

How do negative and imaginary numbers relate to reality?

Part 2 (Dec. 19, 2008): Some mistakes and their consequences

Every expert knows: One must not simply add two complex physical quantities as if they were real ones. Real and imaginary parts are orthogonal to each other. Instead, one has to add their squares according to $\sin^2 + \cos^2 = 1$.

The awareness is not likewise high for another peculiarity: The typical complex physical quantity is rather unrealistic in that, it is monochromatic. Its spectrum contains just one single frequency, wave number, or energy, respectively. The expansion of a realistic function of time is a sum of many sinusoidal components of different frequencies for each of which the complex calculus can only be applied separately.

To what extent were the fathers of quantum mechanics aware of this restriction to just one frequency for each complex frequency domain? At that time, electrical engineering used complex calculus for alternating current systems with only one common frequency. Einstein's father was a manufacturer of electrical equipment. Schroedinger was teaching students of electrical engineering. Dirac was an electrical engineer.

While John C. Slater was possibly the first one who suggested to Bohr and Kramers Fourier components, the famous paper [2] did not yet reveal details. De Broglie [3] still wrote $\exp(-hf/kT)$.

Papers by Heisenberg, Dirac, and Schroedinger show that they made an ansatz which considered the frequency components altogether for instance $n=1,2,3, \dots$ in $\exp(j \omega(n) \alpha t)$ [4] or $\alpha=1,2,3, \dots$ in $\exp(j(\alpha \omega)t)$ [5] or $\omega_{\alpha}=1/2, 3/2, 5/2, \dots$ in $\exp(-j \omega_{\alpha} t)$ [6].

Following Kramers and Heisenberg, Schroedinger and all the others started with such a multiple complex ansatz without mentioning that, strictly speaking, each frequency component belongs to its own complex domain. The presumably lacking awareness did not matter as long as they did not perform operations with their ansatz.

The Hamiltonian point of view is physically correct with real functions of unilateral frequency. So it does not at all match to the traditionally assumed complexity of the n frequency domains, each of which belongs to a real and unilateral counterpart to be superimposed in time domain. Nobody understood that Fourier transform unavoidably joins a physically correct function with an unphysical one.

Incidentally, the Hamiltonian description in terms of energy/frequency is equivalent to the description in terms of time. Therefore time must not be denied.

While Schroedinger's complex picture and Heisenberg's representation by Hermitian matrices are mutually equivalent, real and imaginary parts are not obvious for the latter. Schroedinger noticed that his wave function of time is complex. However, he failed to provide the due correct interpretation when he suggested considering its real part the wave function in time domain. A little bit later the physicists decided to consider the also real squared magnitude instead. Multiplication with the complex conjugate is a common but incomplete practice of returning from a complex frequency domain to the real function of time. One also needs the ignored phase as to correctly return to a realistic unilateral function of time.

As a result of the unfinished inverse transform from complex domain into reality, the world of quantum mechanics was and is still believed to be symmetrical with respect to past and future.

Weyl admitted in [7]:

“The problem of the proton and the electron is discussed in connection with the symmetry properties of the quantum laws with respect to the interchange of right and left, past and future, and positive and negative electricity. At present no acceptable solution is in sight.”

Because past and future are nowhere else symmetrical, Schulman [8] wrote:

“The two greatest puzzles of our age have their origins at this [10^{-6} cm] interface between the macroscopic [not time-symmetric] and the microscopic [time-symmetric] worlds.”

Actually the laws are everywhere invariant against shift and reversal of the abstract time while any process is exclusively influenced from past events.

Presumably, several claimed symmetries in physics are artifacts due to wrong interpretation of sloppy use of transforms into a complex domain and therefore imperfect return. Exact mirror symmetry is rarely to be found in reality.

The illusion that negative and imaginary numbers make the reality richer has perhaps more serious implications. It led via the EPR paper to the idea of entanglement and questionable hope for quantum computing.

[1] C. Gauss: Theorie der biquadratischen Reste (1831)

[2] Bohr, Kramers: Slater Z. f. Phys. 24,69 (1924)

[3] L. de Broglie: Phil. Mag. 47, 446 (1924)

[4] W. Heisenberg: Z. Phys. 33, 879 (1925)

[5] P. Dirac: Proc. Roy. Soc. A 109, 642 (1925)

[6] E. Schrodinger: Naturwiss. 14,664 (1926)

[7] H. Weyl: Gruppentheorie und Quantenmechanik, Preface to 2nd ed. 1931

[8] L. Schulman: Time's arrow and quantum measurement (1997)

To be continued. Dec. 19, 2008