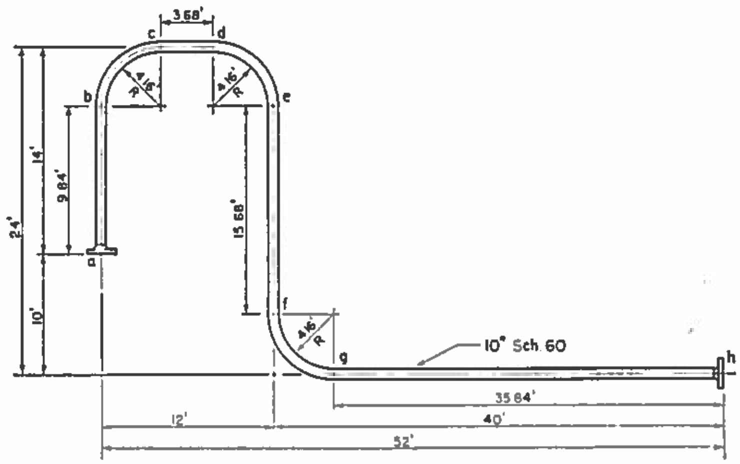
SINGLE PLANE SYSTEM CONTAINING CIRCULAR ARCS



Given: A 10 inch piping system in accordance with the sketch shown above.

Maximum Operating Pressure	400 psi
Maximum Operating Temperature	750° F
Piping Specification	A.S.T.M. A-106 Grade A

Data:

t = 0.500 inches = schedule 60 — page 2 and 14

I_P = 212 inches^4, S_m = 39.43 inches^3, k_{bend} = 1.74, i_{bend} = 1.00 — page 14

$c_{at\ 750°}$ = 996 — page 11

S_A = 17,675 psi — page 3

Find: Reaction forces F_x and F_y at point h. (At point a reaction forces are equal and opposite.)
Reaction moments at points a and h.
Amount and location of Maximum Bending Stress, s_B.

Solution: Determine the location of the centroid and calculate the line inertias in the same manner as outlined on page 50 except that the flexibility factor k must be included for all curved segments of the system.

Solve for F_x and F_y using the equations shown on page 50:

$$L_x = 40 + 12 = 52 \text{ feet}$$

$$L_y = 24 - 14 = 10 \text{ feet}$$

$$F_x = \frac{21{,}369 \times 52 + 10{,}457 \times 10}{9415 \times 21{,}369 - (10{,}457)^2} \times 996 \times 212$$
$$= 2795 \text{ pounds}$$

$$F_y = \frac{9415 \times 10 + 10{,}457 \times 52}{9415 \times 21{,}369 - (10{,}457)^2} \times 996 \times 212$$
$$= 1467 \text{ pounds}$$

Pass these reaction forces through the centroid. The product of these forces and their respective distances from any point gives the bending moment at that point. Assume counterclockwise rotation as + and clockwise rotation as −.

Solve for reaction moments at points a and h as follows:

$$M_{at\ a} = -(2795 \times 0.40) + (1467 \times 17.37)$$
$$= +24{,}364 \text{ ft lb}$$

$$M_{at\ h} = +(2795 \times 9.60) - (1467 \times 34.63)$$
$$= -23{,}970 \text{ ft lb}$$

F, the resultant of F_x and F_y, when passed through the centroid gives the position of the neutral axis. The maximum bending moment occurs at that point which is furthest from the neutral axis. A scale drawing will show that this point is located between f and g at a normal distance of 9.4 feet from the neutral axis. The bending moment and bending stress in this curved section of pipe are determined as follows:

$$M = \sqrt{(2795)^2 + (1467)^2} \times 9.4 = 29{,}672 \text{ ft lb}$$
$$= 29{,}672 \times 12 = 356{,}064 \text{ inch pounds}$$

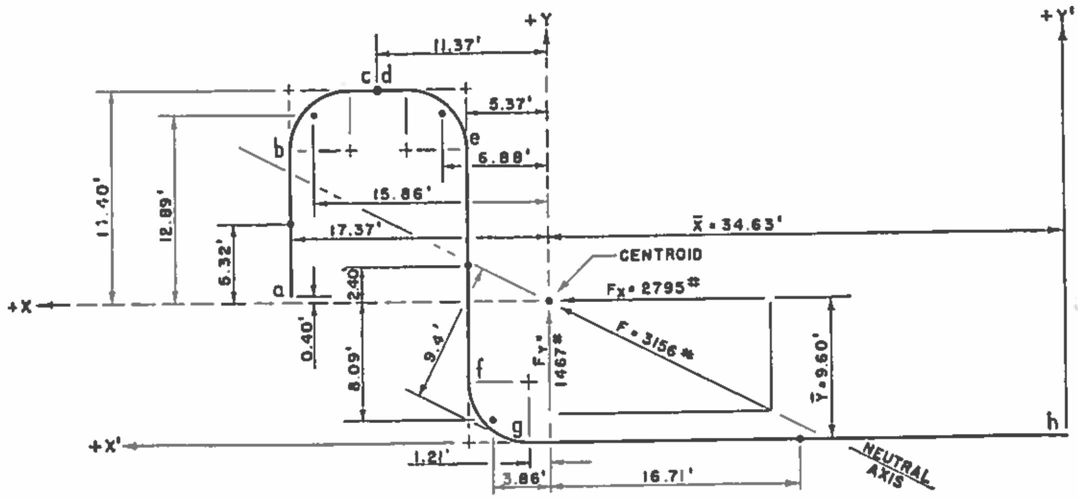
$$s_B = \frac{M}{S_m}\, i = \frac{356{,}064}{39.43} \times 1.00 = 9030 \text{ psi}$$

The maximum bending moment in straight pipe occurs at point g.

$$M_{at\ g} = +(2795 \times 9.60) + (1467 \times 1.21)$$
$$= +28{,}607 \text{ ft lb}$$
$$= 28{,}607 \times 12 = 343{,}284 \text{ inch pounds}$$

$$s_B = \frac{M}{S_m} = \frac{343{,}607}{39.43} = 8714 \text{ psi}$$

The maximum bending stress is 9030 psi, occurring in member fg and is less than the allowable stress range of 22,500 psi.



To find c.g. of each segment see page 46.

ab = 9.84 ft
cd = 3.68 ft
cf = 15.68 ft
gh = 35.84 ft

$\left.\begin{matrix} bc \\ de \\ fg \end{matrix}\right\}$ Radius of bends = 4.16 ft

Centroid (calculated with origin at point *h*)

	Eq. No.	Length L, Ft	x'	Lx'	y'	Ly'
ab	I	9.84	+52	+ 511	+14.92	+147
bc	III	$1.57 \times 1.74 \times 4.16 = 11.37$	+50.49	+ 574.1	+22.49	+255.7
cd	I	3.68	+46	+ 169.2	+24.0	+ 88.4
de	III	$1.57 \times 1.74 \times 4.16 = 11.37$	+41.51	+ 472	+22.49	+255.7
cf	I	15.68	+40.0	+ 627	+12.0	+188
fg	III	$1.57 \times 1.74 \times 4.16 = 11.37$	+38.49	+ 437.6	+ 1.51	+ 17.2
gh	I	35.84	+17.92	+ 643	0	0
		$\Sigma L = 99.15$		$\Sigma Lx' = +3433.9$		$\Sigma Ly' = +952.0$

$$\bar{x} = \frac{3433.9}{99.15} = +34.63 \text{ ft} \qquad \bar{y} = \frac{952.0}{99.15} = +9.60 \text{ ft}$$

I_{xy}

	Eq. No.		
ab	VI	$9.84 \times 17.37 \times 5.32$	= + 909
bc	X A	$-1.74(0.137 \times 4.16^3) + 1.57 \times 1.74 \times 4.16 \times 15.86 \times 12.89$	= + 2307
cd	VI	$3.68 \times 11.37 \times 14.40$	= + 603
de	X B	$+1.74(0.137 \times 4.16^3) + 1.57 \times 1.74 \times 4.16 \times 6.88 \times 12.89$	= + 1025
cf	VI	$15.68 \times 5.37 \times 2.40$	= + 202
fg	X B	$+1.74(0.137 \times 4.16^3) + 1.57 \times 1.74 \times 4.16 \times 3.86(-8.09)$	= − 338
gh	VI	$35.84(-16.71)(-9.60)$	= + 5749
			I_{xy} = +10457

I_x

	Eq. No.		
ab	XIV B	$\frac{(9.84)^3}{12} + 9.84(5.32)^2$	= 358
bc	XVII A	$1.74(0.149 \times 4.16^3) + 1.57 \times 1.74 \times 4.16(12.89)^2$	= 1908
cd	XIV A	$3.68(14.40)^2$	= 763
de	XVII A	$1.74(0.149 \times 4.16^3) + 1.57 \times 1.74 \times 4.16(12.89)^2$	= 1908
cf	XIV B	$\frac{(15.68)^3}{12} + 15.68(2.40)^2$	= 412
fg	XVII A	$1.74(0.149 \times 4.16^3) + 1.57 \times 1.74 \times 4.16(8.09)^2$	= 763
gh	XIV A	$35.84(9.60)^2$	= 3303
			I_x = 9415

I_y

	Eq. No.		
ab	XIV A	$9.84(17.37)^2$	= 2969
bc	XVII B	$1.74(0.149 \times 4.16^3) + 1.57 \times 1.74 \times 4.16(15.86)^2$	= 2879
cd	XIV B	$\frac{(3.68)^3}{12} + 3.68(11.37)^2$	= 480
de	XVII B	$1.74(0.149 \times 4.16^3) + 1.57 \times 1.74 \times 4.16(6.88)^2$	= 557
cf	XIV A	$15.68(5.37)^2$	= 452
fg	XVII B	$1.74(0.149 \times 4.16^3) + 1.57 \times 1.74 \times 4.16(3.86)^2$	= 188
gh	XIV B	$\frac{(35.84)^3}{12} + 35.84(16.71)^2$	= 13844
			I_y = 21369

For equation reference numbers see pages 46 to 50.