How Can Complexity Arise from Minimal Spaces and Systems?

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ABSTRACT

This paper examines the question of how spaces and systems evolve, and how this engenders the possibility for complex forms and phenomena. It is the author's opinion that understanding this matter requires us to examine the most minimal cases and to delineate what tools emerge at each stage in the process, which might allow further evolution to take place. This paper examines several aspects of minimal spaces and systems, explores the connections with Constructive methods in Logic and Mathematics, and Wolfram's "New Kind of Science," and discusses how this might reveal directionality in the evolution of form, which emphasizes those possibilities where complex behaviors might arise. Finally; there is a brief discussion of how this relates to the earliest phases of learning symbolic thinking, as observed in the behavior of very young human children.

1. Introduction

In a space that is empty of objects and observers, we might be tempted to believe that no phenomena can take place. However; this is a questionable assumption, not only in the real world, but also in the theoretical abstract. Accordingly; an absolutely empty space is not demonstrably so, as any observer or probe which might discern this would also create a condition whereby perfect emptiness is changed or disappears. When we examine minimal spaces and systems, therefore, we must refrain from undue (or erroneous) assumptions that arise from our understanding of more conventional systems. The fact that our common experience makes certain assumptions natural to us may not serve us when we try to examine simpler spaces and systems that lack many of the familiar elements. There are some schools of thought who *do* shed considerable light on this topic, however, and understanding the minimal cases well is essential, if we want to have a really thorough understanding of more complex systems.

Though Classical Physics, Math, and Logic can explain the vast majority of what takes place around us, it can't deal with certain subtleties, including Quantum-

Mechanical phenomena, nor do these tools alone give us clear insight into how current conditions evolved from simpler, or more fundamental, conditions. If we wish to examine the simplest objects and spaces, or the most fundamental conditions, the classical formulations yield very little usable information. In order to explore this territory, we need to adopt a different view of things, and the Constructive approach to Math and Logic offers many tools which are useful to examining minimal spaces and systems. In Constructive Mathematics and Constructive Logic, there is ample concern with the actual answer or result, but more emphasis on how a result might be obtained, or how a true statement is verified (Avigad 2000). Although this might appear to yield less precise information, sometimes what it gives us is every bit as precise and more useful. Where a classical proof might be contented with being able to prove that a particular number is, or is not, prime (for example), a constructive proof requires us to devise a way to find, or generate, primes.

2. Constructive Exploration

For exploring minimal spaces and systems, a constructive approach is the most natural, as it allows our knowledge to progress without any initial assumptions (or with truly minimal ones). If we wish to examine a space, and to determine its dimensionality through observation, a single viewpoint (or observation) is clearly not enough. Through successive stages of exploration and observation, or by coordinating several simultaneous observations, a more detailed view may be obtained. Of course; in the case of a minimal space, the number of possible viewpoints is limited, and in the case of a 0-dimensional space, we might be tempted to assume that only one point of view is possible. The reality is that each time one examines such a system is a unique case or instance of the process of observation.

That is; a zero dimensional system, observed twice (or over an interval), now has an extent in time. This makes it a 1-dimensional system (at least). The act of observation itself is normally presumed to have an extent, as well. Employing the visual metaphor, we speak of arcs of observation and a field of view, and this is precisely the terminology and unit of measurement used in astronomy. We find that in navigation, a single field of view can contain enough information to triangulate both position and distance, given observable landmarks of known size. And this sort of information is usually cumulative. So; it is possible to learn quite a lot from just one observation, given the right conditions.

However; in a space that is truly minimal in its extent, these ideas need to be re-worked, as the field of view collapses. All familiar objects have size or extent, and the familiar 3-d space we inhabit is also extended, and expansive. We can speak of an absolutely empty space, but this too portrays a sense of extent. Luckily; we can avoid confusion, if we simply introduce the concept of indeterminacy, showing how this can be exchanged for known extents (in our thinking process), until we gradually bring things into focus. Constructively speaking; we don't know anything about a space or system, until some observation or measurement process allows us to demonstrate that it is so. It is indeterminate, or ambiguous, until observations are made, and a reference frame is established. And since constructive Math doesn't assume the existence of a continuum, we can work toward an understanding (or a system of measurement) in discrete steps. This makes this approach especially well suited for applications in Quantum Mechanics and Computing.

The ambiguity that exists for undefined, or indeterminate, systems is quite a bit deeper than an uninformed observer might imagine, however. We don't know how many extended dimensions a space possesses, or if there are any at all, until we observe and explore it. We don't know if it is discrete or continuous. We don't know if it has a well-defined metric, that allows us to measure objects and distances, or if the geometry of that space is non commutative, making calculations of extent more complicated, or even impossible. The concepts of size and distance need to be expanded, when we consider spaces with non commutative geometry (Connes 2000). And quantum indeterminacy is a bit (or rather a qubit) more complicated than the ordinary kind. But the complication doesn't end there. We now know that objects and spaces can possess dimensional characteristics which are somewhere between whole-numbered values. That is; the spectral and/or Hausdorff dimension of these entities can take on fractional values, making them fractal figures and spaces.

I should mention here that a space needs at least two dimensions, for objects to be measurable, or even topological. Measurable objects with continuous boundaries or surfaces, and independent centers, can only exist in spaces with a dimension of 2 or higher. Since measurability is connected with the calculus of areas, and derives from our ability to cover a surface with an array of rectangles (or other polygons), a 2-d lower limit makes sense. In the constructive formulation, however, spaces do not have an intrinsic dimensionality, but instead their dimension is determined by the array of observable objects and distinct viewpoints manifested in that space. I would argue that this determination is only valid within a range of scale, or magnification, as well (due to natural constraints of observability).

3. Minimal Objects and Spaces

A fully minimal point-particle is smaller than any dimensional space which might contain it, and this makes it essentially scale-free or scale-independent. Normal objects in 3-d space appear smaller when they are further away, and grow to fill our entire field of view, as we come closer and closer. However; minimal objects always appear infinitesimal, if they can be observed at all, and though they can fill minimal spaces. The tricky piece of the story is that the minimal *aspect* of an object can only be observed in (or from) a space which is non-minimal, or extended. We need to be outside of an object, and observe it from a distance, to see it in its entirety, or measure its extent. Ergo; one needs a place in some extended dimension for an observer to be, removed from, or relative to the object under observation, for a detailed measurement (or a distance/size estimate) to be made.

If we study what happens when one starts from a 0-d space, and adds more dimensions, some of this will become clear. In a space of zero dimensions, the spatial state of an object and its observer are superimposed, as there is only one location for both to be. We can say that they are in agreement, or are co-equal, as they are not separated, nor is there any tangible difference. However; there *is* considerable ambiguity, in this situation, as it's uncertain whether what is being observed is the intended object, or the presence of the observer itself. So uncertainty is a definite aspect of reality, for zero-dimensional spaces. Nonetheless; there is a unified nature also, giving observer and object a common, or single, identity, making their state a superposition. This changes, when we enter a 1-d or higher dimensional space, as distance and separation become real, and this allows objects and spaces to have an extent.

Positing that we are a distance away, so that clear observation can take place, will allow us to go further with this idea. The general form for a fixed distance from a given point is what's called an n-sphere, where n is the dimensionality of the figure's surface. An ordinary sphere in 3-d space is called a 2-sphere, because its surface is 2-dimensional. A circle is then called a 1-sphere. But the 0-sphere is a curious entity indeed. This degenerate case of the spheres doesn't have a continuous boundary, or a surface, as the other spheres do. It consists of two points on a line which bracket the point exactly between them. This makes the 0-sphere a dual entity, existing in two places at once, and nowhere else. One could say that it exists in a binary state, except that it's actually in two location states simultaneously.

Generalizing a bit further here, we note that a brane is a figure which can 'wrap around' an object whose surface is a given dimension. Branes are assigned numbers in similar fashion as the spheres, in this instance. The 2-brane is that figure which will wrap around a ball, or filled sphere, in 3-d space, completely covering it (and obscuring it from view). A closed loop of string, or 1-brane, will wrap around a disk, or filled circle (Greene 1999). A 0-brane is then like the 0-sphere, being two distinct points with a point of reference somewhere between them. However; we don't know exactly where (as branes hide their contents) and a 0-brane is vanishingly small too. So it appears to be a point-particle, except at the smallest scales, and it allows one to bracket the location of an object to within an infinitesimal distance.

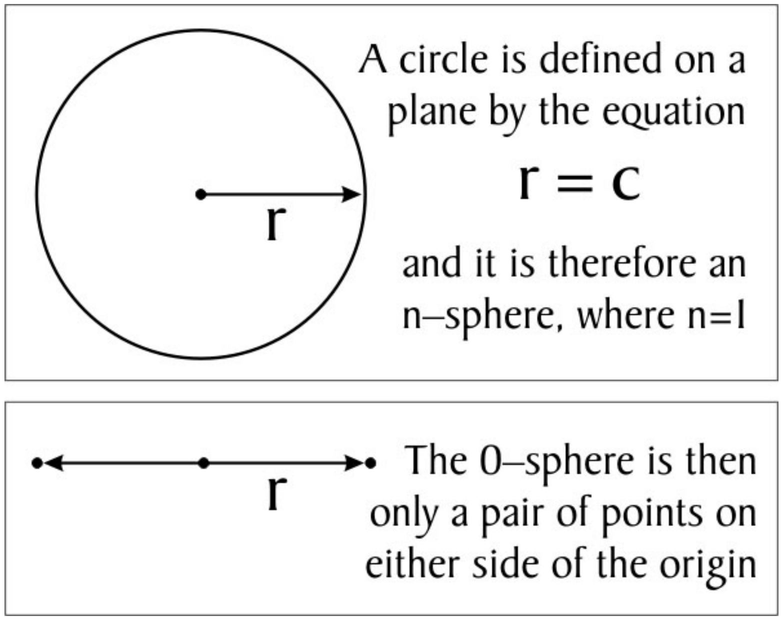

Fig. 1

4. The Dynamics of Measurement

In constructive Math, the statement "there exists" is synonymous with "we can construct a generator, such that…" Therefore; if we wish to follow through in this fashion, we must explain how we make it to each state we occupy along the way to a stated result. So; making a determination about the extent and dimensionality of objects and spaces requires observation, exploration, more observation and comparison, with repetition of this process, and some accumulation of information acquired along the way. In other words; it requires computation with the preservation and transfer of information as we proceed! And this, in turn, requires some kind of structure to contain the information that is evolved.

A network of distance estimating nodes is needed, to determine the size of an object, and the dimension of the space it inhabits. The interesting thing about this statement is that we can interpret the word 'determine' (as used above) to mean either 'measure' or 'define' and it works both ways. This implies that there are parallels between the process of assembling a computational, or perceptual, measurement apparatus, and the process of defining the measurable (or observable) attributes of objects and spaces. And this makes things begin to sound less like were studying a collection of dumb objects and witless observers, inhabiting a space with no sensibility to speak of, and more like there is a condition where everything preserves information, and is intelligent on some level.

Moving our focus from objects and spaces to minimal *systems* thus gives us an interesting new perspective here. If the question is "How large an observational array (or network) must we have, to make an accurate determination of size and dimensionality?" we find that the issues change at each stage in the process, and as we progress from the most minimal systems and spaces, to larger numbers of nodes, dimensions, or objects. We can start with one computational node, with no objects to register, or observe, and no observer or mechanism to read its value. In a 0-d space, there is nothing *to* observe, as the possibility for independent objects does not yet exist (making it essentially an empty space), and any observation we make will be unclear, or indeterminate. The lone node above is like this, because there is nothing it can observe, its state is indefinite, and its observations cannot be communicated to the outside world (as there is no connection).

Truly minimal spaces do not admit objects, per se, but a fully minimal point-particle can theoretically occupy a 0-d space, though we couldn't observe it, and have no way to demonstrate it is so. The state of a zero-dimensional space is likely what Taoists call Wu-Ji; neither light nor dark, neither hot nor cold, and neither large nor small. All these distinctions come later. A concentric observer might detect a difference between the presence and absence of a minimal point particle in a 0-d space, but still have no way of knowing which was which. The ambiguity, in this instance, is about equal in magnitude to the knowledge we can possess.

The same is true for anyone looking from the outside, when attempting to probe, or look into, a minimal or dimensionless space. As we get closer and closer to 0-d, a point is reached where the indeterminacy is as great as our knowledge. This is precisely the situation we encounter at the Planck scale, because the Compton wavelength is roughly equal to the Schwarzschild radius of a Black Hole, at that scale. One encounters a similar situation exploring fractal figures such as the Mandelbrot Set on a computer.

Fig. 2 : *A sunburst from the Mandelbrot Set*

As one zooms in further and further, a point is reached where the distance between adjacent pixels (representing points on the complex plane) is smaller than the least significant bit on that machine. The figure undergoes what is called binary decomposition, where what is displayed is no longer a fractal, but decomposes into squarish pseudo-pointillist or spectral forms.

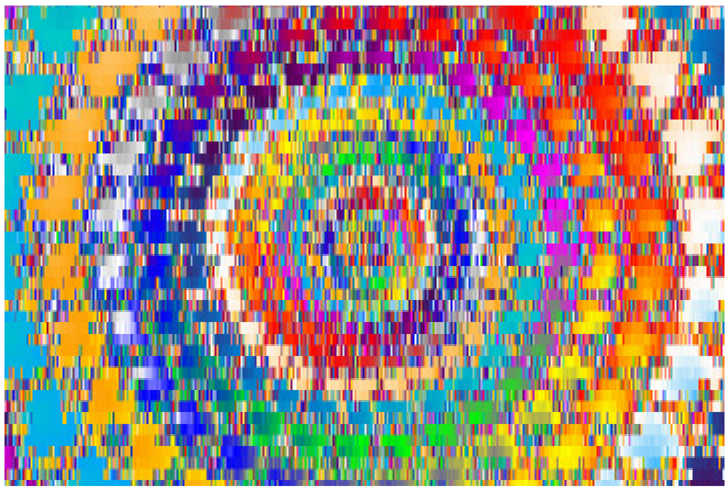

Fig. 3: *Zoom of center shows the onset of binary decomposition*

Working outward or looking upward, from a zero-dimensional space, brings us into the realm of extent, where space has one or more dimensions. Each step along the way is unique, however, and adds to both our knowledge and the array of possibilities we've developed. A step toward the horizon in 1-d space can bring you to 1 or -1 (unit), though you have no way to know which. This is like having a binary computer, with the conventional logic of 'either-or' thinking, but uncertain bearings. As I see it, this basic ambiguity of relation is a root cause of quantum uncertainty, finding expression in complex numbers and quantum fluctuations. And it carries forward, as we continue. Clearly; a point-like observer in a space with more dimensions might have far more to observe, and would be free to move in more directions, but still could not judge the size and distance of objects accurately, without prior knowledge.

Adding a second node gives us the possibility of mutual observation, some form of measurement or estimation, and (if there is a means of sharing information) comparison of results. In this manner, a form of depth perception can be achieved, too. More nodes give more detail, allowing more precise size and distance estimates (or other measurements of observables) to be made. There are several questions that arise here, including what mechanism objects or observers may use to encode measurable parameters, whether forces are involved in conveying information, and so on. For the sake of brevity, I will assume the possibility for objects and observers to possess and exchange information is valid and/or reasonable.

5. Computing Reality

We find that both objects and observers take on properties of computing engines, or systems, in this context. There has been some speculation that physical systems have the properties they do *because* they can function as computers. This would explain the 'unreasonable effectiveness' of Mathematics (Wigner 1960) as a tool for description of the physical universe, and give us numerous powerful new tools for understanding it. We could alter Descartes' famous phrase and state "It computes, therefore it is!" But this way of thinking about things is by no means new, and modern thinkers echo a sentiment that has deep roots.

John Archibald Wheeler coined the phrase "It from Bit" (Wheeler 1990) a number of years ago, to reflect the view that material form arises from its underlying informational content. Since that time, a number of others have expounded, or expanded, upon this concept (Zeilinger 2004; Sarfatti 2004). Paola Zizzi (2001) and David Deutsch (2002) have employed an updated form of this phrase "It from Qubit" to reflect the fact that we are talking about quantum mechanical bits, rather than the conventional kind. In a 1-d space, as I have stated, we find that a step toward the horizon (from one's point of origin) results in a binary decision, taking us to one of two possible destinations. But either/or quickly becomes something else – something far more interesting!

When we expand this idea into higher dimensions, a step toward the horizon brings us to a point on the surface of an n-sphere, where the dimension of space is n+1. Accordingly; our point of origin is also somewhere on the surface of an n-sphere, with respect to our new location, once that step is taken. I believe that we should consider the first step a fundamental length unit, where minimal spaces and systems are concerned, but we can substitute the word brane for sphere in the construction above, if we wish to generalize to an arbitrary or infinitesimal distance. The 0-brane is sometimes called a non-minimal point particle, for that reason.

We see that truly minimal systems have an undeniable simplicity to them, but when we add objects or dimensions, and posit properties which can be possessed by objects, the level of complexity rises swiftly. Information at the minimal level is both grainy (or spectral), and imprecise (or fuzzy), but it becomes more well-defined as systems evolve. When we add observers, or ask what network of information-processors might effectively constitute a knowledge system about those objects and spaces, things get very interesting. In the realm of self-evolving computational systems, Stephen Wolfram states "If one starts from extremely short programs, the behavior one gets is at first quite simple. But as soon as the underlying programs become even slightly longer, one immediately sees highly complex behavior."(Wolfram 2002)

His book “A New Kind of Science” expounds on how the laws of Physics, the nature of objects and forces, and the fabric of space itself, can all emerge from relatively simple computational processes, and computing networks. But a large number of physicists were already convinced, for other reasons, that conserving information might be even more fundamental than the conservation of mass or energy, and that reality may result from computational processes. Nor is Wolfram the only one who has demonstrated that great complexity can arise from simple computations. There are now many compelling examples of this fact. Complexity does not require complicated rules or systems, to emerge robustly.

One particularly well-known example of complex forms that arise from simple formulae is the Mandelbrot Set, whose generating equation ($\mathbf{z}_n = \mathbf{z}^2_{n-1} + \mathbf{z}_0$ | where $\mathbf{z}$ is of the form a+bi) involves simply squaring complex numbers, adding the original number, and repeating the process with the result, until some limit is reached. Once we assemble a computing network capable of measuring the size of objects, and finding the dimension of the space they inhabit, we have sufficient computing power to calculate the Mandelbrot Set, and other fractal figures. So the possibility for relatively simple systems to generate complex behavior is easily demonstrated, at this point, and the progression from minimal spaces and systems has been enumerated. It is important for the reader to note that the objects and spaces we’ve contemplated here, and the knowledge systems proposed to understand them, show a similar and parallel evolution.

6. Information, Knowledge, and Cognition

This suggests that it is indeed *because* objects and dimensional spaces function as information receptacles, and sense, process, and exchange observations about their surroundings (or contents), that they exhibit the complexity we observe in the universe at large. The most exciting piece of this story, however, is that it relates powerfully to the manner in which sentient beings learn about the world. If we observe the behavior of very young human children, we see that they build their internal grids in a manner that is very similar to the progression I described above, for building dimensional spaces and creating knowledge systems to measure them, starting from the minimal cases. We know where the story of children begins, but we can only guess at their point of awakening, or their first observations.

A baby in the womb might open its eyes, but have little to take in. It might have the sense of being a unique observer, but there would also be the sense of being merged with its surroundings, or being a part of its mother. There is no way for it to know the difference, as the sense of separation would not be a reality for the little one yet. However; we have no way to get that information, as a baby in the womb has little means to communicate with us. That makes this stage of our development similar to the 0-d spaces and single node systems described earlier. Although this can’t be proven, it cannot be easily disproved either, and there are compelling reasons to make this connection.

Very young children learn about separation rather quickly, but it takes them a while to acquire a sense of object constancy. A game of peek-a-boo is endlessly fascinating to an infant; as they really are surprised to see you each time you peer out from behind an object. But putting a toy away can be a tragedy, because the very young need to learn it still exists even when they can't see it. The binary sense of "there or not-there" is gradually replaced by a sense that objects can be a procedural step away (opening the toy chest), and still be real, or persistent. After this point there are still confusions, however, according to the research of Judy S. DeLoache, as a child may mistake a life-sized picture for a real object and try to put on the shoe in a photograph, or talk to the 'kid' in the mirror, or and so on (DeLoache 2005).

Gradually; an understanding of dimension, size, and proportion emerges. Before this happens, though, a child might try to sit on a doll chair or get into a model car, as they apprehend its purpose (or recognize its form), but don't seem to grasp perspective, in terms of how distant things appear smaller, or perhaps they don't know how big they are themselves, quite yet. But with continued observation, it appears that a sense of varying degrees eventually replaces the child's earlier binary distinctions. And it may well be that acquiring a sense of proportionality is what allows us to assign fractional weights to our observations, rather than having the binary logic of a world with strict either-or, yes-no, black-white, distinctions. Being able to 'put things in perspective' and having a 'sense of proportion' could be literal, as well as figurative.

A young child will observe, then explore, observe some more and compare, explore some more and observe again, examine objects from various angles or distances, and so on. The reader should recognize the methodology described before, for determining the size of objects and the dimensionality of spaces. We also observe a strong connection with the constructive approach (in Math and Logic), at this phase, as the very young make no assumptions and display openness without qualifications. As more and more observations are made, however, they know that things in their world are a particular way, and the child develops a sense of the relative sizes (and spatial arrangements) of different spaces and objects.

This marks an important step forward. Only once this phase where they learn about dimension and proportion has been mastered can young children begin to learn the real power of symbols, as forms which represent something else. In DeLoache's research, children were shown a scale model room, with a toy version of a hidden object, clearly visible, then released into the actual room, and told to find the object. Those younger than a certain age could not make the connection, but it was obvious to their older peers (around age three). After this point, symbolic thinking develops quickly, along with a range of complex behaviors.

7. Conclusions

Complexity arises when we try to squeeze more information than will fit into a system. As systems evolve beyond a threshold, there is more to represent than there are ways to show it. An information explosion occurs, after a certain amount of observation and comparison takes place, or after a certain amount of rules-based evolution from a simple basis. And we see that the threshold for producing complexity is actually rather low, in that simple systems executing relatively simple calculations and procedures can evolve exceedingly complex forms and behaviors. What this indicates to me is that information, or knowledge, often evolves faster than the structures (and rules or behaviors) which exist to contain, or express, them.

The Mandelbrot Set requires fairly simple arithmetic, and it resides on a 2-d surface within a circle, but it appears almost endlessly varied, in the range of form it contains. And in general, we see that more complicated systems do not necessarily exhibit more complex behaviors. Wolfram echoes this sentiment, stating "above a fairly low threshold, adding complexity to an underlying program does not fundamentally change the kinds of behavior it can produce." (Wolfram 2002) Rather; it could be the bounding surfaces reigning in (or containing) evolving form, which force systems that contain too much information to become complex.

It appears that there will always be fundamental limits for evolving systems to push against. One might imagine that a filled n-sphere would increase in volume (or hyper-content) as more dimensions are added, but this is not the case. Instead, according to MathWorld, "the 5-dimensional unit-ball has maximal content,"(Weisstein 2003) and what is contained decreases toward 0 as we add still more dimensions. So; more is not necessarily better, for adding content, or building complexity. Rather; the fact that possibilities multiply quickly, once a certain level of orderly arrangement is established, makes studying the minimal cases important to understanding how both physical and knowledge systems evolve.

To understand how complex forms and behaviors emerge, we must discern the levels of abstraction that arise in the creative process. As constructive Math and Logic so aptly illustrate, the process of creating a knowledge system to study something (e.g. - prime numbers) can also give one the means to generate that thing. Recent work on a theory called Causal Dynamical Triangulation (Loll et. al 2006; Ambjørn et. al 2005 and 2006; Zizzi 2003 and 2005; Markopoulou 2002), suggests that studying how space can be measured and approximated with a simplicial fabric, at the smallest level of scale, allows us to chart its evolution. This theory yields the interesting result that spacetime is 2-d at the Planck scale, displays fractal structure for constant time-slices, and resembles the familiar 4-dimensional spacetime, at the cosmic scale. Working from minimal cases, building on first principles and moving forward, gives us a uniquely useful perspective for studying all manner of systems, even highly complex ones, and the constructive approach gives us tools for discerning the levels of abstraction that arise along the way.

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