Different Infinities and The Universe

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Abstract

For a long time, many people had thought that the nature should be smooth. It looks as if smoothness describes the universe in more aesthetical and stable way. 'Analog', as being closely related to smoothness and infinity, seemed to provide the world eternity and stability. On the other hand, 'Digital', although the concept that everything has its fundamental discrete elements had sprung from ancient time, looks rather bizarre and mortal to our analog-lly tamed eyes. It is only when atom was discovered, eventually something 'digital' became genuine part of our physical reality. Unlike our physical conception, 'digital' objects played main characters in mathematics. Mathematicians have struggled to build 'analog' object as a rigous subject. Knowing that objects are composed of discrete elements raise us questions about discreteness of the fabric of space. This article will borrow its ground from set theories which can deal with both analog and digital properly. We will see what the nature should look like in set theories with finite elements and infinite elements. And see how we can use different infinities to understand the physics.

1 Introduction

1.1 Outline

Our purpose is to reveal what kind of universes can be built from certain ground of set theories. What we are mainly interested is how the principles of physics behave, as we assume fabric of space is locally 'digital' or 'analog'. We will work with minimum principles of physics, in order to make the context as compact as possible. We try to make the contents understandable to wider range, by reducing technical issues as much as possible. In the following subsections, we will clarify the notations we are going to use, and present minimum preliminary of set theory of infinity and principles of physics for this paper. In section 2 and 3 we will examine principles of physics within the foundation of finite set and infinite set. And in last section, we sum up and conclude.

1.2 Definitions and Preliminary

Definitions

The meaning of notions 'analog' and 'digital' are rather vague. We will clarify its definition in the language of set theory. To satisfy mathematical rigour, we define 'digital' first and let

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the definition of 'analog' as the opposite. What's digital is some phenomona or object that can bisect finite times, and reach at countable elements which are called 'digits'. To make the definition appropriate for set theory and as simple as possible, we define 'digital' as the set composed of countable elements.[1] Thence, We define 'analog' as uncountable set.

Cardinality

It was noted more than a century ago by Georg Cantor that infinite set has different sorts. For example, integer set and real number set cannot be 1-1 corresponded to each other. In this case, we give diffenrent cardinality to both sets. There is a theorem that says if $\beth_n$ is an infinite set with certain cardinality then $2^{\beth_n} = \beth_{n+1} > \beth_n$. So we know there are infinite number of cardianlity. For simplicity, we will only deal with interger set and real number set throughout the text.[2]

Continuum Hypothesis

Familiar infinite sets, the set of integer and the set of real numbers, were first shown to have different cardinality. Continuum Hypothesis says that they have cardinal number next to each other. Simply,

$$\aleph_0 < 2^{\aleph_0} = \aleph_1$$

$\aleph_0$ is the cardinality of integer and $2^{\aleph_0}$ real numbers. And $\aleph_1$ is the smallest cardinality greater than $\aleph_0$. It was shown that the hypothesis is independent of standard set theory. Which means, with standard axioms of set, it is not both provable and disprovable.

Physical Principles

Only couple of principles of physics will be mainly considered for simplicity. Those are from quantum theory and big bang cosmology. The principle of uncertainty and the observation of expanding universe will be presummed. And also the space will be supposed to be locally conneted. We use these concepts without clarification assuming it is familiar enough.

Our set will contain the position of the space mostly. (it directs the position of particles sometimes.) There is an axiom needed to be chosen when one deals with infinite sets, which is Axiom of Choice. Informally saying, the axiom states whether one can pick one element out of infinite set or not. This can be important if one wants to build legitimate theory of space with quantum theory.

1.3 Axiom of Choice and Uncertainty Principle

When we talk about the space as a set of points, we should be careful with what we mean by saying a point. Classically, picking up certain point of geometry doesn't make problem. But with quantum theory, or uncertainty principle, we are not sure what it should mean to pick up an element out of points of space. For example, if one select one point out of infinite set (AC(Axiom of Choice) is on) and any particle turns out to exist at that point, uncertainty principle breaks down. Because we are now certain that a particle was at that position, the momentum of the particle should blow up. So if we are allowed to pick up an element then the point should not have physical meaning by itselt. Only interval of space, or

[1] Infinite countable set is strictly digital by our definition. But it will be treated as a special case. Because, although it is countable, it does not derive digits in finite bisection.

[2] Continuum hypothesis issue would not harm our argument, since we can just select those two sets for convenience without concerning the order of cardinality.

subsets excluding the ones with sole element, should be used to represent physics appropriate with uncertainty principle.

Therefore, if one wants to talk about quantum physics in the infinite set with AC, there needs some care. Also set without AC or with negation of AC can be used, even though they are less familiar than AC as an axiom. We will see this issue again in section 3.

2 Locally digital universe

We used term local because there could be the set (ex. integer set) which is finite within certain subset but as a whole, *globally*, infinite. but with the big bang cosmology, we know that the universe had a start and it has finite size now. So once we assume the fabric of space is locally finite set then the universe is also finite object.

In this scheme, just like matters composed of particles, space also composed of discrete elements. In this universe, there's nothing analog can happen. what looks like smooth is just an illusion because of vast number of possibility. So we can expect that in the scale involving enormous number of elements behave quasi-analog, and in the scale involving quite small number of elements show digital phenomena.

It seems this setting provide consistent background for principles of physics. However it is not so. In a normal set theory, there is a theorem that you can pick up an element from finite set. As we discussed before, one position element cannot have physical meaning because of uncertainty principle. If we want to talk physically meaningful, we need to say that there exists the least subset which contains enough points to provide degree of uncertainty. Physically saying, no matter how hard we try, our observation is confined to detect certain scale. If the universe is stable, then this argument doesn't collide with physical priciples. However, since we know the universe is expanding, assuming the points are not nonlocal, the number of elements in the least physical subset should be shrinking. As a result, certainty to detect position get increased. At critical point, where the least subset contain only one element, uncertainty principle breaks down. If we think expanding universe and nonlocality as principles of the nature, then quantum theory is time dependent. In other words, In locally finite space, quantum theory, expanding universe and nonlocality of position of space cannot be altogether correct description of the universe.

Moreover, if this is the way the fabric of the space is weaved, then field theories are all mere approximation of reality, since field is analog mathematical form. In addition, string theory also doesn't fit into this discrete world because string is analog object, unless it only portraits approximate picture of reality.

3 Locally analog universe

Unlike locally digital universe, this model doesn't degenerate global structure.[1] Once we are working in this setting, we don't have to care the consistency of space depending on time. As we claimed, Axiom of Choice is not required to describe quatum theory. Let's see what

[1]At least in a real number set, any subset has the same cardinality with the set itselt, if we exclude sole element and union of them as subsets. Although the set with different cardinality can be chosen, as long as it is uncountable set, we treat only real number set for convenience.

the universe is like with and without AC.[1]

3.1 Without AC

By occam's razor, if the axiom doesn't affect the physical reality, we first drop it and see how one can build mathematical structure for physics. This regime is more general than set theory with AC. Actually, frequently used mathematical formalisms which spell the physical reality depend on AC as an axiom in many cases. It might be rather technical to build strict mathematical formalism of reality without AC. Our intetion is not to try to establish the technicality. Instead, interest will be constrained to think the fabric of space of infinity within physical sense.

For example, let's work with the real number set representing the fabric of space. Without AC, this space is perfect analog world. Since we can't pick a point, all subsets have same cardinality to each other. This space itself seems to have nothing quirky, no matter which scale you look into it. However, we know our physical world has wild behavior at some scale. What could make such an effect occur at certain scale which is not varying whereever you are examining it? what standard was it there to border where to change the reality abruptly?

One way of thinking about it is to consider that quantum scale is chosen with respect to the global set. Although we may not know why and how it was selected, there should exist special scale where the quantum world starts to dominate. However, unless every subsets know the information of it's position with respect to the global set, in other words unless every part of space is nonlocally connected, the existence of special size of space can't be dertermined. That's because this space is like an infinite Russian Matryoshka doll. If you are an local doll and don't know how far you are from the original mother doll, then there is no way to decide to start producing quantum doll. So you should know whether you are under critical point or not, and in order to do so every dolls should contain the information of the position with respect to the original doll.

Therefore, It seems that if we want to build physical space which is proper with quantum regime, there should be mysteriously selected scale and also the space should be nonlocally connected to entire space of the universe. We are not sure if this is the only way to resurrect quantum theory as a solid principle of physics in real number set without AC. Anyway, this method gives uncomfortable feeling due to not only the existence of special scale, but also nonlocal behavior of space. If we want to work with local space and with no such a special selection, quantum theory shoudn't be fundamental or there's yet undiscovered knowledge about the physics of the universe.

3.2 With AC

We argued that AC is not an essential assumption for making the mathematical building blocks relevant to quantum theory. The advantage of working with AC is that instead of replacing the basic set theoretical regime, we may still work with familiar set theoretical setup. However, once it is possible to pick up an element, the set is no longer locally analog. Because one can choose subset which contain finite elements. To make the set locally analog and to hurdle the physical interpretation of determinacy, new meaning can be given to one element of the set. Here we utilize the idea of different infinite sets.

[1]The case with negation of AC will not be covered in this text.

The trick is to replenish one element with physical meaning by inscribing different infinite set in it. This different infinite set is opened, only when sellected one component is under our physical consideration. Dealing with normal infinite subsets, which is uncountable, doesn't open up new degree of freedom. Only choosing an element gives another infinite degree of freedom. Since, now any subset has infinite degree of freedom the set is locally analog.[1] The reason why different infinite sets are used is to distinguish between choosing an element and choosing intervals. In other words, we separate the subsets into two, and give different physical interpretations. One type of subsets is normal interval, or uncountable(infinite) subsets, and the other for countable(finite) subsets.

Structural leap

As we observe, peculiar transformation of our setting is occured when we choose an element. Let's call the action of choosing an element in set theory as 'structural leap' in physical sense. The nature of space looks like classical world, facing only those intervals of sets. However, when one look at one point of space and see what the nature is like, because of structural leap, the physics is not the same as before.

For example, assuming the space is represented by set of real numbers. And structural leap occurs into integer number set.[2] when we choose a point of the space, we are given integer set of choices. This is different from working with interval of real number, still representing infinite number of degree of freedom which prevent uncertainty principle from breaking down. And quantum geometry is represented in the integer set.

It seems that this manipulation of infinite sets can provide framework which does not conflict with big bang cosmology and quantum theory. Since the set is locally analog, expansion of universe do not destroy the fabric of space. Also by imbuing physical meaning with a choice of an element, one may construct peculiar physics. However, we are not sure which infinite set is best for understanding the fabric of space. Even though real number-integer set could be the most wishful combination, it is not so desirable in mathematical sense. We are not sure real number set is the next cardinal number to the set of integer. Even worse, so far we know that it is not provable or disprovable with current set theory. This limitation prohibits us from inducing preferable infinite sets as tool kits. If the space can really be represented by these specific sets, we confront questions such as why those infinite sets are important in nature and what it means to set theory.

3.3 Amusement park of infinities

From now on, let's enjoy all those possibilities sprining out of infinite sets. This subsection would deal rather unphysical cases and bring some imaginations. Even though we don't yet fully understand which order different cardinal sets be and how each set behave, we know that different cardinality exist and they are comparable to each other. In a mathematical notation,

$$\aleph_0 < \aleph_1 < \aleph_2 < \ldots < \aleph_n < \aleph_{n+1} < \ldots$$

[1]Strictly saying, it is analog, only when new degree of freedom has uncountable infinity. According to our definition, countable infinite set is digital.

[2]Integer number set is not strictly analog. So we can't say the real number universe inscribed by integer set is locally analog set. However, for integer set is infinitely divisible, we may loosen restriction and call it is semi-analog. Working with real number set-integer number set is not an obligation. this is chosen only for convenience.

The example of the universe mentioned in last subsection can be represented by $\aleph_1$ with structural leap into $\aleph_0$.[1]

We can speculate further by thinking structural leap occuring from higher cardinality to lower cardinality, and extend it to arbitrary case. Which means, the space of $\aleph_n$ has degree of freedom inscribed on one element of $\aleph_{n+1}$. In this case, if one observe bigger scales, inverse structural leap keeps occuring and different worlds appear. And if one observe smaller scales, structural leap occurs but has the end. Physically saying, there is the universe and multiverse which contain the universe as an element and multi-multiverse which contains multiverse and so on. And by streaming through smaller structural leap, one will meet countable set in the end. If we compose the case the other way around, similarly we will experience infinite structural leaps as going down to scale. But one will meet the end of evolution as going up.

There is no reason why structural leap should occur in order. It may occur aribitrarily or only one leap is allowed. There is such a vast possibilities if we follow structural leap philosophy. Whichever is the combination of the reality, it is hopeless to examine it. Our possible task is to concentrate on determining proper sets and build geometry out of them to depict the physical space.

3.4 Geometries of infinite sets and physics

Geometry has played major role in building mathematical theory of physics. Most of current knowledge of geometry is built on real number set. Almost without doubt, physicists treat space as if it is a set of $\aleph_1$. It is because real numbers set is familiar and works well with nature. Actually, this arrangement has not fallen us seriously. In dealing with classical case, it works almost perfectly. Although some peculiarity arise when quantum principles come in to play, It's efficiency is still remarkable.

There's no alternative well established geometrical theory of set of other cardinality, yet.[2] Following structural leap argument, geometry of other cardinality could yield better understanding of the universe.

4 Conclusions and thoughts

Theoretical reasoning and experimental verifications are two cornerstones of physics. In theoretical thinking, mathematics is inevitable due to its clarity and efficiency. Now it is believed that experiments will face its limit in verifying direct evidence of fundamental concept of physics, such as time and space in planck length. In this era, mathemtical tool could be most reliable weapon to cut down the difficulties of investigating physical world. Since, direct experimental observation is rarely possible as we watch smaller scale, we should invent smart way of verifying it indirectly. To become smart, understanding its mathematical nature is first option. The more one can spell the reality with mathematics, the better chance we can get to understand the world by knowing where to see and what to expect leaning on the expectation of the calculation.

We are not decisive whether the space is digital or analog in this paper. It may depend on one's favor to have certain stance and one can keep matching her model with the worldview

[1] Let's assume Continuum Hypothesis is correct throughout this subsection.

[2] If there exists one.

about physics. Mother nature seems to be sometimes calm and sometimes whimsical. She shows such a grand panorama of the universe but leaves detail in secrets. For a long time, we've witnessed many miracles of the nature. But the most miraculous thing is that she has us to think about her world.