Non-local non-singular gravity and its consequences for cosmology and blackhole physics

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In this essay we argue that a ghost-free non-local higher derivative extension of General Relativity may be able to render gravity asymptotically free in the deep ultra-violet, while recovering the Newtonian gravitational potential in the far infrared. Such a construction has fundamental consequences for the way we think about gravity at short distances and times – we will show how in such models cosmological and blackhole singularities could be resolved providing us with encouraging signs for a possible consistent ultraviolet completion of gravity.

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I. INTRODUCTION

The gravitational interaction in four dimensions is universal – it couples to all inertial masses with equal strength. The equivalence principle has been tested to an unprecedented level of one part in 10^{12} [1, 2]. The Einstein's theory of General Relativity (GR) has been extremely successful in explaining some of the most intriguing physical phenomena such as the perihelion shift of mercury, bending of light, gravitational lensing, and understanding the relativistic dynamics of binary pulsars, see [3]. GR is also the tool to understand the expansion history of our universe and aspects of the hot big bang cosmology such as the Big Bang Nucleosynthesis and the formation of the Cosmic Microwave Background Radiation [4], and the large scale structures seen in our universe [5].

However as it stands the theory of gravity is pathological, it admits space-time singularities: For instance, if r is the distance from a gravitational source (massive object), the gravitational potentials are well behaved for a distant observer exhibiting the classic Newtonian $1/r$ fall off, but the curvatures/potentials blow up near $r = 0$. Therefore, though no departures from GR predictions has been noticed in terrestrial laboratory which has probed gravity up to distances as short as 10μm [6], theoretical consistency strongly suggests that gravity must be modified at some fundamental short distance scale, commonly conceived to be the string scale or the Planck scale.

Many GR singularities that have been found, such as the black hole singularities, however are believed to be covered by a horizon, which had lead Penrose to label that nature prefers covered singularities [7], perhaps the singularities are not that problematic after all? Unfortunately, in GR there also exists cosmological singularities, such as the Big Crunch/Bang singularity which is certainly devoid of any horizon. For any equation of state obeying the strong energy condition $p > -\rho/3$, regardless of the geometry (flat, open, closed) of the universe, the scale factor of the universe in a Friedmann Robertson Walker (FRW) metric vanishes at a finite time, say at t = 0, and the matter density diverges. In fact, all the curvature invariants, such as R, $R\Box R$, become singular – the basic fabric of space-time ceases at the singularity [8], further corroborating the need to modify GR in the ultra-violet (UV).

Thus one of the essential challenges in gravitational physics is to find a way to consistently modify GR which would resolve all it's classical singularities, but will recover GR in the low energy limit. The other outstanding challenge for gravitational physics is that we do not yet have a completely satisfying quantum version. The problem is that the Einstein Hilbert action is non-renormalizable; in 4 dimensions the higher loops keep generating new divergences implying that the theory requires an infinite number of counter terms thereby loosing predictability.

GR is based on fundamental principles of physics, such as general covariance, the equivalence principle, locality, etc. Gravity is a gauge theory, the gauge symmetry being the diffeomorphisms. In particular, this precludes having a mass term for gravity, or for that matter any non-derivative polynomial interactions that we are so familiar with in other quantum field theories. This means that if one wants to preserve general covariance (for recent attempts to modify gravity which abandons general covariance, see [9]), one has little choice but to modify the kinetic operators in the action. Or in other words, look at higher (more than two) derivative theories of gravity [1]. Intriguingly, higher derivative theories have a rather appealing feature, their propagator is more convergent in the UV as compared to the usual $1/p^2$ fall-off. Indeed, it is well known that the inclusion of fourth order gravitational terms in the action makes

[1] It is always possible to modify gravity by including extra degrees of freedom such torsion or extra scalars, but we are primarily interested in modifying the physics of the graviton.

the theory renormalizable [10], albeit it does so at the cost of introducing the "Weyl" ghost.

To see why the existence of ghost-like states which violates unitarity and/or makes the vacuum unstable [11], is a rather generic feature of covariant "finite-order" higher derivative theories, let us consider a simple scalar field theory model of the form

$$S = \int d^4x \; [\phi\Gamma(\Box)\phi - V_{\texttt{int}}(\phi)] \tag{1}$$

where Γ is any finite polynomial. One can always write Γ as:

$$\Gamma(-p^2) \sim (p^2+m_1^2)(p^2+m_2^2)\dots(p^2+m_n^2)\,. \tag{2}$$

If there are at least two discrete single poles (say $m_1 \neq m_2$), then at least one of them is ghost like, *i.e.* , one of the residues has to be negative:

$$\frac{1}{(p^2+m_1^2)(p^2+m_2^2)} \sim \frac{1}{p^2+m_1^2} - \frac{1}{p^2+m_2^2} \tag{3}$$

A double pole can be represented as the convergence of two simple poles with opposite residues, and suffers from similar problems [12]. Similar arguments follow for higher order poles, it seems that higher derivative theories are simply inconsistent theories.

Now, causality of space like separated quantum operators should commute in a quantum field theory. However, in gravity the question of whether operators are space-like separated becomes a dynamical issue and the causal structure can fluctuate due to quantum effects. Perhaps this gives a first hint that gravitational interactions need not be local at high energies. The main reason why one is wary of nonlocal theories is because traditionally physicists believe in a "Markovian philosophy": that one should be able to predict the future if one knew the present state of the universe. With a nonlocal theory, the past history also becomes important, the present is not enough. However, the advent of quantum mechanics questions this very deterministic way of thinking about the universe. According to Feynman's formulation, every trajectory (evolution) is possible and has a certain probability of occurring which is all that we can hope to calculate. When considering the universe it is natural to think of these evolutions "beginning" in the infinite past and "ending" in the infinite future, and the action simply let's us assign a probability to each of these possibilities. When you start thinking about quantum mechanics this way, the entire evolution history of our universe is important anyway, and nonlocality poses no particular obstacle to blend in at all. Given a particular infinite time evolution, the nonlocal action can assign a probability uniquely and unambiguously, as will become progressively clear.

Another intriguing suggestion in favor of nonlocality comes from string theory according to which higher derivative corrections appear already classically (*i.e.* at the tree level), and even non-perturbatively (e.g., as expected in the $1/N$ expansion in some Yang-Mills theory [13]). Moreover, these corrections often lead to actions of the form (1) where $\Gamma(\Box)$ is an exponential such as in p-adic stringy toy models [14], or strings on random lattice [15, 16]. From string field theory [17, 18] (either light-cone or covariant) the form of the higher-derivative modification can again be seen to be Gaussian: There are $e^{\Box}$ factors appearing in all vertices (e.g., $(e^{\Box}\phi)^3$), which can be transplanted to kinetic terms by field redefinitions ($\phi \to e^{-\Box}\phi$). And we suddenly realize a remarkable fact, if $\Gamma(\Box)$ contains an infinite series of box operators, it can bypass problems such as (3) because it can still have only a single zero or none at all! For instance, a gaussian being an entire function has none. The theory becomes unitary and at most has a single perturbative degree of freedom. It is worth pointing out that such string theory inspired nonlocal scalar field theory models have gained popularity in recent years, in their applications to String theory [18–22], cosmology [23] and particle physics [24]. So, the upshot is that if we are willing to give up locality and consider an infinite series of higher derivative terms in the action, as opposed to finite-order higher derivative actions, then we may be able to consistently and covariantly modify gravity [2].

In this essay we will discuss how to construct such nonlocal actions of gravity which nevertheless recovers Newtonian gravity in the low energy limit. We will go on to develop criterion so that gravity can be made *asymptotically free* in the ultraviolet. In this regard gravity can be thought of as an emergent phenomena at low energies and large distances. Such an *asymptotically free* gravity has many profound consequences ranging from blackhole physics to cosmology and the aim of this essay is to provide a glimpse of these results, summarizing to a large extent the work done in [25–28] along with providing some new directions we are currently investigating [29, 30].

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[2] Gravity being a gauge theory, the issue of ghosts is a bit subtle as some ghosts are beneficial Fadeev-Popov type that appears in non-abelian gauge theories. This admits the presence of some higher derivative terms such as involving the Gauss-Bonnet invariant and functions of the Ricci scalars. However these modifications are not known to be able to resolve the singularity problems.

II. GHOST FREE NONLOCAL GRAVITY ON MINKOWSKI BACKGROUND

In order to understand both the asymptotic behavior of the Newtonian potentials and the issue of ghosts, we require only the graviton propagator. Thus it is sufficient to perturb the metric fluctuations around the Minkowski background

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \,, \tag{4}$$

and consider terms in the action that are up to $\mathcal{O}(h_{\mu\nu}^2)$. Since $R_{\mu\nu\lambda\sigma}$ vanishes around Minkowski background, only terms that are products of at most two curvature terms are relevant:

$$S = \int d^4x\sqrt{-g}\left[\frac{R}{2} + R_{\mu_1\nu_1\lambda_1\sigma_1}\mathcal{O}^{\mu_1\nu_1\lambda_1\sigma_1}_{\mu_2\nu_2\lambda_2\sigma_2}R^{\mu_2\nu_2\lambda_2\sigma_2}\right], \tag{5}$$

where $\mathcal{O}$ is a differential operator containing covariant derivatives and $g_{\mu\nu}$, and we have set $M_p = 1$. Using the symmetry properties of the Reimann tensor and the Bianchi identities, it turns out that the most general action can be captured by 3 arbitrary functions, $\mathcal{F}_i(\Box)$'s [27],

$$S = \int d^4x\,\sqrt{-g}\left[\frac{R}{2} + R\mathcal{F}_1(\Box/M^2)R + R_{\mu\nu}\mathcal{F}_2(\Box/M^2)R^{\mu\nu} + C_{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box/M^2)C^{\mu\nu\lambda\sigma}\right]\,. \tag{6}$$

where $C_{\mu\nu\lambda\sigma}$ is the Weyl tensor. Note that the higher derivatives are suppressed by some mass scale M which could potentially lie anywhere between approximately $100mev \sim (10\mu m)^{-1}$, and the Planck scale $\sim 10^{19}GeV$.

Substituting the background (4), we obtain the following action

$$S_q = -\int d^4x\left[\tfrac{1}{2}h_{\mu\nu}a(\Box)\Box h^{\mu\nu} + h_\mu^\sigma b(\Box)\partial_\sigma\partial_\nu h^{\mu\nu} + hc(\Box)\partial_\mu\partial_\nu h^{\mu\nu} + \tfrac{1}{2}hd(\Box)\Box h + h^{\lambda\sigma}\tfrac{f(\Box)}{\Box}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\mu\nu}\right]. \tag{7}$$

where

$$a(\Box) = 1 - \frac{1}{2}\mathcal{F}_2(\Box/M^2)\Box - 2\mathcal{F}_3(\Box/M^2)\Box \tag{8}$$

$$b(\Box) = -1 + \frac{1}{2}\mathcal{F}_2(\Box/M^2)\Box + 2\mathcal{F}_3(\Box/M^2)\Box \tag{9}$$

$$c(\Box) = 1 + 2\mathcal{F}_1(\Box/M^2)\Box + \frac{1}{2}\mathcal{F}_2(\Box/M^2)\Box \tag{10}$$

$$d(\Box) = -1 - 2\mathcal{F}_1(\Box/M^2)\Box - \frac{1}{2}\mathcal{F}_2(\Box/M^2)\Box \tag{11}$$

$$f(\Box) = -2\mathcal{F}_1(\Box/M^2)\Box - \mathcal{F}_2(\Box/M^2)\Box - 2\mathcal{F}_3(\Box/M^2)\Box. \tag{12}$$

From the explicit expressions we observe the following relationships:

$$a + b = 0; \quad c + d = 0; \quad b + c + f = 0\,, \tag{13}$$

so that we are left with only two independent arbitrary functions. The field equations can be written in the form

$$a(\Box)\Box h_{\mu\nu} \;+\; b(\Box)\partial_\sigma(\partial_\nu h_\mu^\sigma + \partial_\mu h_\nu^\sigma) + c(\Box)(\eta_{\mu\nu}\partial_\rho\partial_\sigma h^{\rho\sigma} + \partial_\mu\partial_\nu h) + \eta_{\mu\nu}d(\Box)\Box h + f(\Box)\Box^{-1}\partial_\sigma\partial_\lambda\partial_\mu\partial_\nu h^{\lambda\sigma} = \kappa\tau_{\mu\nu}$$

or equivalently, $\quad \Pi_{\mu\nu}^{-1\lambda\sigma}h_{\lambda\sigma} = \kappa\tau_{\mu\nu}$ (14)

where $\Pi_{\mu\nu}^{-1\lambda\sigma}$ is the inverse propagator.

While the matter sector obeys stress energy conservation, the geometric part is also conserved as a consequence of the generalized Bianchi identities:

$$-\kappa\tau\nabla_\mu\tau_\nu^\mu = 0 \;=\; (a+b)\Box h^\mu_{\nu,\mu} + (c+d)\Box\partial_\nu h + (b+c+f)h^{\alpha\beta}_{,\alpha\beta\nu}\,. \tag{15}$$

It is now clear why eqs.(13) had to be satisfied. What is also remarkable is that these same conditions ensure that the different spin degrees of the metric (a spin 2, a vector and two scalars) decouple as well as eliminates the vector and one of the scalars which are typically ghost like [31]:

$$\Pi = \frac{P^2}{ak^2} + \frac{P^0}{(a-3c)k^2} \tag{16}$$

where P^2 and P^0 are the spin-2 and the remaining scalar projection operator respectively, see [27] for details. Further, since we want to recover GR in the infrared, we must have

$$a(0) = c(0) = -b(0) = -d(0) = 1 \ , \tag{17}$$

corresponding to the *massless* graviton propagator with the Einstein-Hilbert action. This also means that as $k^2 \to 0$ we have only the *massless* graviton propagator:

$$\lim_{k^2\to 0} \Pi^{\mu\nu}{}_{\lambda\sigma} \ = \ \frac{P^2}{k^2} - \frac{P^0}{2k^2} \ . \tag{18}$$

Thus, we finally conclude that provided (17) is satisfied, the $k^2 = 0$ pole just describes the physical graviton state, the negative ghost-like residue of the scalar propagator has precisely the coefficient to cancel the unphysical longitudinal degrees of freedom in the spin-2 part [31]. Secondly, the condition that the theory be ghost free boils down to simply requiring that $a(\Box)$ is an entire function, and $a(\Box) - 3c(\Box)$ has at most a single zero, the corresponding residue at the pole would necessarily have the correct sign, this is in fact what happens in the simple $F(R)$ gravity models.

If further one does not want to introduce any extra degrees of freedom, one is left with only a single arbitrary entire function, $a(\Box)$:

$$a(\Box) = c(\Box) \Rightarrow 2\mathcal{F}_1(\Box) + \mathcal{F}_2(\Box) + 2\mathcal{F}_3(\Box) = 0 \tag{19}$$

While several different $\mathcal{F}$'s can satisfy the above relation, a particularly simple class which mimics the stringy gaussian nonlocalities is given by

$$a(\Box) = e^{-\frac{\Box}{M^2}} \text{ and } \mathcal{F}_3 = 0 \Rightarrow \mathcal{F}_1(\Box) = \frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} = -\frac{\mathcal{F}_2(\Box)}{2} \tag{20}$$

leading to a *ghost free* action of the form:

$$S = \int d^4x \ \sqrt{-g} \left[\frac{R}{2} + R \left[\frac{e^{\frac{-\Box}{M^2}} - 1}{\Box} \right] R - 2R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box} \right] R^{\mu\nu} \right] \tag{21}$$

By construction the above action contains only the graviton as physical degrees of freedom as in GR, but contains an exponentially damped propagator in the UV which, as we shall now argue, can have profound consequences for the gravitational singularities.

III. BEYOND QUADRATIC CURVATURE TERMS

Is there a way to extend the above algorithm to include terms which are higher than quadratic in curvatures? We know that the classical background space-time responds to the matter content of the universe, and one would imagine that a truly consistent theory of gravity should be free from ghosts and other instabilities around any such realizable background. This in fact would be a way to impose further restrictions on the allowed terms going beyond the quadratic curvatures. While analyzing the issue of ghosts and instabilities around arbitrary classical backgrounds is well beyond the present scope, (anti)de Sitter space-times serves as a relatively tractable playground. For instance, the facts that the Weyl tensor vanishes on (A)dS space-times, that the Ricci tensor is proportional to the metric, and finally that the metric is always annihilated by covariant derivatives, allow one to limit oneself to only actions of the form [27]

$$S = \int d^4x\sqrt{-g} \left[\frac{R}{2} + \alpha_0(R, R_{\mu\nu}) + \alpha_1(R, R_{\mu\nu}) R\mathcal{F}_1(\Box)R + \alpha_2(R, R_{\mu\nu}) R_{\mu\nu}\mathcal{F}_2(\Box)R^{\mu\nu} + \alpha_3(R, R_{\mu\nu}) C_{\mu\nu\lambda\sigma}\mathcal{F}_3(\Box)C^{\mu\nu\lambda\sigma} \right] \tag{22}$$

while studying fluctuations.

To get an idea about how the higher curvatures may enter the arena, let us consider a simple subclass of the above action which is a generalization of the stringy nonlocal gravity action (21):

$$S = \int d^4x \ \sqrt{-g} \left[\frac{R}{2} + \alpha_1(R) R \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box/M^2} \right] R - 2\alpha_2(R) R_{\mu\nu} \left[\frac{e^{-\frac{\Box}{M^2}} - 1}{\Box/M^2} \right] R^{\mu\nu} - \Lambda \right] \tag{23}$$

with

$$\alpha_1(0) = \alpha_2(0) = 1 \ , \tag{24}$$

so that the action is equivalent to (21) as far as the fluctuations around the Minkowski space-time ($\Lambda = 0$) is concerned.

Now, in order to have a consistent (A)dS vacuum we need to make sure that the linear variation of the action around the (A)dS metric, $\bar{g}_{\mu\nu}$:

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \ , \tag{25}$$

vanishes. Since

$$\bar{R}_{\mu\nu} = \lambda \bar{g}_{\mu\nu} \ ; \ \bar{R} = 4\lambda \text{ and } \bar{\nabla}_\mu \bar{g}_{\nu\rho} = 0 \tag{26}$$

all terms in the action that contain the $\Box$ operators do not contribute to the linear variation: one of the curvature terms, either to the left or to the right of $\Box$'s must take on the background values which then gets annihilated by the action of the Box operators, directly acting to the right or via integration by parts to the left. We thus have

$$\delta S = \delta \left\{ \int d^4x \ \sqrt{-g} \left[\frac{M_p^2}{2} R - \alpha_1(R) R^2 - 2\alpha_2(R) R_{\mu\nu} R^{\mu\nu} - \Lambda \right] \right\} \tag{27}$$

Setting this variation to zero yields rather straight-forwardly

$$\lambda \left[1 - \frac{32\lambda^2 \alpha_1'(\lambda)}{M_p^2} - \frac{16\lambda^2 \alpha_2'(\lambda)}{M_p^2} \right] = \frac{\Lambda}{M_p^2} \tag{28}$$

determining λ in terms of the arbitrary functions α_1, α_2.

To study the issue of ghosts we need to compute the action up to $\mathcal{O}(h^2)$, and it is clear that this will again depend only on the values of the functions α_1, α_2, and their derivatives at λ given by (28). While the criteria for the absence of ghosts and general stability of the system is currently being investigated [29], what this suggests is that we will end up constraining the functions $\alpha_1(R), \alpha_2(R)$ which encodes the deviations from the quadratic curvature Lagrangian. How far the requirement of the absence of ghosts on general space-time backgrounds can constrain the action is a matter of conjecture, but at least this seems a powerful way to select a subset of nonlocal actions which are theoretically allowed. This is similar in spirit to the way the criterion of renormalizability can be used to narrow down the range of interactions allowed in local quantum field theories.

Let us end with a slight digression to a very different application of higher derivative gravity theories, that of explaining the current acceleration of our universe. There have been a vigorous investigation on whether one can modify gravity in a way that would cause our universe to accelerate, but will not contradict with the results of the various tests of GR. In particular extra terms containing various combinations of the Reimann tensor have been looked into, but they almost invariably contain the Weyl ghost [37]. In our derivation of the deSitter vacuum solution above (28) what we saw is that it is only these $\Box$-independent terms that enter into the calculations, but on the other hand we now know how the addition of an infinite series of $\Box$ terms can alleviate the problem of ghosts. Thus nonlocalizing the modified gravity actions that have been studied in the context of the dark energy problem may provide a way to salvage theoretical consistency of some of these models.

IV. BLACK HOLES

Let us now study the short distance behavior of the metric for a point source:

$$\tau_{\mu\nu} = \rho \delta_\mu^0 \delta_\nu^0 = m \delta^3(\vec{r}) \delta_\mu^0 \delta_\nu^0 \ , \tag{29}$$

for the action (21). We can compute the two potentials, $\Phi(r)$, $\Psi(r)$, corresponding to the metric

$$ds^2 = -(1 + 2\Phi)dt^2 + (1 - 2\Psi)dx^2 \ . \tag{30}$$

Due to the Bianchi identities [32, 33], we only need to solve the trace and the 00 component of (14). Since the Newtonian potentials are static, the trace and 00 equation simplifies considerably to yield

$$\begin{aligned} (a(\Box) - 3c(\Box))\Box h + (4c(\Box) - 2a(\Box) + f(\Box))\partial_\mu \partial_\nu h^{\mu\nu} &= \kappa\rho \\ a(\Box)\Box h_{00} + c(\Box)\Box h - c(\Box)\partial_\mu \partial_\nu h^{\mu\nu} &= -\kappa\rho \, , \end{aligned} \tag{31}$$

For $f = 0$ and $a(\Box) = c(\Box)$, the Newtonian potentials are solved easily:

$$4a(\nabla^2)\nabla^2\Phi = 4a(\nabla^2)\nabla^2\Psi = \kappa\rho = \kappa m\delta^3(\vec{r}). \tag{32}$$

Taking the Fourier components of (32), in a straight forward manner one obtains

$$\Phi(r) \sim \kappa m \int d^3p \, \frac{e^{i\vec{p}\vec{r}}}{p^2 a(-p^2)} = \frac{\kappa m}{r 2\pi^2} \int \frac{dp}{p} \frac{\sin p\, r}{a(-p^2)}. \tag{33}$$

We note that the $1/r$ divergent piece comes from the usual GR action, but now it is ameliorated. For (21) we have

$$\Phi(r) \sim \kappa m \int \frac{dp}{p} e^{-p^2/M^2} \sin{(p\, r)} = \kappa \frac{m\pi}{4\pi^2\, r} \mathtt{erf}\left(\frac{rM}{2}\right) = \frac{Gm}{r} \mathrm{erf}\left(\frac{rM}{2}\right) = \frac{m}{4\pi M_p^2 r} \mathrm{erf}\left(\frac{rM}{2}\right) \tag{34}$$

and the same for $\Psi(r)$. For $r \to \infty$, $\mathtt{erf}(r) \to 1$, and we recover the GR limit. On the other hand, as $r \to 0$, $\mathtt{erf}(r) \to r$, making the Newtonian potential converge to a constant $\Phi \sim mM/M_p^2$. Thus, although the matter source has a delta function singularity, the Newtonian potentials remain finite!

A rather interesting and fundamental consequence of the above result is that there are no singularities or horizons for mini-blackholes: Provided $mM \ll M_p^2$, our linear approximation can be trusted all the way to $r \to 0$, the geometry remains smooth and Φ, Ψ remains small so that no horizon can form. In other words, they are not blackholes at all! For $M \sim M_p$, we can trust our solution for tiny masses, $m \leq M_p \sim 10^{-5}$g, and for $M \sim 1$ TeV, the corresponding mass of order $m \sim 10^{11}$g. This tells us that there would be no blackholes formed at the LHC from the proton-proton collision at the center of mass of 14 TeV. Typical parton mass is obviously much smaller than the Planck scale and therefore even if $M \sim M_p$, one cannot form blackholes.

Unfortunately, we cannot say anything concrete about the astrophysical black holes as for such large masses our linear approximation breaks down, and one would need to solve the full nonlinear gravitational equations to get any insights into the physics of the horizon. The mini blackhole analysis does however gives one hope that even the large black hole solutions would be singularity-free, indeed large blackholes must evolve into mini-black holes via Hawking radiation, and Hawking radiation has never been associated with production (in the time-reversed sense) of singularities.

V. BIG CRUNCH/BANG SINGULARITIES

We have seen how by looking at small fluctuations around a given vacuum one can gain considerable insights into some of most fundamental properties of the theory, such as the (non)existence of ghost like states and the fate of black hole singularities. Can we however go beyond the linearized approximations? Remarkably, it turns out that one can, especially while studying cosmological FRW backgrounds. Cosmology of a subset of actions (6) containing only the Ricci scalar:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} + \frac{1}{2}R\mathcal{F}(\Box/M^2)R - \Lambda + \mathcal{L}_{\mathrm{M}}\right), \text{ with } \mathcal{F}(\Box/M^2) = \sum_{n\geqslant 0} f_n \left(\frac{\Box}{M^2}\right)^n \tag{35}$$

has been studied in [25, 26, 28].

There exist a particularly simplifying ansatz viz:

$$\Box R = r_1 R + r_2\ , \tag{36}$$

where r_1, r_2 are constants, that in fact allows one to obtain exact cosmological solutions [26]. To see how this works let us write down the trace of the modified Einstein equations for the nonlocal action (35):

$$\widetilde{G} \equiv g^{\mu\nu}\widetilde{G}_{\mu\nu} = G + \sum_{n=0}^{\infty} \frac{f_n}{M^{2(n+1)}} \widetilde{G}^n = g^{\mu\nu} T_{\mu\nu}\ , \tag{37}$$

where

$$\widetilde{G}^n = 6\Box^{n+1}R + \sum_{m=1}^{n} \left[(\Box^{n-m}R)_{;\mu}(\Box^{m-1}R)^{;\mu} + 2(\Box^{n-m}R)(\Box^m R)\right] \tag{38}$$

In the presence of radiation and a suitable cosmological constant, courtesy the ansatz, the trace equation simply reduces to two algebraic equations determining the parameters r_1, r_2:

$$\mathcal{F}'(r_1) = 0 \qquad \& \qquad r_2 = -\frac{r_1[M_P^2 - 6\mathcal{F}_1 r_1]}{2[\mathcal{F}_1 - f_0]}, \; ; \; \mathcal{F}_1 \equiv \mathcal{F}(r_1) \; , \tag{39}$$

and a constraint specifying the required value of Λ:

$$\Lambda = -\frac{r_2 M_P^2}{4r_1}. \tag{40}$$

So, what kind of cosmological backgrounds do we get? It turns out that if $r_2 < 0$ and $r_1 > 0$, one obtains nonsingular bouncing solutions which are stable attractors [26], and asymptotes to a deSitter space-time in the future and the past. Moreover, it was recently shown that these solutions are also stable under arbitrary spatial fluctuations [28]. The evolution of the spatial fluctuations, in fact, preserve the inflationary mechanism of generating near scale-invariant perturbations and therefore can serve as a way to geodesically complete models of inflation. A particulary simple analytical solution of this type is the hyperbolic cosine bounce:

$$a(t) = \cosh\left(\sqrt{\frac{r_1}{2}}t\right) \tag{41}$$

which was first discovered in [25].

We are still quite far from providing a completely satisfactory resolution of the cosmological singularities: (i) We found that these theories also admit singular attractor solutions along with nonsingular ones [26], (ii) so far we have only been able to analyze the cosmology in the presence of radiation and a cosmological constant, or when the stress energy tensor completely vanishes [27], how can we generalize the results? (iii) we haven't yet been able to incorporate anisotropic dynamics which is known to lead to dangerous chaotic Mixmaster type behavior Nevertheless, the nonlocal models certainly seems an encouraging way to try and resolve all the classical singularities in GR.

VI. QUANTUM DIVERGENCES

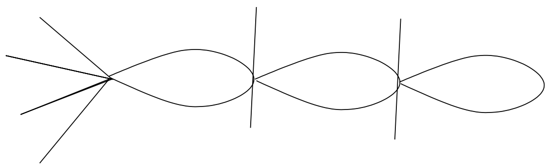

FIG. 1: An 8-point graph for ϕ^6 vertex.

What is still not clear to us is the status of these nonlocal theories. One can treat these theories as effective theories, (6) as an effective action obtained by performing quantum calculations of some hitherto unknown fundamental theory. While the issue of ghosts in this context is still pertinent, apart from the quantum unitarity problem the presence of ghosts also make the theories classically unstable, there is certainly no reason to compute quantum loops in this case. On the other hand, one can take a more ambitious approach and view these theories as full-fledged quantum theories where one is able to consistently compute quantum loops. Indeed, according to string theory these nonlocal actions account for the α' corrections as a series expansions, but one is still required to perform quantum loop calculations to obtain a perturbative expansion in the string coupling, g_s. In fact, these theories posses a unique advantage as compared to their local counterparts, the quantum loop integrals typically yield finite results due to the exponential damping factors.

Consider for instance, an 8-point scattering amplitude in a ϕ^6 local quantum field theory, see fig. above. This is quadratically divergent and it is easy to construct similar quadratically divergent higher point graphs as well. Indeed this is the reason why such a theory is said to be non-renormalizable, it keeps generating higher point divergent graphs. Once we "nonlocalize" it,

$$S = \int d^4x \, \left[\phi\Box e^{\frac{-\Box}{M^2}}\phi - \frac{\phi^6}{M_6^2}\right] \; , \tag{42}$$

all diagrams become finite. The quadratically divergent loop now yields

$$\frac{1}{M_6^2}\left[\int d^4p\frac{e^{\frac{-p^2}{M^2}}}{p^2}\right]\sim\frac{M^2}{M_6^2} \tag{43}$$

The result is finite and the appearance of the parameter M/M_6 is a rather generic feature of these diagramatic calculations, please note that M_6 is a new scale governing the strength of the ϕ^6 interactions. If $M \ll M_6$, as one goes to higher and higher loops, one obtains a perturbative expansion in M/M_6 providing therefore a consistent way of incorporating quantum corrections in these theories. These kinds of calculations have been useful in understanding several quantum features of string theory, such as Regge behavior [16] and thermal duality [19]. In many ways the nonlocal parameter M simply acts as a Lorentz invariant "physical" regulator.

Of course, the million dollar question is whether these algorithms can be transferred to gravitational theories? The immediate problem one encounters is that gravity is a gauge theory and therefore the free kinetic action is tangled with the interactions. Effectively, the same exponential suppression in the propagators that makes the graphs convergent now appears as an exponential enhancement in the interactions! Thus it is not at all clear whether such nonlocalization can really help us make progress towards a calculable theory of quantum gravity, but it was possible to make progress in gauge theories [34] and we believe that these theories certainly merits a thorough investigation [30].

[1] P. Roll, R. Krotkov, and R. Dicke, Ann. Phys. (N. Y.) 26 (1964) 442.
[2] V. Braginsky and V. Panov, Zh. Eksp. Teor. Fiz. 61 (1971) 873.
[3] S. G. Turyshev, Ann. Rev. Nucl. Part. Sci. **58**, 207 (2008) [arXiv:0806.1731 [gr-qc]].
[4] K. A. Olive and J. A. Peacock,
[5] P. J. E. Peebles, Princeton, USA: Univ. Pr. (1993) 718 p
[6] D. J. Kapner, T. S. Cook, E. G. Adelberger, J. H. Gundlach, B. R. Heckel, C. D. Hoyle and H. E. Swanson, Phys. Rev. Lett. **98**, 021101 (2007) [hep-ph/0611184].
[7] R. Penrose, Phys. Rev. Lett. 14, 5759 (1965)
[8] S. W. Hawking, R. Penrose, Proc. R. Soc. Lond. A 27 January 1970 vol. 314 no. 1519 529-548
[9] P. Horava, Phys. Rev. D **79**, 084008 (2009) [arXiv:0901.3775 [hep-th]].
[10] K. S. Stelle, Phys. Rev. **D16**, 953-969 (1977).
[11] J. M. Cline, S. Jeon and G. D. Moore, Phys. Rev. D **70**, 043543 (2004) [hep-ph/0311312].
[12] A. V. Smilga, Phys. Lett. B **632**, 433 (2006) [hep-th/0503213].
[13] G. 't Hooft, Nucl. Phys. B **72**, 461 (1974).
[14] P. G. O. Freund and M. Olson, "Nonarchimedean Strings," Phys. Lett. B **199**, 186 (1987).
P. G. O. Freund and E. Witten, "Adelic String Amplitudes," Phys. Lett. B **199**, 191 (1987).
L. Brekke, P. G. O. Freund, M. Olson and E. Witten, "Nonarchimedean String Dynamics," Nucl. Phys. B **302**, 365 (1988).
[15] M.R. Douglas and S.H. Shenker, Nucl. Phys. **B 335** (1990) 635;
D.J. Gross and A.A. Migdal, Phys. Rev. Lett. **64** (1990) 127;
E. Brézin and V.A. Kazakov, Phys. Lett. **236 B** (1990) 144.
[16] T. Biswas, M. Grisaru and W. Siegel, "Linear Regge Trajectories from Worldsheet Lattice Parton Field Theory," Nucl. Phys. B **708**, 317 (2005) [arXiv:hep-th/0409089].
[17] E. Witten, Nucl. Phys. **B268**, 253 (1986).
[18] V. A. Kostelecky and S. Samuel, Phys. Lett. **B207**, 169 (1988).
V. A. Kostelecky and S. Samuel, Nucl. Phys. **B336**, 263 (1990).
[19] N. Moeller and B. Zwiebach, JHEP **0210**, 034 (2002).
[20] J. A. Minahan, JHEP **0103**, 028 (2001).
[21] Robert Bluhm, Phys. Rev. D **43**, 4042 (1991).
[22] T. Biswas, J. A. R. Cembranos and J. I. Kapusta JHEP **10**, 048 (2010).
T. Biswas, J. A. R. Cembranos and J. I. Kapusta Phys. Rev. Lett. **104**, 021601 (2010).
[23] N. Barnaby, T. Biswas and J. M. Cline, JHEP **0704**, 056 (2007).
J. E. Lidsey, Phys. Rev. D **76**, 043511(2007).
N. J. Nunes and D. J. Mulryne, AIP Conf. Proc. **1115**, 329 (2009).
N. Barnaby and J. M. Cline, JCA **0707**, 017 (2007).
N. Barnaby and J. M. Cline, JCA **0806**, 030 (2007).
[24] P. F. Mende and H. Ooguri, Nucl. Phys. B **339**, 641 (1990). K. -y. Oda and N. Okada, Phys. Rev. D **66**, 095005 (2002) [hep-ph/0111298]. J. W. Moffat, Eur. Phys. J. Plus **126**, 53 (2011).
[25] T. Biswas, A. Mazumdar, W. Siegel, JCAP **0603**, 009 (2006). [hep-th/0508194];
[26] T. Biswas, T. Koivisto, A. Mazumdar, JCAP **1011**, 008 (2010). [arXiv:1005.0590 [hep-th]].
[27] T. Biswas, E. Gerwick, T. Koivisto and A. Mazumdar, Phys. Rev. Lett. **108**, 031101 (2012)
[28] T. Biswas, A. S. Koshelev, A. Mazumdar and S. Y. .Vernov, JCAP **1208**, 024 (2012) [arXiv:1206.6374 [astro-ph.CO]].
[29] T. Biswas, E. Gerwick, T. Koivisto, A. Kosheylev and A. Mazumdar, work in progress.
[30] T. Biswas, A. Mazumdar and L. Modesto, work in progress.
[31] P. Van Nieuwenhuizen, Nucl. Phys. **B60**, 478-492 (1973).
[32] I. Quandt, H. -J. Schmidt, Astron. Nachr. **312**, 97 (1991). [gr-qc/0109005].
[33] S. Nesseris and A. Mazumdar, Phys. Rev. D **79**, 104006 (2009) [arXiv:0902.1185 [astro-ph.CO]].
[34] D. Evens, J. W. Moffat, G. Kleppe and R. P. Woodard, Phys. Rev. D **43**, 499 (1991).
[35] G. 't Hooft and M. J. G. Veltman, Annales Poincare Phys. Theor. A **20**, 69 (1974).
[36] T. Clifton, P. G. Ferreira, A. Padilla, C. Skordis, [arXiv:1106.2476 [astro-ph.CO]].
[37] T. Chiba, JCAP **0503**, 008 (2005). [gr-qc/0502070]. A. Nunez, S. Solganik, Phys. Lett. **B608**, 189-193 (2005). [hep-th/0411102].