A2)	Two Bedrooms (EQ. A3)	Three Bedrooms (EQ. A4)
Constant	–61.21	–18.23	12.77	38.23
	(–2.12)**	(–2.90)***	(3.69)***	(2.31)***
Rents	–0.67	–0.66	–0.41	–0.98
	(–2.78)***	(–3.53)***	(–1.08)	(–3.66)***
Income	7.96	4.19		–3.28
	(2.56)***	(6.04)***		(–1.82)**
Affordability			–2.46	
Index			(–3.98)***	
Mortgage	0.44	0.62	0.53	0.42
	(3.96)***	(19.58)***	(12.14)***	(3.79)***
Vacancy	0.004	–2.93	–0.008	–5.47
	–0.003	(–7.22)***	(–0.01)	(–5.81)***
Trend	–8.94	–7.3	(–0.65)	1.03
	(–4.41)	(–18.28)***	–1.05	–0.82
D. W.	1.65	2.54	2.85	2.93
R&ADJ R^2	0.88/53	98/96	93/83	92/79

* Significance at the 90 percent confidence level
** Significance at the 95 percent confidence level
*** Significance at the 99 percent confidence level

NEW EXHIBIT B
Cumulative Demand Factors
for New Multifamily Housing

	0-br	1-br	2-br	3-br
0	1	1	1	1
10	0.67	0.66	0.75	0.98
20	0.45	0.44	0.56	0.96
30	0.3	0.29	0.42	0.94
40	0.2	0.19	0.32	0.92
50	0.14	0.13	0.24	0.9
60	0.09	0.08	0.18	0.89
70	0.06	0.05	0.13	0.87
80	0.04	0.04	0.1	0.85
90	0.03	0.02	0.08	0.83
100		0.02	0.06	0.82
110			0.04	0.8
120			0.03	0.78
130			0.02	0.77
140			0.02	0.75
150			0.01	0.74
160			0.01	0.72
170				0.71
180				0.7
190				0.68
200				0.67
210				0.65
220				0.64

APPENDIX 2

The Transferable Belief Model (TBM) and Belief Function and Demand for Elderly Care Housing

In this section we summarize the innovative use of the belief functions to analyze elderly need factors for the estimate and forecast of the housing demand. The availability of only bundles of need data, and the necessity to combine different sources of information such as the U.S. Census, and NCHS data has made this analysis complex. TBM functions offer an additional point of view that piggy backs on the market analysts' "degree of belief" about the level and relevance of need factors for a local area. This paper focuses on the essential elements that would make such an effort practical.

I. Introduction to Belief Functions

Let us consider a typical situation where a private or a social planner is trying to assess the demand for assisted living facilities for the elderly population. As we learned earlier, the elderly population of 75 and over is the most likely market segment. The market also is separated into three age cohorts: 75–79, 80–84, and 85 plus, with the latter being the most favored target market. Once the analyst has collected population, income, and wealth data, a critical decision point is reached, regarding: 1. How to reconcile various sources of need factors with differing "degrees of belief" of support, and 2. How to deal with cases if the data is available for only a set of need elements which cannot be disaggregated for individual elements. In cases where a private planner is a developer of housing, and a social planner is a (government) reviewer of the developer's package for a loan, then the situation can result in depriving an unsaturated market area of the utility of a housing project. The purpose of this note is to show how transferable belief models can be helpful in such situations.

We can trace this technique back to Carnap (1945), who tried to distinguish between a belief and a chance. Carnap's work allows us to make statements of the form that "You believe to degree 27 percent that the chance of heads is 50 percent" (Lewis 1980, 266). Through a set of axioms of von Neumann-Morgenstern (1944), and Savage (1954), risk and uncertainty enter the decision making process. In the former, Suppes and Winet (1955, 259) argued that the combination of utility with probability ". . . makes it difficult to make unequivocal measurements of either one or the other." In reviewing Savage's work, Shafer (1986 464) found it hard to produce a setting that ". . . permits a disentanglement of belief and value," let alone well-defined preferences. For instance, an investor has certain "beliefs" about the outcome of a project, and attaches "values" to different amounts of money. In real estate analysis a common practice is to write off overbuilding as "speculative" investment without an attempt to assess the beliefs and values of the investor (Ricks 1972, 220). The TBM was specifically engineered to leave probability out of the picture, and to deal with the data in a set theoretic form. TBM essentially offers a non-probabilistic way to transfer information from one set of beliefs to another. In this paper, we show that if the estimate and forecast of demand has been carried out to an effective demand (that is demand based on income) level, then the "degree of belief" about the need factors can become a way of selecting them to complete the demand estimate.

The literature on TBM is vast, while its computing algorithm is complex. This may explain why the literature is not in widespread usage. According to Shafer (1986, 155), belief function ". . . uses mathematical probability in a more general way than the better known Bayesian theory does." They are about "theory of evidence . . . weights of evidence . . . and numerical degrees of support based on evidence . . . The theory does not focus on acts of judgment . . . It focuses instead on . . . the combination of degrees of belief or support based on one body of evidence and those based on an entirely distinct body of evidence" (Shafer 1976, 3). Shafer has shown how such beliefs can be combined using a formula by Dempster (1967). Among the many manifestations of the now famous Dempster-Shafer (DS) theory in the literature, we shall use the several pragmatic specializations of Smets to find the need factor for our analysis. Smets (1977, chapters 8 and 12) argued that the DS approach is suited for working with imperfect information, imprecision and uncertainty, and has carefully steered it in a direction that avoids the use of probability theory. The essence of TBM remains that the "degrees of belief" are assigned to a set-theoretic (e.g., the need for assistance with three or more daily living factors such as bathing and shopping) rather than a point value or atomic (referring to only one factor in the set) measure or proposition. Krause and Clark (1993, 69) said that ". . . beliefs may be assigned to sets of propositions without there being a necessary requirement to distribute belief with finer granularity among the individual propositions in the set."

The set-theoretic nature of our need factor problem follows from the fact that the U.S. Census data reports only on mobility and on self-care need values for the elderly. However, the information such as for bathing, walking, shopping, and taking prescriptions for singles is not available. What is more significant are the subgroups of those singletons, such as persons with three or more ADLs or IADLs factors. Practitioners of market analysis hold varying beliefs on how these factors apply to local market areas such as a county, city, MSA, or any other sub-market definition. Information from the local surveys may indicate a more refined factor for the target market, and therefore, a need often exists to combine survey results with the Census information.

The rest of this paper is divided into three sections. Section II gives an intuitive symbolic guide of the DS theory. Section III surveys the most important sources in terms of the need factors in the U.S., and Section IV identifies and combines the set-theoretic data that the practitioners of the "invisible college" of elderly demand use to approximate that demand.

II. An Intuitive Approach to DS theory

We wish to give a symbolic guide for the literature on the belief functions. Because the notations are not quite standard yet, we attach a description of them and some of their sources.

- Ω A frame of discernment, universe of all propositions that contain the truth (Krause and Clark 1993, 77–103).
- $\varnothing$ Empty set for a proposition that cannot be believed.
- 2^{Ω} Power set, representing all the subsets of Ω.
- θ_i A set of atoms. Selfcare limitations, mobility limitations, and persons in ALF facilities with 3 or more ADL or IADL factors (Smets and Kruse 1997, 362).
- ω_i A set of worlds mapping to the interpretations of the universe. $\Omega = \{\omega_1, \omega_2, \omega_3, \ldots . \omega_n)$
- A_i A collection of subsets of atomic elements. $A_1 = \{\omega_6, \omega_8\}$, $A_2 = \{\omega_3, \omega_4, \omega_7\}$ etc.
- $|A|$ Represents the number of elements of A, or distance from A. We will qualify this usage as we need to.
- ϖ The truth. $\varpi \in A_1$ or $\varpi \in A_2$ etc. If $\varpi \varepsilon A \subseteq \Omega$, the universe is close (Smets 2001, 5).
- B Refers to subsets of A, ie $B \subseteq A$
- m, A basic probability assignment function (bpa), m: $2^{\Omega} \rightarrow [0,1]$ (Smets 1998, 299; 2001, 6).
- Bel, A belief function that maps the subsets to the unit interval, Bel: $2^{\Omega} \rightarrow [0,1]$ (Smets 1998, 299; 2001, 5).

A first step in the use of the TBF model is to assign a measure (bpa) to the various subsets. Intuitively, we know that 1. $m(\varnothing) = 0$ or $m(\varnothing) > 0$ in an open universe were belief states and are in contradictions, (Smets 2001, 6), and 2. $\Sigma_{A \subseteq \Omega}\, m(A) = 1$. The intuition follows from the knowledge that propositions that cannot be believed must be assigned zero, and that the universe of propositions containing the truth must be assigned the full value of one. Correspondingly, the support or evidence for believing in those propositions should be 1. $Bel(\emptyset) = 0$, and the 2. $Bel(\Omega) = 1$. To make assignments for other subsets, we have to evaluate "focal" elements and study their combinations to form a "Core," according to the rules of Boolean algebra. Broadly speaking, the relationship between Bel and bpa is not as simple. We have to show that we can transfer the support for beliefs for some propositions to the bpa function ($m: 2^{\Omega} \rightarrow [0,1]$), which requires a complex numerical algorithm. It is theoretically supplied by the Mobius Inverse function. We illustrate the Mobius Inverse function for a vacant and a full set.

Following Smets (2001, 1), the support for $\omega_0 \varepsilon$ A is from the subsets contained in A. Denote those subsets by B. Assuming that the zero set is not in B, then we can relate Bel (A) and m(B) thus:

$$Bel(A) = \Sigma_{\emptyset \neq B \subseteq A} m(B), \text{ for all } A \subseteq \Omega,$$

In order to find the values for m(B), we must invert the above equation. Such an inverse is guaranteed by the Mobius transformation to be:

$$m(A) =$$

$$\Sigma_{B \subseteq A} (-1)^{|A|-|B|} Bel(B), \text{ for all } A \subseteq \Omega, A \neq \emptyset$$

$$m(\emptyset) = 1 - Bel(\Omega)$$

We look at the cases for $\emptyset \in A$, and $\emptyset \notin A$. The proof for the null and A sets is given by Shafer, and therefore only summarized here. For $\emptyset \in A$, $\Sigma_{B \subset A}(-1)^{|B|} = (-1)^{|A|} = (-1)^{|\emptyset|} = 1$. But for $\emptyset \notin A$ not empty, we use the Binomial theorem to represent : $\Sigma_{B \subset A}(-1)^{|B|} = (-1)^{\|\emptyset\|} + (-1)^{\|\theta i\|} + (-1)^{\|\theta i, \theta i\|} + \ldots + (-1)^{|A|} = (1-1)^n = 0$. The latter follows from the Binomial theorem.

III. Practitioners' data

We start out by listing what comprises the need data. Although it is convenient to divide the need data into categories, such partitioning is not necessary for the analysis at hand. The reason behind it is that all ADL and IADL factors that fall into convenient categories such as physical problems with walking, bathing, dressing, eating, getting in and out of bed, and problems with home management such as meal preparation, cleaning, shopping, money management, medication management and transportation (Stone 2000, 5) are valuable but not available. The U.S. Census reports on the self care and mobility limitations. Each elderly person would have a preference for a service based on his/her ranking of these subsets. It is noteworthy to mention, that a traditional economic analysis would treat such information as a partial order and would be able to compute cardinal utility functions for the agents, such as by Savage (1954), von Neumann-Morgenstern (1944), or a modified Hernstein-Milnor set of axioms(1953). However, such functions succeed in simplifying the data in terms of the values that only indicate order, which is true up to a linear transformation. For example, given a utility function U(x), a transformation such as $G(U(x))^2$ is also true. Our focus, however, is on values that have some evidence to support our beliefs. The latter also distinguishes the subject tool from Bayesian models where we have only point-set information. Hence we resort only to the belief functions.

IV. Combination of Need Data: A Practical Application

In this section, we state a formal problem for a county in the US, and proceed to show the usefulness of the belief function in a decision making process. We will focus on the assisted living market, and will use the NCHS factors for all living arrangements. Table A3 shows the available data in percentage form. We emphasize that the data is in a set-theoretic form. For example, the 0.092 percent in row 3 column 1 describes walking, shopping, or any response that the respondent felt was appropriate to the census questionnaire.

We now look at some reasons for not relying on the proportions in Table A3 for an estimate. One reason is that practitioners tend to use rule of thumb ratio. Another reason is that to come up with scientific factors is technically demanding, both in the probability and mathematical senses. A common practice is to adopt the NCHS 13.7 percent of the 75–84 cohort , and 24.3 percent of the 85 plus cohort as a more reliable estimate. The first value of the DS technique is that it enables us to use the Dempster's combination formula to merge the set-theoretic information. Dempster's formula is a grid with the unit intervals, that will separate the two vectors of proportions to be combined, resulting in their intersection, namely, $m_i \cap m_j$. The works of Gordon and Shortliffe (1985, 279), Cohen (1985,

TABLE A3
Mobility and Selfcare Factors
1990 Census: A County Data

	Age		
Description	75–79	80–84	85 Plus
No Limitation	0.778	0.671	0.502
Mobility	0.092	0.143	0.219
Selfcare	0.05	0.051	0.044
Mobility and Selfcare	0.08	0.125	0.235

40), and Voorbraak (1991, 173) exemplify the use of this grid. Table A4 illustrates this for the 85 plus cohort for the 1990 Census data in Table A3 as row elements, and the NCHS ratios as column elements. A distinct feature of this table is that it does not get rid of any information in the rows that is not allocated to the 3+ ADLs column. It reallocates the unused information to the exhaustive set. A simple analogous example of Smets (1997, 355) may help us better understand this matter. We are given $\{\omega 1\} = 0.2$, $\{\omega 2\} = 0.3$, and $\{\omega 1, \omega 2, \omega 3\} = .5$. These data were combined with a better reliability factor of 0.7, and an unreliability factor of 0.3. The results for the first two are straightforward; we simply multiply them by the reliability factor to get 0.14, and 0.21, respectively. The result for the last combination will then pick up the residual data from the former two combinations. In this example, the combination has discounted our factor to a more reliable level.

Table A5 displays the results for the three age cohorts for which we have Census data combined with NCS data. Table A6 presents the Census data for comparison. Among the basic probability assignment factors, it shows other related computations of the DS model. The universal set, Ω, shows the set-theoretic representations of each atom, which is also indicated by a dummy variable of either 1 or 0 under the heading of the atoms, namely, columns, a, b, c. To reiterate, the "a" atom represents mobility; "b"—self-care, and "c"—persons with more than 3 ADLs. Focusing on the bpa (or "m" column), the entry for the zero row represents the null set, while the other entries are the combined data for the three set-theoretic atoms.

The entries for position 0–3 are from the 1990 Census, STP 14 data series combined with the NCHS data of 13.7 percent for the 75–84 cohort, and 24.3 percent for the 85 plus cohort (NCHS 1993, Table 1). This percentage is the best available figure and it is taken to represent persons with 3 or more ADLs and IADLs with a high degree of belief. We combine it with the Census data of Table 3 to yield m-values.

V. Results

The "m," Bel, Pl, and q functions are describing beliefs, but are specifically referred to as belief, plausibility, and commonality functions, respectively (Walley 1441).These functions represent beliefs based on the evidence and not on the probability. Bel(A) and Pl(A) refers respectively to "the total weight of evidence supporting A," and the "the total weight of evidence not in contradiction with A" (Orponen 1990, 246–247). In general, Pl(A) = 1 – Bel(not A). As Shafer puts it, we have "a report on how well A is supported and a report on how well its negation . . . (not A) is supported . . . the degree of plausibility is obviously analogous to the upper probability" (Shafer 1976, 144). Another way to

TABLE A4

Arthogonal Grid Examplar		
	$mj(3+ \text{ADLs})_{85+} = 0.243$	$m_j(^{\Omega}) = 0.766$
$m_i(\text{mobility and Selfcare})_{85+} = 0.235$	$0.235*0.243 = 0.057$	$mi(\text{mobilityandSelfcare})_{85+} =$ $0.235*0.766 = 0.18$
$mi(^{\Omega}) = 0.765$	$mj(3+ \text{ADLs})_{85+} =$ $0.765*0.235 = 0.179$	$\Omega =$ $0.765*0.766 = 0.586$

TABLE A5
Census Data Combined with NCHS Reliability Data

position	3 2 1	m	b	bel	pl	q	w	n_m	W	Ω
Cohort: 75–79										
0	0.000	0.000	0.000	0.000	0.000	0.897	1.000	0.000	0.000	∅
1	0 0 1	0.013	0.013	0.013	0.887	0.887	0.985	0.013	0.989	Mobility {a}
2	0 1 0	0.010	0.010	0.010	0.884	0.884	0.989	0.010	0.986	Selfcare {b}
3	0 1 1	0.874	0.897	0.897	0.897	0.874	1.000	0.874	1.000	{a,b}
Cohort: 80–85										
0	0.000	0.000	0.000	0.000	0.000	0.910	1.000	0.000	0.000	∅
1	0 0 1	0.020	0.020	0.020	0.900	0.900	0.978	0.020	0.989	{a}
2	0 1 0	0.010	0.010	0.010	0.890	0.890	0.989	0.010	0.978	{b}
3	0 1 1	0.880	0.910	0.910	0.910	0.880	1.000	0.880	1.000	{a,b}
Cohort: 85 Plus:										
0	0 0 0	0.000	0.000	0.000	0.000	0.878	1.001	0.000	0.000	∅
1	0 0 1	0.053	0.053	0.053	0.867	0.867	0.939	0.053	0.987	{a}
2	0 1 0	0.011	0.011	0.011	0.825	0.825	0.987	0.011	0.940	{b}
3	0 1 1	0.814	0.878	0.878	0.878	0.814	1.000	0.814	1.000	{a,b}

look at it is that as more evidence favoring A is attained, it can be transferred to the Bel(A), and therefore PL(A) represents the maximum potential support for A (Smets and Kruse 1999, 8). In response to some comments on the use of Bel and PL, Shafer later remarked that we can interpret the Bel(A) and Pl(A) results as upper and lower limit betting rates. These are "bounds for some true but somehow unknown probabilities" (Halpern and Fagin 1992, 278). Finally, the commonality function, q, describes "the weight of evidence equally supporting of all the elements of A" (Orponen, Ibid).

Interpreting Table A5 from the upper and lower support levels, we notice first that all the normalized values for the Pl(A) function in the last column of the table are higher, and the Bel(A) and Pl(A) functions are consistent in the sense of upper and lower bounds. The normalized Pl(A) data highly supports the {a} = mobility atomic proposition in Table A6, but equally supports the Dempster's combined bpas of Table A5. The advantages here might very well have come from the fact that the Pl(A) function "attends to singletons or other relatively small subsets" (Shafer 1976, 145).

Since beliefs represent a body of evidence, we can consider combining them in a generalized

TABLE A6
Census Data Uncombined with NCHS Reliability Data

position	3 2 1	m	b	bel	pl	q	w	n_m	n_pl	Ω
Cohort: 75–79										
0	0 0 0	0	0	0	0	0.222	1.259	0	0	∅
1	0 0 1	0.092	0.092	0.092	0.172	0.172	0.465	0.092	0.775	{a}
2	0 1 0	0.05	0.05	0.05	0.13	0.13	0.615	0.05	0.586	{b}
3	0 1 1	0.08	0.222	0.222	0.222	0.08	1	0.08	1	{a,b}
Cohort: 80–84										
0	0 0 0	0	0	0	0	0.319	1.183	0	0	∅
1	0 0 1	0.143	0.143	0.143	0.268	0.268	0.466	0.143	0.84	{a}
2	0 1 0	0.051	0.051	0.051	0.176	0.176	0.71	0.051	0.552	{b}
3	0 1 1	0.125	0.319	0.319	0.319	0.125	1	0.125	1	{a,b}
Cohort: 85 Plus:										
0	0 0 0	0	0	0	0	0.498	1.082	0	0	∅
1	0 0 1	0.219	0.219	0.219	0.454	0.454	0.518	0.219	0.912	{a}
2	0 1 0	0.044	0.044	0.044	0.279	0.279	0.842	0.044	0.56	{b}
3	0 1 1	0.235	0.498	0.498	0.498	0.235	1	0.235	1	{a,b}

sense, rather than updating them in the probabilistic sense. The guiding light is to put emphasis on the evidence that we think is more reliable. This implies that the practitioner may have some prior notions from which some learning can take place. In general, "we construct a belief function to represent the new evidence and combine it with our "prior" belief function." Practitioners of housing market studies have been known to harbor prior beliefs such as "most of the weight of belief favors demand from the 85 plus cohort," or "Nationally, only x-percent of elderly are placed in assisted living facilities" (Shafer 1976, 25). The problem then becomes to choose elements of the three cohorts (age groups of 65–74, 75–84 and 85 plus) to meet these requirements.

$$m_i(\text{mobility and selfcare})_{85+} =$$
$$0.498 \Rightarrow m_i(\Omega) = 1 - 0.498 = 0.502$$

$$m_j(3+\text{ ADLs})_{85+} = 0.234 \Rightarrow$$
$$m_j(\Omega) = 1 - 0.234 = 0.766$$

$\mathrm{Bel}_i \oplus \mathrm{Bel}_j$ (mobility and selfcare)$_{85+}$ = $m_i \oplus m_j$ (mobility and selfcare with 3+ ADLs)

What we have shown is that TBF is not a preferred method to use in the situation where the following statement is true: "as there are many persons, so there are many need factors." One has to examine other elements that support the "degree of belief" in the factors, such as its plausibility, upper and lower bounds, and commonality. By the same token that there is no room for a blind belief in science, there is no room for a blind belief in this analysis either. The analysis we have performed can be carried to different levels, and the failed categories data from these levels can be combined with the Census need data.

Notes

1. This need definition is adopted from the National Center for Health Statistics (NCHS, January 1993), while the age definition is in accordance with the Housing and Urban Development's (HUD) practice.
2. Usually non-farm employment, or labor force surrogates for saving and investment as the driver of the family model. The elderly markets, RSCs, ALFs, BCs, NHs, and ALZs, leap and bound with ADLs and IADLs need driven factors.
3. We are not addressing regulated housing markets.
4. Qualitative demand is not integral to this analysis in the FHA technique. It is done from mainly extentional analysis. However, we illustrate qualitative demand for the elderly model that follows.
5. The standard error for these variables is usually given by the U.S. Census (see 1990 *Census of Population and Housing for the U.S.*, Appendix C, C1–C11).
6. Some other formulation of utility functions exists. Sickles and Taubman (1986) summarize a utility function that includes consumption, leisure, and health factors only. We adopted the above version because the decision to retire has already been made in the case of assisted living.

References

American Association of Retired Persons. *Across the States 1998: Profiles of Long-Term Care Systems,* 3rd ed., (Washington, D.C.: AARP, 1998a).

American Association of Retired Persons. *Boomers Approaching Midlife: How Secure a Future?*, (Washington, D.C.: AARP, 1998b).

Anas, Alex and Richard J. Arnott., "Dynamic Housing Market Equilibrium with Taste Heterogeneity, Idiosyncratic Perfect Foresight, and Stock Conversions," *Journal of Housing Economics,* 1(1), 1991, 2–32.

Anderson, R., and H. Sonnenschein, "On the Existence of Rational Expectation Equilibrium," *Journal of Economic Theory,* 26, 1982, 261–278.

Arrow, Kenneth, and F. H. Hahn, *General Competitive Analysis,* (San Francisco, CA: Holden-Day, Inc., 1971).

ASPE: The Assistant Secretary for Planning and Evaluation and AOA: The Administration on Aging. *Informal Caregiving: Compassion in Action,* (Washington, D.C.: 1998).

Carnap, Rudolf, "The Two Concepts of Probability," *Philosophy and Phenomenological Research,* Vol. 5, 1945, 513–532.

Chu, C.Y. C and Hui-Wen Koo "Intergenerational Income-Group Mobility and Differential Fertility," *American Economic Review,* 8, No. 5. Dec., 1990, 1125–1138.

Cohen, Paul R., *Heuristic Reasoning about Uncertainty: An Artificial Intelligence Approach,* (Boston, MA: Pitman Advanced Publishing Program, 1985).

Dempster, A. P., "Upper and lower probabilities induced by a multivalued mapping," *Annals of Mathematical Statistics,* 38, 1967, 325–339.

Debreu, G. and H. Scarf, "A Limit Theorem on the Core of an Economy," *International Economic Review,* 4, 1963, 235–246.

Dornbusch, Rudiger, Stanley Fisher, and Richard Startz, *Macroeconomics,* (New York: Irwin-McGraw-Hill, 1998.

Engelhardt, Gary V., and James M. Poterba, "Housing Prices and Demographic Change: Canadian Evidence," *Regional Science and Urban Economics,* 21, 1991, 539–546.

Feinstein Jonathan, and Daniel McFadden, "The Dynamics of Housing Demand by the Elderly: Wealth, Cash Flow, and Demographics Effects," NBER Working Paper # 2471, December 1987; Reprinted in *The Economics of Aging,* D. Wise, ed., (University of Chicago Press, 1989, 55–86).

FHA, "Techniques of Housing Market Analysis," Department of Housing and Urban Development, Techniques Developed by FHA Economic and Market Analysis Division, Revised, August, 1970.

Fair, Ray C. and Dwight M Jaffee, "Method of Estimation for Markets in Disequilibrium," *Econometrica,* 40, No. 3, 1972, 497–514.

Friedman, Milton, *A Theory of the Consumption Function,* (Princeton University Press 1957).

Fuchs, Victor, R., "Medicare Reform: The Larger Picture," *Journal of Economic Perspectives,* 14(2), Spring 2000, 57–70.

Gordon, Jean and Edward H. Shortliffe, "The Dempster-Shafer Theory of Evidence," in Bruce G. Buchanan and Edward H. Shortliffe, *Rule-Based Expert Systems: The MYCIN Experiments of the Stanford Heuristic Programming Project,* (New York: Addison-Wesley Publishing Company, 1985).

Granger, Clive W. J., *Empirical Modeling in Economics: Specification and Evaluation,* (Cambridge University Press, 1999, and 2002).

Halpern, Joseph Y., and Ronald Fagin, "Two Views of Belief: Belief as Generalized Probability and Belief as Evidence," *Artificial Intelligence,* 54, 1992, 275–317.

Hamilton, Bruce W., "The Baby Boom, the Baby Bust, and the Housing Market: A Second Look," *Regional Science and Urban Economics,* 21, 1991, 547–552.

Hawes, C., M. Rose, C.D. Phillips, and V. Iannacchione. *A National Study of Assisted Living for the Frail Elderly: Results of a National Telephone Survey of Facilities,* (Beachwood, OH: Menorah Park Center for the Aging, 1999).

Hawes, C., J.B. Wildfire, and L.J. Lux. *Regulation of Board and Care Homes: Results of a Survey in the 50 States and the District of Columbia: National Summary,* (Washington, D.C.: American Association of Retired Persons, 1993).

Hawes, Catherine, Mariam Rose and Charles D. Phillips, *A National Study of Assisted Living for the Frail Elderly,* Myers Research Institute Study, December 14, 1999.

Herbert, Liesi E., Paul A. Scherr, Laurel A. Beckett, Marilyn S. Albert, David M. Pilgrim, Marilyn J. Chown, H. Harris Funkenstein, and Denis A. Evans, "Age-Specific Incidence of Alzheimer's Disease in a Community Population," *JAMA,* May 3, 1995, 1354–1360.

Hernstein, I. N., and J. Milnor, "An Axiomatic Approach to Measurable Utility," *Econometrica,* 21, 1953, 291–297.

Hoeger, Thomas J., Gabriel A. Picone, and Frank A. Sloan., "Public Subsidies, Private Provision of Care and Living Arrangements of the Elderly," *The Review of Economics and Statistics,* LXXVII, 3, 1996, 428–440.

Klein, L. R., and Kosobud, R. F., "Some Econometrics of Growth: Great Ratios of Economics," *Quarterly Journal of Economics,* LXXV, 2, 1961, 173–198.

Kreps, D., "A Note on 'Fulfilled Expectations' Equilibrium," *Journal of Economic Theory,* 14, 1977, 14, 32–43.

Krause, Paul and Dominic Clark, *Representing Uncertain Knowledge: An Artificial Intelligence Approach,* (Oxford: Intellect Books, 1993).

Lee, Tong Hun, "The Stock Demand Elasticities of Non-Farm Housing," *The Review of Economics and Statistics,* XLVI, 1964, 82–89.

Leeuw, Frank de, *The Review of Economics and Statistics,* 52(1), 1971, 1–10.

Lewis, David, "The Subjectivist's Guide to Objective Chance," in Richard C. Jeffrey, ed., *Studies in Inductive Logic and Probability,* volume II, (Berkeley, CA: University of California Press, 1980).

National Center for Health Statistics (NCHS), "Health Data on Older Americans," Series, 3, No. 27, January 1993.

Mankiw, N. Gregory, and David N. Weil, "The Baby Boom, The Baby Bust, and the Housing Market," *Regional Science and Urban Economics,* 19, 1989, 235–258.

McAllister, Patrick H. "Rational Behavior and Rational Expectation," *Journal of Economic Theory,* 52, No. 2, 1990, 332–363.

Megbolugbe, Isaac, J. Sa-Aadu and James D. Shilling, "Elderly Female-Headed Households and the Decision to Trade Down," *Journal of Housing Economics,* 8(4), 1999, 285–300.

Montgomery, Claire A., "A Structural Model of the U.S. Housing Market: Improvement and New Construction," *Journal of Housing Economics,* 5(2), 1996, 166–192.

Muth, Richard F., "The Demand for Non-farm Housing," in Arnold C. Harberger, ed., *The Demand for Durable Goods* (University of Chicago Press, 1960).

National Center for Health Statistics (NCHS), "Health Data on Older Americans," Series, 3, No. 27, January 1993.

Orponen, P., "Dempster's Rule of Combination is #P-complete," *Artificial Intelligence,* 44, 1990, 245–253.

Pitkin, John R., and Dowell Myers. "The Specification of Demographic Effects on Housing Demand: Avoiding the Age-Cohort Fallacy," *Journal of Housing Economics,* 3(3), 1994, 240–250.

Price, Waterhouse and Coopers, "The Assisted Living Industry: An Overview-1998," Survey in 1997.

Ricks, R. Bruce, "Comment on Fair, Ray C., Disequilibrium in Housing Models," *Journal of Finance,* XXVII, 1972, 220.

Savage, L. J., *The Foundations of Statistics,* (New York: John Wiley and Son, 1954).

Schiff, Eric, "Family Size and Residential Construction," *The American Economic Review,* 36(1), 1946, 97–112.

Shafer, Glenn, "The Combination of Evidence," *International Journal of Intelligent Systems,* 1, 1986, 155–179.

Shafer, Glenn, *A Mathematical Theory of Evidence,* (Princeton University Press, 1976).

Sickles, Robin C., and Paul Taubman, "An Analysis of the Health and Retirement Status of the Elderly," *Econometrica,* 54(6), November 1986, 1339–1356.

Smets, Phillippe, "Belief Function in Decision Making," in R. Srivastaca, ed., *Survey of Decision Making Within the Upper-Lower Probability and the TBM Frameworks.* (Physica-Verlag, 2001, forthcoming).

Smets, Phillippe and Rudolf Kruse, "The Transferable Belief Model for Belief Representation," July 27, 1999. Web: http://iridia.ulb.ac.be/~psmets/AAPapers.html.

Smets, Phillippe, "Imperfect Information: Imprecision and Uncertainty," in Amihai Motro and Phillippe Smets, ed., *Uncertainty Management in Information Systems: From Needs to Solution,*(Boston, MA: Kluwer Academic Publishers, 1997).

Smets, Phillippe and Rudolf Kruse, "The Transferable Belief Model for Belief Representation," in Amihai Motro and Phillippe Smets, ed., *Uncertainty Management in Information Systems: From Needs to Solution,* (Boston, MA: Kluwer Academic Publishers, 1997).

Smets, Phillippe "Numerical Representation of Uncertainty," in *Handbook of Defeasible Reasoning and Uncertainty Management System,* vol. 3, (Boston, MA: Kluwer Academic Publishers, 1998).

Smith, Lawrence B., "Disequilibrium in Housing Models," *Journal of Finance,* Discussion, XXVII, 1972, 228.

Smith, Lawrence B., "A Note on the Price Adjustment Mechanism for Rental Housing," *American Economic Review,* 64, June, 1974, 478–481.

Stone, R.I., *Long-term Care for the Elderly with Disabilities: Current Policy, Emerging Trends, and Implications for the Twenty-First Century,* by Robyn I. Stone, Funded by Milbank Memorial Funds, August 2000.

Suppes, P. and M. Winet, "An Axiomatization of Utility Based on the Notion of Utility of Differences," *Management Science,* 1, 1955, 259–270.

Swan, Craig, "The Markets for Housing and Housing Services: Comment," *Journal of Money, Credit and Banking,* 5(4), 1973, 960–972.

Swan, Craig, "Labor and Material Requirements for Housing," *Brookings Papers on Economic Activity,* 1971(2), 347–377.

von Neumann, J., and O. Morgenstern, *Theory of Games and Economic Behavior,* (Princeton University Press, 1944, Third Edition, 1953).

Voorbraak, Frans, "On the justification of Dempster's rule of combination," *Artificial Intelligence,* 48, 1991, 171–197.

Walley, Peter, "Belief Function Representations of Statistical Evidence," *The Annals of Statistics,* 15(4), 1987, 1439–1465.

U.S. Congress, Office of Technology Assessment, "Loosing a Million Minds: Confronting the Tragedy of Alzheimer' Disease and Other Dementias," OTA-BA-323 (Washington, DC: U.S. Government Printing Office, April, 1987).