Mathematics, the oracle of physics

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Abstract. Mathematics studies abstract patterns, either instantiated in structured system of things given to perception or in imagination, or freely invented and characterized axiomatically. The method of modern physics, which defines it, consists in studying the reality man perceives by means of abstract patterns extracted from it and idealized as mathematical patterns, which, then, can be freely extended. In this way, modern physics invites mathematics in. By an elaborate interplay between symbolic mathematics and creative semantics of mathematical systems, or by purely formal analogies between the mathematical surrogate science substitutes for perceptual reality and no matter which mathematical domain, mathematics manages to play a fundamental role in scientific heuristics. The mystery of the applicability of mathematics in empirical science is a consequence of the belief that physical and mathematical realities are two completely isolated and self-sufficient domains which can only communicate by mysterious means. Here, I strive to show that this belief is false.

We may not be able to read our fate in the cards but we can foretell futures events by reading mathematical symbols; mathematics allows us to predict the outcome of future experience. Besides predictive, mathematics has also *heuristic* powers; that is, one can discover how the world works by mathematics means, independently of *observing* how it works, and this is surely puzzling. Mathematics is to a large extent created without much attention to how the world is; the world is what it is independently of our mathematical creations. How is it possible that mathematics has *anything* to say about the world, let alone disclosing its innermost secrets? How mathematics came to hold the keys to the secrets of reality? All sorts of dubious explanations have been offered, including pre-established harmony and God Himself.

Often, great mysteries are born out of great prejudices or *idées reçues* that go either unquestioned or unnoticed. This is a case in point. The belief, so ingrained in us so as to pass for established truth, that the world physicists investigate exists "out there", in itself, simply *given* to us, ready-made, as an object of inquiry, and that mathematics, a creation of man, *just happens* to be our best instrument to investigate the world must be called into question if the usefulness of mathematics as a tool to explore physical reality ceases to be a mystery or a gateway to the mystic.

Since the usefulness of man-made mathematics in natural science is an unquestionable fact, one must look with suspicion to the belief that physical reality is something that we simply stumble upon. The alternative that I propose is the following: nature, *as conceived in the empirical sciences*, is something that we *constitute* - and by "we" I mean the scientific

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community as a whole -, a *substitute*, built on the basis of what the senses offer but going well beyond it. Physical reality, in short, is a scientific construct. This view, advanced among possibly others by the philosopher Edmund Husserl[2], but embraced by important physicist such as Hermann Weyl[3], offers a natural, simple and historically well-founded naturalist solution for the problem of the applicability of mathematics in science.[4] In few words the solution is this: man investigates the reality he experiences with the senses, or rather, the reality he *constitutes* from the testimony of his senses[5], by *substituting* it by a *mathematical surrogate*. Mathematics is applicable in science because the object of science is *not* reality as *perceived* (and much less reality as *sensed*) but reality as *conceived*, and our scientific *conception* of physical reality is mathematical through and through. From this perspective the mystery surrounding the "unreasonable effectiveness" of mathematics in science utterly vanishes, becoming nothing but an instance of the applicability of mathematics in mathematics itself, a much less momentous phenomenon. But in order to correctly appreciate this view we must first clarify what mathematics and physics are actually up to.

The nature of mathematics. Mathematics is the a priori science of the abstract structural aspects of structured systems of things. Things that we find in our experience or, as is often the case, intentionally posit for the sole purpose of supporting interesting abstract structures. Mathematical entities in general are *nothing* beyond relata in systems of abstract relations characterizing particular structural patterns. They do not exist before being invented and exist only insofar as are coherently conceived. In other words, their existence is purely *intentional*, they only exist in relation with other "things" of the same kind whose defining properties are established in complete mathematical freedom. Sometimes, however, mathematical structures - usually simple ones - offer themselves naturally as formal-abstract aspects of our *immediate* experience of reality. Perception is a *structured* system of things (perception is constituted out of sense data but it is not reducible to them) and who says structure says mathematics.

The concept of number, for example, certainly downed on us by observing the world. We often consider things of our experience collectively, the books on the table, the days of the week, and the like. Collections are entities of experience, but the perception of a collection differs from the perception of its members. To look at Peter *and* to look at John is a different experience from

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[2] See Husserl 1954, especially chapter II, section 9.
[3] See, for instance, Weyl 2009.
[4] By "naturalist" I mean a solution that does not give man a privileged position in the natural scheme of things.
[5] Not everything that we perceive comes through the senses, an important part of it is a contribution of built-in intentional systems that constitute *percepts* out of sense data.

that of looking at Peter *and* John. Whereas the first has *two* objects, Peter and John, the second has only *one*, the *collection* whose members are Peter and John. This shows that *different* perceptions can be based on the *same* data. Now, we can look at a collection and imagine some or all of its members being substituted in a one-to-one manner by different objects and ask ourselves what these two collections have in common. Someone must have raised this question thousands of years ago and come up with a new notion and a revolutionary technology, the notion of quantity and the technology of counting, that is, determining quantity. Inventing symbols for specific quantities is the following natural step. Maybe some things were too precious to be counted just like other things, and quantities of special things deserved special names or symbols, but when men learned how to completely abstract content from quantity, giving the same quantity-name to all quantitatively identical collections of things, he gave the first step into creating numbers and inventing arithmetic. But not quite yet.

Creating numerical terms and inventing a clever technique for finding out the quantity of elements of collections obtained from other collections by certain operations (adding collections, for example) is a technology, but not a science; logistics, maybe, not arithmetic. Only when numerical terms were taken as *names* of determinate *objects* - numbers -, and numbers, or, rather, the structure of the numerical domain, taken as objects of theoretical, not mere practical interest, arithmetic proper was born.

An important step into the constitution of arithmetic as a science was the creation of flexible enough symbolic system that could generate numerical terms systematically independently of the direct experience of any specific collection of things having that quantity; for example, the Hindu-Arabic positional system. The invention of a symbol to mark an empty place in the notation (invented, apparently, by the Babylonians), later raised to the dignity of denoting the *number* zero, was a major advance.

We could, maybe, have posited only a *finite* amount of numbers, say, up to ten to the tenth to the tenth power, since our experience of quantities cannot conceivably go beyond this quantity. But this would make the arithmetical calculus cumbersome and artificial (for example, the sum of numbers that exist could be a non-existing number). Much better to postulate an infinity of numbers. By positing an infinite domain of abstract objects and carrying out the systematic investigation of its structural properties man does arithmetic proper. Many questions can be raised about collections of things in the world as to their quantities; the nice thing about arithmetic is that we can answer these questions by arithmetical means without having to care about the collections themselves.

This is as good an example as any of the applicability of mathematics in reality. Certain abstract aspects of things of experience (quantities) are subsumed under ideal concepts (numbers); the science of numbers can then be applied in our experience of the world insofar as the notion of quantity is concerned. It is interesting to observe here a certain "dialectics" between perception and free mathematical creation. Perception *suggests* the notion of quantity, but we freely *create* the concept of number; the mathematical science of numbers (also our creation) justifies itself practically by offering more sophisticated instruments of quantity evaluation that those that are available in pre-mathematical experience.

Mathematics was born in the dawn of civilization, thousands of years ago, out of necessity, and the first mathematical structures that interested man were discernable patters of organization of systems of objects of perception (the preeminence of geometry testifies to this). But man was not content with patterns he could actually perceive; he also *imagined* patterns freely. As a rule, *and in this resides the power of mathematics for our understanding of structures of possible experience*, the mathematical investigation of more complex abstract structures, often *invented* ones, helps throwing light on less complex structures, including those that we *meet* in experience.

The nature of (*modern*) *physics*. Physics is the science of physical reality *as we perceive it*. But the reality that we perceive is not only what meets the senses; perception is a joint contribution of sensorial and psycho-physical intentional systems; sensation provides the raw material, the hyletic data, which these systems *organize* (a key world!) into perception proper. Brute sensorial data are abstractions; what we actually perceive are already organized clusters of sensations. "Red patch here now" may be an atomic element of a particular analyses of perception, but what we actually perceive is a thick red stain on the carpet that looks like blood. Perception is organized; there are patterns of organization in what we perceive. But, and this is important, to organize perception is not a task for higher-level intellectual systems but, rather, lower-level perceptual ones. The way perception is organized is not a conscious choice either, but a consequence of us having the perceiving systems that we have. We cannot perceive differently from the way we do unless we alter the way perceptual systems function; by the action of drugs, for instance. Our perceptual systems were naturally selected in evolution in order to give us a representation of reality convenient enough to keep us alive until we reproduce.[6]

The idea that our perceptual systems are "mirrors of nature" and, consequently, that the

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[6] I said convenient, not necessarily accurate, if this notion is even applicable, which I'm not sure it is.

patters we perceive in reality exist independently of us and our perceptual apparatuses is the first move into fabricating the puzzle of the applicability of mathematics in physics (but, as we shall see, not the last). What we perceive is not always real, for we are prone to misperception, but misperception can only be corrected by further perception. Perceptual reality is in a sense a limit idea, a *maximal*, *coherent* and *stable* whole of what can in principle be perceived. Despite occasional events of misperception, perception is our privileged means of accessing perceptual reality. The philosopher Immanuel Kant believed that there are two realities, noumenal, unknowable reality-in-iself *and* phenomenal, perceptual reality. Modern physics, however, works with a different duality; there is reality *as we perceive it* and there is *physical* reality. For scientific purposes, physical reality is an abstract and idealized version of perceptual reality prone to mathematical treatment. It cannot *as a matter of principle* be *adequately* perceived; perception of physical reality is essentially imperfect and fragmentary, mathematics only has adequate access to it. In fact, physical reality (as scientifically conceived) is *expressly constituted* as an ideal realm that is either to some extent already mathematical or capable of being mathematized. To realize this is the key to understand how mathematics has anything to say about physical reality.

But, as I have already stressed, perceptual reality already displays patterns of organization that can elicit mathematical interest, and a substantial amount of mathematics was created in this way. They, however, are usually mathematically too poor to invite mathematical treatment on a grander scale. And here is where idealization comes in. Idealization is a way of "polishing" perceptual reality into something more perfect that cannot, as a matter of principle, not mere fact, be adequately perceived. We *perceive* a ball but *conceive* it as a perfect sphere, we perceive the trajectory in space of a projectile and conceive it as a segment of a parabola. Patterns detected in perceptual reality are systematically idealized as mathematical patterns proper. *Physical* reality *as conceived* in modern physics, from the XVI and XVII centuries on, is an *exactification* of perceptual reality that in a curious reversion of ontological priority is taken as the *only real* reality that there is, a being in itself existing independently of us and our perceptions.

Physical reality (again, as conceived in science) presents itself as a closed domain of being submitted to strict legality which mathematics only can adequately express and where all facts are already in themselves determined. The role of science being that of finding out what the facts are and express them in mathematically formulable laws and principles. This project is a product of modernity (by which I mean the period that follows the Renaissance) and was born at the same time that mathematics was undergoing radical changes that greatly improved its power.

Galileo, Descartes and Newton were its fathers. What makes mathematics relevance for physics a mystery is "forgetting" the true nature of physical reality and the reversal of ontological priority that enthrones physical reality as *real* reality and perceptual reality as only a *necessarily* distorted copy of it.[7]

It may be instructive to pause for a second to appreciate this hierarchy of increasingly *mathematical* realms of things that we have discerned. From the messy realm of raw sensations, to significantly better organized but still mathematically simple realm of perception, to idealized physical reality suitable for full-blown mathematical treatment. Mathematics gradually makes its way in by the action of both lower-level perceptual and higher-level cognitive processes. Among the latter, idealization stands out. To idealize is to exactify. Another is a sort of refurnishing of the world of perception. In perception things have colors and flavors, feel warm or cold, smooth or rough, but in physical reality all these things disappear, confined to the interior of subjectivity, having no place in the *objective* reality which physics is concerned with. These so-called secondary qualities can survive in physical reality only by being given *mathematical* substitutes. Colors, for example, are transmuted from something that we *see* to mathematically expressible patterns of interaction between reflecting bodies and incident light. By stripping the essence of physical bodies to their *mathematical* extension, Descartes played a pivotal role in this process of substituting things that we see by things that we do not.

Quantification is another important moment in the process of mathematization of perceptual reality, depending on idealization and the refurnishing that I have already mentioned. We perceive, for example, that the sensation of warmth of a body can have different degrees, and we can *roughly* compare two bodies as to their respective sensations of warmth, but this is far from *measuring* the *temperature* of a body. First, sensations are, or so physics claims, subjective, temperature is an objective property; second, measurement requires a *presupposition*, namely, that the temperature of a body can, in principle, vary *continuously* (and so, that the arithmetical continuous of real numbers is an appropriate tool to express temperature), even though our senses cannot distinguish minute variations of the sensation associated with minute changes of temperature. This presupposition, however, does not have the status of a physical hypothesis, since it cannot be put to test. Rather, it expresses a fundamental *fiat* in the constitution of our conception of physical reality (which may at any moment be reconsidered if perception or theoretical considerations so suggest).

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[7] Perception counts as an experience of *physical* reality only if conveniently idealized.

The contrast between perceptual reality and its mathematical exactification is brought out dramatically in an exchange between the 1952 Nobel Prize winner Felix Bloch and Werner Heisenberg, of whom he had been the first doctorate student. Bloch tells: "We were on a walk and somehow began to talk about space. I had just read Weyl's book *Space, Time and Matter*, and under its influence was proud to declare that space was simply the field of linear operations. "Nonsense," said Heisenberg, "space is blue and birds fly through it.""[8]

The applicability of mathematics. Mathematics has three major applications in the empirical sciences, physics particularly. Surrogatory (or representational), predictive and heuristic. The first has already been discussed; mathematics, one could say, provides physis with a context of *representation* of perceptual reality. But to say this would be misleading, for it might induce the wrong picture that mathematics *re*-presents what perceptual reality *presents*. The truth is that physics *substitutes* perceptual reality by a mathematical surrogate that contains, simultaneously, more *and* less than what perception (not to mention the senses) provide. In the words of Weyl, physics substitutes perceptual reality by a *symbolic reconstruction* that only at few points can be checked against conveniently idealized perception, standing or falling as a whole. For future reference, let's call W physical reality as conceived in science. W provides the standard semantics for the theories of mathematical physics.

The *predictive* role of mathematics is a consequence of the surrogatory one. By investigating W scientists may discern general relations, expressible in mathematical formulas, that offer themselves naturally as laws of nature in mathematical garb. These laws can be used to foretell events in perceptual reality by "decoding" W in terms of possible perception. "Predictions" can serve either to reinforce (but never logically prove) the validity of general laws, if effectively verified, or to falsify them, if not verified. Any relevant mathematical fact discerned in W indicates an empirical fact *in principle* perceivable. Unverified mathematical "previsions" lessens the reliability of W as a representative of perceptual reality.

The *heuristic* role of mathematics in science follows from the trivial fact that W, as *any* mathematical realm, can be mathematically enriched into new realms M by the introduction of new *mathematical* entities (analogously to the enrichment of the domain of natural numbers with the introduction of negative integers). This allows for the use of more powerful mathematical methods and, consequently, more in-depth investigation of the sub-domain W. Now mathematics can display its power in full, for by its means *we can investigate the mathematical surrogate of*

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[8] See Bloch 1976

perceptual reality (W) *by investigating ideal words* (M) *in which not all entities necessarily represent anything in perceptual reality*.

Suppose now that by investigating M with adequate mathematical methods we manage to discern properties of W that are verified when decoded into perceptual reality. This, of course, *suggests* that *some* of the theoretical entities of M-W *may* have correspondents in possible perception. If they are found, which is not guaranteed, for M may have *formal* relevance for W without having any ontological import on reality, mathematics has played a relevant heuristic role in science.[9] But note, mathematics simply *suggests,* without *guaranteeing*, that certain formal entities and facts *may* correspond to contents of possible perception. Mathematics is an oracle, but like that of Delphi, it does not say everything that we might want to know, nor is it always reliable.

Suppose now that we find a mathematical realm K that is *isomorphic* to W, which, moreover, can be more easily or thoroughly investigated than W. This is a very convenient situation indeed, for we may forget all about W and set out to investigate K instead. Any assertion that is true in K is also true in W if conveniently reinterpreted under the isomorphic correspondence. The isomorphism may even be only *partial*. In cases such as these, W is investigated, we can say, by *formal* analogy with K. K is, we can also say, a *formal model* of W. A very interesting instance of this procedure is Maxwell's discovery of displacement currents by reasoning analogically in mechanical term.[10] By inquiring how mechanical energy could be stored in dielectrics in a *mechanical model* of electromagnetic phenomena, Maxwell was led into postulating the existence of displacement currents, a new type of electric current besides conduction currents.[11] The electromagnetic and mechanical contexts are, respectively, our W and our K, both outright *idealized mathematical* domains.[12] By boldly assuming that displacement currents could also produce magnetic effects, Maxwell was able to foresee the existence of electromagnetic waves, later experimentally verified by Hertz.

Another way mathematics can display its heuristic virtues is by assuming the validity in W or its mathematical extensions of very general principles, suggested by experience or by purely mathematical convenience. Principles such as those of conservation (of electric charge, energy, etc - of course, when I say "electric charge", "energy", etc, I mean some mathematical

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[9] It may even happen that more than one interpretation offer themselves to the inquiring scientist; choosing those that enrich our conception of reality adequately is more a matter of physical intuition than mathematics.
[10] See Longair 2003, p. 77-102.
[11] Under the hypothesis of conservation of electric charge, displacement currents are *mathematically required* to exist.
[12] In this case, the isomorphism is only partial.

entity in W that *may* correspond to something experienceable) or that of minimum action, for example. These principles usually have neat mathematical formulations. Now, by drawing their consequences in the context of some theory, mathematics can disclose mathematical facts that *suggest* correspondents in the realm of possible experience. One example is the discovery of the neutrino.[13] In order for energy to be conserved in processes of β-disintegration, a particle, the neutrino, was conjectured to exist that was firstly envisaged as nothing but a package of mathematical properties *required* by principles of conservation, i.e. mathematical facts of a sort. Later, it was experimentally detected. Thus mathematics plays its apparent, but only apparent "unreasonable" role in physics - by simply drawing the consequences of mathematical hypotheses in mathematical contexts.

Conclusions. Mathematics studies abstract patterns, either instantiated in structured system of things given to perception or imagination, or freely invented and characterized axiomatically. Man created mathematics in order to understand such patterns. In its origins, mathematics was a child of perception, but man quickly extrapolated the given, inventing ever more elaborated mathematical patterns. Modern physics was born when man decided to study the reality he perceives by means of abstract patterns he extracts from perceptual reality and idealize in terms of mathematical patterns. As a consequence perception is substituted by mathematics as the privileged means of access to reality. The book of nature, in Galileo's famous words, is written in geometrical characters. This, however, is not the statement a fact, but the establishment of a program. By so doing, modern physics invites mathematics as a whole to turn its attention again to reality. By an elaborate interplay between symbolic mathematics and creative semantics of mathematical systems, or by purely formal analogies between the mathematical surrogate science substitutes for perceptual reality and no matter which mathematical domain, mathematics manages to play a fundamental role in scientific heuristics. The mystery of the applicability of mathematics in empirical science is a consequence of the view that physical and mathematical realities are two completely isolated and self-sufficient domains, which can only communicate by mysterious means. But this, as I strived to show here, is a false view.

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[13] See Paty 1988, p. 329-335.

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