How do negative and imaginary numbers relate to reality?

Negative as well as imaginary numbers relate to the natural numbers like extensions.
May we conclude from this that they immediately extend realistic models of the real world?
Gauss wrote [1]:
"Positive und negative Zahlen koennen nur da eine Anwendung finden, wo das Gezaehlte ein Entgegengesetztes hat, was mit ihm vereinigt der Vernichtung gleichzustellen ist. Genau besehen findet diese Voraussetzung nur da statt, wo nicht Substanzen (fuer sich denkbare Gegenstaende), sondern Relationen zwischen je zweien Gegenstaenden das Gezaehlte sind.

"Positive and negative numbers can only be applied where the counted has an opposite, the union with which equals annihilation. Strictly speaking, this is only the case where the counted is not substances (things that can be thought per se) but relations between two things each."

The alternative to positive as well as negative numbers is numbers without a sign rather than formally positive numbers. Gauss is obviously still correct. Virtually all basic physical quantities have an absolute zero, for instance temperature, mass, spatial and temporal distance. This does not exclude derived quantities like for instance sound pressure to alternate between positive and negative values superimposed to the much larger average pressure of air. Likewise, the alternating electric current can be imagined as a small component superimposed to a larger average motion of elementary electric charges. The arbitrarily defined alternating components have to have an arbitrarily chosen reference-point zero. Negative temperature on the Celsius scale relates to the arbitrarily chosen criterion that water has been frozen to ice. When Descartes introduced the spatial Cartesian coordinates x, y, z, he strived for as little as possible arbitrariness. Ironically, the absence of natural reference points for them requires arbitrary choices of the zeroes. The same is true for the abstract notion time. Even ubiquitous agreement on a world time is not natural but arbitrary. One can neither measure a positive and negative radius nor a positive and negative elapsed time. Likewise, the corresponding quantities wave number and measurable frequency do not need a sign. While our ordinary notion of time cannot be thought per se, elapsed time is obviously a measure that does not need a relation to another thing. What is still future time cannot be measured in advance.
Elapsed time is therefore basic to the ordinary time abstracted from it.

Don't worry about the negative sign of the elementary electric charge. It has no physical bearing. When Lichtenberg coined the denotations positive and negative electricity, he could not yet know that electrons are responsible for what he called negative electricity. It is also quite logical while seemingly baffling that the magnetic North Pole is located at the geographic South Pole and that elapsed time corresponds to negative ordinary time. Basic physical quantities can be attributed to the unilateral (one-sided) ray of all sign-less numbers IR+ between x=0 and the arbitrarily large rather than to the bilateral (two-sided) line of all real numbers IR between minus infinity and plus infinity.
The entity IR of all real numbers reads blackboard bold R.

Accordingly, virtually all functions f(x) of basic physical quantities are in principle unilateral functions f(x>0) even if it is often more reasonable to consider them as if they were bilateral in order to benefit from Fourier transform.

Fourier transform (FT) is a complex integral transform. Its application on F(x>0) demands four carefully performed steps:

At first, the unilateral original function f(x) of the variable x>0 must be adapted to the range of integration extending from –oo to +oo. This is usually done by assuming f(x<0) = 0 and splitting the extended f(-oo<x<+oo) into even and odd parts.

The second step is the multiplication of the even and odd parts with the complex kernel of the integral. It tacitly corresponds to a transform into a special artificial complex domain.
As a result, the Fourier transform F(-oo<y<+oo) of f(x>0) is necessarily twice unphysical: bilateral and complex. It exhibits Hermitian symmetry with trspect to its variable y: Its real part is mirror-symmetrical to y=0, its imaginary part is anti-symmetrical to y=0. The variables x and y are called Fourier transform pairs.

It is sloppy but common practice to consider for instance time t and angular frequency omega such a pair.
Actually, in an original function f(x=Omega t), the variable t is real and unilateral while Omega is considered a constant parameter. In the belonging complex Fourier transform F(y=omega T), the somewhat strange variable omega is bilateral. Strictly speaking, the pair must consist of the physically correct t and the twice unphysical omega.
This clarification is necessary because one can also chose a physically correct original function f(omega T). In this case, F(Omega t) is the twice unphysical function of the somewhat strange bilateral variable t.
Accordingly there are doublets of two possible FT pairs: unilateral t and bilateral omega or unilateral omega and bilateral t. Either x or y must be unphysical. The complex frequency domain cannot simultaneous be the complex time domain and vice versa.

The reason for the unavoidable unphysical behavior is the tacit transform into a particular and arbitrarily chosen complex domain, which must not be confused with complex plane of mathematics.
Let's start with Euler's identity:
2 cos(x) = exp(jx) + exp(-jx).
Here j stands for the imaginary unit. Electrical engineers prefer j in order to avoid confusion with the symbol i for electric current. In English language, j may prevent unwelcome automatic correction of i into I.
The two exponential functions can be interpreted as rotating phasors, sometimes also called vectors in the complex plane. Transform of cos(x) into the complex x domain means "simplification" by arbitrary omission of one of the two phasors. Some disciplines preferred to omit the anti-clockwise rotating phasor, others the clockwise rotating one.
The other way round, the omission can also be seen as addition of a positive or negative imaginary part sin(x) to the original cosine: cos(x)<- ->> cos(x)+ j sin(x) and its omission with inverse transform.

Let's not yet call calculus in complex x-domain the third step and also abstain from premature interpretation of results in complex x-domain. The third step is the inverse FT. If all three steps were performed correctly, then the imaginary part will vanish with correct inverse transform.

While almost everybody used to frequently sin by skipping some of the trivial steps, almost nobody used to perform the fourth and last step before final interpretation of the result: Even and odd part of the result from inverse FT have to be added as to eventually get back a fully physically correct unilateral result. Sloppiness typically results in a seemingly time-symmetrical result.

While negative frequencies in a complex domain are just mathematical artifacts without physical meaning, this does not mean they may be neglected. They convey the encoded unilateral origin and its sign.

Unqualified or utmost self-confident users of complex FT skip all steps, make an ansatz as to immediately enter into an unspecified complex domain and do not even shy back from wild guesses concerning the due interpretation immediately on this shaky ground. Some mathematicians who were not familiar with the transform into a special complex domain fostered such careless practice by thoughtlessly using the misleading words "in general complex".

In principle, spectral analysis does not always require integration from –oo to +oo. The essential condition is completeness between two absolutely reflecting boundaries, no matter whether such a boundary is of Dirichlet or Neumann type, an electric open circuit or a short circuit. Fourier himself dealt with infinitely periodic heat conduction in a ring. One may imagine the ring cut and replaced with periodically repeated identical structures each of which located between two absolutely reflecting mirrors for cosine or sine functions, respectively.

As explained above, physical reality fits to f(x>0). Prediction fits to f(t>0). It is a matter of taste whether one continues it by assuming f(x<0) = f(x>0) or already excludes all negative x from the very beginning. In both cases cosine transform yields the spectral decomposition. Admittedly, cosine transform would fail for the unphysical exception f(x) = sin(x) marking a dead point, which is fortunately irrelevant in practice.

Comparison between complex and real-valued spectra yields the surprising result that the seemingly richer complex representation of reality does not include more information as compared with the simpler result of cosine transform except for information about the arbitrarily chosen reference point t=0. Complex Fourier transform yields a twice redundant result with four times as much data as essential. One must not expect additional mathematical extensions to influence physics due to the seemingly enlarged degree of freedom with introduction of negative or even complex numbers. Positive numbers and cosine transform are tailor-made for basic physical quantities. The redundancy with complex FT gives frequently rise to non-causality and ambiguity.

Cosine transform (CT) does not only offer benefits in practice of coding signals. It is also a touchstone for possibly questionable theories or interpretations. While FT-pairs are always twin pairs of one physically correct and one unphysical variable and vice versa, as explained above, CT-pairs like radius(>0) and radial wave number (>0) or elapsed time (>0) and frequency(>0) consist of two physically correct variables.

Can position q (in m) and momentum p (in Js/m) also be considered like a CT pair? At first, we may clarify that Planck's constant h is a factor that can be ascribed to the momentum. We arrive at the insight that uncertainty is based on the same purely mathematical reason for the CT-pairs frequency/elapsed time and momentum/distance. Both the exponential kernel of FT and the cosine kernel of CT are restricted to one for any physically correct input. Simply speaking, the number of periods must be integer. Bandwidth has a lower limit.

Heisenberg's so called quantization condition pq-qp= -j h_bar turns out to be an artifact of FT. With CT, i.e. without j, it simply reads pq-qp=0 as to be expected.

Meaningful complex physical quantities like complex impedance and complex permeability often describe the relation between two different physical quantities. For instance:
Complex impedance equals to voltage divided by current. In this case, the rotating phasors of complex voltage and complex current cancel each other, relative phase remains.
Reactive power is the average of the real product of purely imaginary components of voltage and current.
Elderly physicists like Ellis and engineers like me learned that real parts and imaginary parts must not be mingled.
The advantage of representation in complex time domain is the option to perform d/dt and int dt for a given constant Omega by means of multiplication with j Omega and division by j Omega, respectively.
Imaginary components are always fictitious counterparts to real ones. In reality there is neither a purely imaginary resistance nor a purely real permeability. Evanescent modes are physically meaningful imaginary field components. However they are never complete fields per se.

To be continued. Dec. 17, 2008